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THE RECURRENT AND METRIC CONNECTION AND F-STRUCTURES IN GAUGE SPACES OF THE SECOND ORDER

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Abstract. *Lately a big attention has been paid to the second order gauge connection, but the investigations are mostly restricted to the d-connection. Here this connection is generalized and a recurrent gauge connection is given on the manifold* E^{n+m+l}.
Let us denote by $T_H(E), T_{V_1}(E), T_{V_2}(E)$ *the subspaces of* $T(E)$ *spanned by adapted bases, then* $T(E) = T_H(E) \oplus T_{V_1}(E) \oplus T_{V_2}(E) = T_H(E) \oplus T_V(E)$.
If an almost complex structure J on the tangent space $T(E)$ *of the gauge* E^{2n} *manifold and the* $f_v\,(2k+1,1)$ *-structure on* $T_V(E)$ *are given, then the* $f_h\,(2k+1,1)$ *-structure on the horizontal subspace is defined in the natural way. We can define the* $F(2k+1, 1)$*-structure on* $T(E)$ *using* $f_v\,(2k+1,1)$ *and* $f_h\,(2k+1,1)$*. The condition for the reduction of the structural group of such manifolds is given.*

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1. ADAPTED BASIS IN $T(E)$

Let E be an $n+m+l$ dimensional C^∞ manifold. Some point has coordinates (x^i, y^a, z^p) and the allowable coordinate transformations are given by the equations

$$x^{i'} = x^{i'}(x) \quad i,j,k,h, = 1,...,n \qquad y^{a'} = y^{a'}(x,y) \quad a,b,c,d,e = n+1,...,n+m \tag{1.1}$$

$$z^{p'} = z^{p'}(x,z) \quad p,q,r,s,t = n+m+1,...,n+m+l$$

where

$$\operatorname{rank}\left[\frac{\partial x^{i'}}{\partial x^i}\right] = n, \operatorname{rank}\left[\frac{\partial y^{a'}}{\partial y^a}\right] = m, \ \operatorname{rank}\left[\frac{\partial z^{p'}}{\partial z^p}\right] = l.$$

Proposition 1. 1. *The coordinate transformations of type (1.1) form a group.*

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If the functions $N_{i'}^{b'}(x',y')$ and $M_{i'}^{p'}(x',z')$ satisfy the following law of transformation:

$$N_i^b(x,y) = N_{i'}^{b'}(x',y')\frac{\partial x^{i'}}{\partial x^i}\frac{\partial y^b}{\partial y^{b'}} + \frac{\partial y^{a'}}{\partial x^i}\frac{\partial y^b}{\partial y^{a'}}, \tag{1.2}$$

$$M_i^p(x,z) = M_{i'}^{p'}(x',z')\frac{\partial x^{i'}}{\partial x^i}\frac{\partial z^p}{\partial z^{p'}} + \frac{\partial z^{p'}}{\partial x^i}\frac{\partial z^p}{\partial z^{p'}}, \tag{1.3}$$

then the adapted basis of $T(E)$ is $B(N,M)=\{\frac{\delta}{\delta x^i},\frac{\partial}{\partial y^a},\frac{\partial}{\partial z^p}\}$, where

$$\frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - N_i^b(x,y)\frac{\partial}{\partial y^b} - M_i^p(x,z)\frac{\partial}{\partial z^p}, \frac{\delta}{\delta x^i} = \frac{\partial x^{i'}}{\partial x^i}\frac{\delta}{\delta x^{i'}}. \tag{1.4}$$

Let us denote by $T_H(E), T_{V_1}(E), T_{V_2}(E)$ the subspaces of $T(E)$ spanned by $\{\frac{\delta}{\delta x^i}\},\{\frac{\partial}{\partial y^a}\},\{\frac{\partial}{\partial z^p}\}$ respectively; then $T(E) = T_H(E)\oplus T_{V_1}(E)\oplus T_{V_2}(E)$.

Putting $\delta y^a = dy^a + N_i^a(x,y)dx^i, \delta z^p = dz^p + M_i^a(x,y)dx^i$, the adapted basis $B^* = \{dx^i, \delta y^a, \delta z^p\}$ of $T^*(E)$ is formed [3].

Orthogonality of the subspaces of *T*(*E*). The metric tensor G in E is a symmetric, positive definite tensor of type (0, 2).

In the adapted dual basis $B^* = \{dx^i, \delta y^a, \delta y^b\}$of $T^*(E)$ the metric tensor G has the following components

$$\begin{aligned} G = {} & g_{ij}dx^i\otimes dx^j + g_{ib}dx^i\otimes\delta y^a + g_{iq}dx^i\otimes\delta z^q + g_{aj}\delta y^a\otimes dx^j + g_{ab}\delta y^a\otimes\delta y^b \\ & + g_{aq}\delta y^a\otimes\delta z^q + g_{pj}\delta z^p\otimes dx^j + g_{pb}\delta z^p\otimes\delta y^b + g_{pq}\delta z^p\otimes\delta z^q. \end{aligned} \tag{1.5}$$

Proposition 1.2. *If $T_H(E)$, $T_{V_1}(E)$ and $T_{V_2}(E)$ are mutually orthogonal spaces with respect to the metric tensor G, then*

$$g_{ij} = \overline{g}_{ij} - \overline{g}_{cj}N_i^c - \overline{g}_{ic}N_j^c - \overline{g}_{rj}M_i^r - \overline{g}_{ir}M_j^r + g_{pq}M_i^pM_j^q + g_{ab}N_i^aN_j^b, \tag{1.6}$$

$$0 = \overline{g}_{ib} - g_{ab}N_i^a, 0 = \overline{g}_{aj} - g_{ab}N_j^b, 0 = \overline{g}_{iq} - g_{pq}M_i^p, 0 = \overline{g}_{pj} - g_{pq}M_j^q \tag{1.7}$$

where $\overline{g}_{ij},\overline{g}_{cj},\overline{g}_{ic};\overline{g}_{rj},\overline{g}_{ir},\overline{g}_{ib},\overline{g}_{aj},\overline{g}_{iq},\overline{g}_{pj}$ are components of the metric in the natural basis.

Theorem 1.1. *If $T_H(E)$, $T_{V_1}(E)$ and $T_{V_2}(E)$ are mutually orthogonal with respect to the metric tensor G given by* (*1.5*)*, then:*

$$N_i^c = \overline{g}_{ib}g^{bc}, M_j^r = \overline{g}_{pj}g^{pr} \tag{1.8}$$

Proposition 1.3. *The nonlinear connections N_i^c and M_j^r determined by* (1.8) *satisfy the transformation laws* (1.2) *and* (1.3).

2. GAUGE COVARIANT DERIVATIVES OF THE SECOND ORDER

Let $\nabla : T(E) \times T(E) \to T(E)$ ($\times$ is Descarte's product) be a usual linear connection, such as that $\nabla : (X,Y) \to \nabla_X Y \in T(E)$, $\forall X,Y \in T(E)$ The operator ∇ is called the generalized gauge connection of the second order. It is called *d*-gauge connection of the second order if $\nabla_X Y$ is in $T_H(E), T_{V_1}(E), T_{V_2}(E)$ if Y is in $T_H(E), T_{V_1}(E), T_{V_2}(E)$ respectively $\forall X \in T(E)$ It has been studied by many authors, mostly Romanian geometers.

We shall suppose that on E the metric tensor G is given by (1.5) ([5], [6]).

If we form the adapted basis B^* using the nonlinear connection coefficients determined by (1.8), as functions of the metric tensor G and suppose that $T_{V_1}$is orthogonal to T_{V_2}, then according to Theorem 1.1 in this basis the metric tensor (1.5) has the form :

$$G = g_{ij} dx^i \otimes dx^j + g_{ab} \delta y^a \otimes \delta y^b + g_{pq} \delta z^p \otimes \delta y^q. \tag{2.1}$$

Definition 2.1. *The generalized gauge connection* ∇ *of the second order is defined by*

$$\nabla_{\partial_\alpha} \partial_\beta = \Gamma^\gamma_{\beta\alpha} \partial_\gamma, \tag{2.2}$$

where $\alpha, \beta, \gamma, ... = 1, ..., n+m+l$, *and* ∂_α *are elements of the adapted basis B* [4].

Theorem 2.1. *If the vector field X, Y expressed in B have the form*

$$X = X^a \partial_a = X^i \delta_i + X^a \partial_a + X^p \partial_p, Y = Y^\beta \partial_\beta = Y^j \delta_j + Y^b \partial_b + Y^q \partial_q,$$

then

$$\nabla_Y X = X^\alpha_{,\beta} Y^\beta \partial_\alpha, \tag{2.3}$$

where

$$X^\alpha_{,\beta} = \partial_\beta X^\alpha + \Gamma^\alpha_{\gamma\beta} X^\gamma = \partial_\beta X^\alpha + \Gamma^\alpha_{i\beta} X^i + \Gamma^\alpha_{a\beta} X^a + \Gamma^\alpha_{p\beta} X^p. \tag{2.4}$$

Theorem 2.2. *The covariant derivatives are transformed as tensors if all connection coefficients are transformed as tensors except*

$$(a) \qquad \Gamma^k_{ji} = \Gamma^{k'}_{j'i'} \frac{\partial x^{i'}}{\partial x^i} \frac{\partial x^j}{\partial x^{j'}} \frac{\partial x^k}{\partial x^{k'}} + \frac{\partial^2 x^{k'}}{\partial x^i \partial x^j} \frac{\partial x^k}{\partial x^{k'}}$$

$$(b) \qquad \Gamma^c_{bi} = \Gamma^{c'}_{b'i'} \frac{\partial x^{i'}}{\partial x^i} \frac{\partial y^{b'}}{\partial y^b} \frac{\partial y^c}{\partial y^{c'}} + \frac{\partial^2 y^{c'}}{\partial x^i \partial y^b} \frac{\partial y^c}{\partial y^{c'}} - N^a_i \frac{\partial^2 y^{c'}}{\partial y^b \partial y^a} \frac{\partial y^c}{\partial y^{c'}}$$

$$(c) \qquad \Gamma^r_{qi} = \Gamma^{r'}_{q'i'} \frac{\partial x^{i'}}{\partial x^i} \frac{\partial z^{q'}}{\partial z^q} \frac{\partial z^r}{\partial z^{r'}} + \frac{\partial^2 z^{r'}}{\partial x^i \partial z^q} \frac{\partial z^r}{\partial z^{r'}} - M^s_i \frac{\partial^2 z^{r'}}{\partial z^s \partial z^q} \frac{\partial z^r}{\partial z^{r'}}$$

$$(d) \qquad \Gamma^c_{ba} = \Gamma^{c'}_{b'a'} \frac{\partial y^{b'}}{\partial y^b} \frac{\partial y^{a'}}{\partial y^a} \frac{\partial y^c}{\partial y^{c'}} + \frac{\partial^2 y^{c'}}{\partial y^a \partial y^b} \frac{\partial y^c}{\partial y^{c'}}$$

$$(e) \qquad \Gamma^r_{qp} = \Gamma^{r'}_{q'p'} \frac{\partial z^{q'}}{\partial z^q} \frac{\partial z^{p'}}{\partial z^p} \frac{\partial z^r}{\partial z^{r'}} + \frac{\partial^2 z^{r'}}{\partial z^q \partial z^p} \frac{\partial z^r}{\partial z^{r'}}.$$

3. RECURRENT GAUGE CONNECTION OF SECOND ORDER

In the following we shall use such adapted basis B^* in which the nonlinear connections are given by (1.8) and the metric tensor G has the form (2.1).

Definition (3.1) *The generalized gauge connection* ∇ *of the second order is recurrent (metric) if*

$$g_{\alpha\beta,\gamma} = \omega_\gamma g_{\alpha\beta} \quad (g_{\alpha\beta,\gamma} = 0), \tag{3.1}$$

where $\omega = \omega_h dx^h + \omega_a \delta y^a + \omega_s \delta z^s$ *is a 1-form in* $T^*(E)$ *and*

$$g_{\alpha\beta,\gamma} = \partial_\gamma g_{\alpha\beta} - \Gamma^\kappa_{\alpha\gamma} g_{\kappa\beta} - \Gamma^\kappa_{\beta\gamma} g_{\alpha\kappa}. \tag{3.2}$$

Theorem 3.1. *The connection coefficients of the recurrent gauge connection of the second order are determined by*

$$2\Gamma^\gamma_{\alpha\beta} = g^{\kappa\gamma}(\gamma_{\alpha\kappa\beta} - \omega_{\alpha\kappa\beta} + \widetilde{T}_{\alpha\kappa\beta}), \tag{3.3}$$

where

$$\gamma_{\alpha\kappa\beta} = \partial_\beta g_{\alpha\kappa} + \partial_\alpha g_{\kappa\beta} - \partial_\kappa g_{\alpha\beta}, \ \omega_{\alpha\gamma\beta} = \omega_\alpha g_{\gamma\beta} + \omega_\beta g_{\alpha\gamma} - \omega_\gamma g_{\alpha\beta}, \tag{3.4}$$

$$\widetilde{T}_{\alpha\gamma\beta} = \widetilde{T}^\rho_{\alpha\gamma} g_{\rho\beta} + \widetilde{T}^\rho_{\beta\gamma} g_{\rho\alpha} + \widetilde{T}^\rho_{\alpha\beta} g_{\rho\gamma}, \ \widetilde{T}^\rho_{\alpha\gamma} = \Gamma^\rho_{\alpha\gamma} - \Gamma^\rho_{\gamma\alpha}. \tag{3.5}$$

From (2.1) and (3.1) follows $g_{aj,\gamma} = 0$, $g_{pj,\gamma} = 0$, $g_{aq,\gamma} = 0$, where $\gamma = i$ or $\gamma = b$ or $\gamma = s$.

Theorem 3.2. *The connection coefficients of the metric gauge connection of the second order are given by* $2\Gamma^\gamma_{\alpha\beta} = g^{\kappa\gamma}(\gamma_{\alpha\kappa\beta} + \widetilde{T}_{\alpha\chi\beta})$.

4. $f(2k+1,1)$-STRUCTURES

Let in (1.1) $m + l = n$, then $\dim T_H(E) = n = \dim(T_{V_1}(E) \oplus T_{V_2}(E)) = \dim T_V(E)$ $T_H(E)$ is the subspace spanned by $\{\delta_i\}$, $i = 1,...,n$ and the subspace $T_V(E)$ is spanned by $\{\partial_{\bar{i}}\}, \bar{i} = n + i$.

Let $X \in T(E)$ then $X = X^i\delta_i + X^{\bar{i}}\partial_{\bar{i}}$ and the automorphism $P : \aleph(T(E)) \to \aleph(T(E))$ defined by $PX = X^{\bar{i}}\delta_i + X^i\partial_{\bar{i}}, P(\partial_{\bar{i}}) = \delta_i, P(\delta_i) = \partial_{\bar{i}}$, is the natural almost product structure on $T(E)$i.e. $P^2 = I$ If we denote by v and h the projection morphism of $T(E)$ to $T_V(E)$ and $T_H(E)$ respectively, we have $P \circ h = v \circ P$ [1].

The automorphism $JX = -X^{\bar{i}}\delta_i + X^i\partial_{\bar{i}}, J(\delta_i) = \partial_{\bar{i}}, J(\partial_{\bar{i}}) = -\delta_i$ is the natural almost complex structure on $T(E)$.

Definition 4.1. *We call metric vertical* $f_v(2k+1,1)$ *-structure of rank r on* $T_V(E)$ *a non-null tensor field* f_v *of type* (1.1) *and of class* C^∞ *such that* $f_v^{2k+1} + f_v = 0, k \in N$, *and rank* $f_v = r$, *where r is constant everywhere.*

Definition 4.2. *We call metric horizontal $f_h(2k+1,1)$ structure on $T_H(E)$ a non-null tensor field f_h on $T_H(E)$ of type (1.1) of class C^∞ satisfying $f_h^{2k+1}+f_h=0, k\in N$, rank $f_h = r$ where r is constant everywhere.*

An $F(2k+1,1)$ -structure on *T(E)* is a non-null tensor field F of type $\binom{11}{11}$ such that $F^{2k+1}+F=0,\ k\in N$ rank = F = *2r const.*

For our study it is very convenient to consider f_v or f_h as morhisms of vector bundles.

$$f_v:\aleph T_V(E)\to \aleph T_V(E), f_h:\aleph T_H(E)\to\aleph T_H(E).$$

Let f_v be a metric vertical $f_v(2k+1,1)$ -structure of rank r. We define the morphisms

$$1=-f_v^{2k}, m=f_v^{2k}+I_{T_V(E)}$$

where $I_{T_V(E)}$ denotes the identity morhism on $T_V(E)$ It is clear that **1 + m** = *I.* Also we have **1m = m1 =** $-f_v^{4k}=f_v^{2k}=-f_v^{2k-1}(f_v^{2k+1}+f_v=0), m^2 = m, \mathbf{1}^2=\mathbf{1}$. Hence the morphisms **1, m** applied to the $\aleph(T_V(E))$ are complementary projection morphisms, then there exist complementary distributions *VL* and *VM* corresponding to the projection morphisms **1** and **m** respectively such that dimVL = r and dimVM = n – r.

It is easy to see that

$$1f_v=f_vl=f_v, mf_v=f_vm=0, f_v^{2k}m=0, f_v^{2k}1=-1. \tag{4.1}$$

Proposition 4.1. *If a metric $f_v(2k+1,1)$ -structure if rank r is defined on $T_V(E)$,then the horizontal $f_h(2k+1,1)$ structure of rank r is defined on $T_H(E)$ by the natural almost product structure of T(E), as f_p or by the natural almost complex structure of T(E), as f_j*

Proof. If we put

$$f_pX=Pf_vPX, \forall X\in T_H(E), f_jX=-Jf_vJX, \forall X\in T_H(E), \tag{4.2}$$

it is easy to see that

$$f_p^{2k+1}X=Pf_v^{2k+1}PX, f_j^{2k+1}X=-Jf_v^{2k+1}JX \text{ and } f_p^{2k+1}+f_p=0, f_j^{2k+1}+f_j=0$$

and rank f_p = rank $f_j = r$. It is easy to see that $f_p = f_j = f_h$

Proposition 4.2. *If a metric $f_v(2k+1,1)$ -structure of rank r is defined on $T_V(E)$, then an $F_p(2k+1,1)$ -structure or $F_j(2k+1,1)$ -structure are defined on T(E) by the natural almost product or natural almost complex structure of T(E).*

Proof. We put $F_p = f_p h + f_p v$, $F_j = f_j h + f_j v$ where f_p, f_j are defined by (4.2) and *h,v* are the projection morphism of *T(E)* to $T_H(E)$ and $T_V(E)$. Then it is easy to check that

$$F_p^2=f_p^2h+f_p^2v, F_p^{2k+1}=f_p^{2k+1}h+f_p^{2k+1}v.$$

Thus $F_p^{2k+1}+F_p=0$. Similarly $F_j^{2k+1}+F_j=0$. It is clear that rank F_p = rank $F_j = 2r$

If 1_p, $m_p$are complementary projection morphisms of the horizontal $f_p(2k+1,1)$ - structure, which is defined by the natural almost product structure of *T*(*E*)*,* we have

$$1_pX=-f_p^{2k}X=-Pf_v^{2k}PX=P1PX, \forall X\in T_H(E)$$
$$m_pX=f_p^{2k}+I_{T_v(E)}X=Pf_v^{2k}PX+PI_{T_V(E)}PX=PmPX, \forall X\in T_H(E).$$

If L_p, M_p are complementary projection morphisms of the $F_p(2k+1,1)$ structure on $T(E)$, then we have

$$L_p = -F_p^{2k} = -f_p^{2k}h - f_v^{2k}v = 1_p h + 1v,$$
$$M_p = F_p^{2k} + I_{T(E)} = f_p^{2k}h + f_v^{2k}v + I_{T_H(E)}h + I_{T_V(E)}v = m_p h + mv. \quad (4.3)$$

Thus, if there is a given metric $f_v(2k+1,1)$ -structure on $T_V(E)$ of rank r, then there exist complementary distributions HL_p, HM_p *of* $T_H(E)$, corresponding to the morphisms 1_p, $m_p$such that

$$HL_p = PVL, HM_p = PVM. \quad (4.4)$$

Thus we have the decomposition $T(E) = T_H(E) \oplus T_V(E) = PVL \oplus PVM \oplus VL \oplus VM$.

If TL_p, TM_pdenote complementary distributions corresponding to the morphisms L_p, M_p respectively, then from (4.3) and (4.4) we have

$$TL_p = PVL \oplus VL, TM_p = PVM \oplus VM \text{ [2].}$$

Let $\overline{g}$ be a pseudo-Riemannian metric tensor, which is symmetric, bilinear and non-degenerate on $T_V(E)$.

$$\overline{g} : \aleph(T_V(E) \times \aleph(T_V(E)) \to \Phi(T(E)).$$

(For example $\overline{g}$ can be the vertical part of metric structure *G*).

The mapping

$$a : \aleph(T_V(E)) \times \aleph(T_V(E)) \to \Phi(T(E))$$

which is defined by

$$a(X,Y) = \frac{1}{2}[\overline{g}(1X,1Y) + \overline{g}(mX,mY)] \forall X,Y \in \aleph(T_V(E))$$

is a pseudo-Riemannian structure on $T(E)$ such that

$$a(X,Y) = 0, \forall X \in \aleph(T(VL), Y \in \aleph(T(VM)$$

Theorem 4.1. *If a metric $f_v(2k+1,1)$ –structure $k \geq 1$ of rank r is defined on $T_V(E)$ then there exists a pseudo-Riemannian structure of $T_V(E)$ with respect to which complementary distributions VL and VM are orthogonal and f_v is an isometry on $T_V(E)$*

Proof: If we put

$g(X,Y) = \frac{1}{2k}[a(X,Y) + a(f_vX, f_vY) + ... + a(f_v^{2k-1}X, f_v^{2k-1}Y)]$, it is easy to see that $g(X,Y) = 0 \;\; \forall X \in \aleph(VL),\; Y \in \aleph(VM)$.

Using (4.1) we get

$$g(f_vX, f_vY) = \frac{1}{2k}[a(f_vX, f_vY) + a(f_v^2X, f_v^2Y) + ... + a(X,Y)].$$

Thus f_v is an isometry with respect to g.

Let $X \in \aleph(T(VL))$, then $f_vX, f_v^2X, ..., f_v^{2k}X \in \aleph(T(VL))$ and

$g(X, f_v^k X) = g(f_v X, f_v^{k+1} X) = ... = g(f_v^k X, f_v^{2k} X) = -g(f_v^k X, X).$

Consequently $g(X, f_v^k X) = g(f_v X, f_v^{k+1} X) = ... = g(f_v^k X, f_v^{2k} X) = 0$ and r = 2ks.

Thus we can choose in $\aleph(T(VL))$ r = 2ks mutually orthogonal unit vector fields such as that $f(X_\alpha) = X_{\alpha+s}$ $\alpha = 1,2,...,(2k-1)s$, $f(X_\alpha) = -X_{-(2k-1)s+\alpha}$, $\alpha = (2k-1)s+1,...,2ks.$

An adapted frame of the metric $f_v(2k+1,1)$ -structure on $T_V(E)$ is the orhogonal frame $R = \{X_\sigma, X_\rho\}$ where X_ρ is an orthogonal frame of $\aleph(T(VM))$.

Let $\overline{R} = \{\overline{X}_\sigma, \overline{X}_\rho\}$ be another adapted frame of the metric $f_v(2k+1,1)$ -structure and $\overline{R} = AR$, then orthogonal matrix A is an element of the group $U_{(ks)} \times O_{(n-2ks)}$ [2].

Theorem 4.2. *A necessary and sufficient condition for* $T_V(E)$ *to admit metric* $f_v(2k+1,1)$ *-structure,* $k \geq 1$ *of rank r is that r=2ks,* $k = 2^q$ *and the structure group of the tangent bundle of the manifold is to be reduced to the group* $U_{(ks)} \times O_{(n-2ks)}$.

We can define a mapping g_p:

$$g_p(X,Y) = g(PX, PY), \qquad \forall X, Y \in \aleph(T_H(E)).$$

g_p is a metric structure on $T_H(E)$ Using (4.4), the distributions HL_p, HM_p are orthogonal with respect to g_p and the horizontal $f_p(2k+1,1)$ -structure which is defined by $f_p X = P f_v PX$, $\forall X \in \aleph(T_H(E))$ is an isometry on $T_H(E)$ with respect to g_p.

Proposition 4.3. *If* $\{X_\alpha, X_\beta\}$ *is an adapted frame of a given metric* $f_v(2k+1,1)$ *-structure* f_v *on* $T_V(E)$ *with respect to g, then the frame* $\{PX_\sigma, PX_\rho\}$ *is an adapted frame of the horizontal* $f_p(2k+1,1)$ *-structure with respect to g.*

It is clear that the frames $\{PX_\sigma, PX_\rho, X_\sigma, X_\rho\}$ are adapted frames to the decomposition

$T(E) = HL_p \oplus HM_p \oplus VL \oplus VM.$

Theorem 4.3. *Of a metric* $f_v(2k+1,1)$ *- structure is defined on* $T_V(E)$ *with pseudo-Riemannian structure g, then the structure group of the tangent bundle on* $T(E)$ *is reduced to* $U_{(ks)} \times O_{(n-2ks))} \times U_{(ks)} \times O_{(n-2ks)}$.

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REKURENTNE I METRIČKE KONEKSIJE I F-STRUKTURE U METRIČKIM PROSTORIMA DRUGOG REDA

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Skoro kompleksna struktura J je data na tangentnom prostoru mnogostrukosti E dimenzije 2n i data je u vertikalnom tangentnom prostoru $T_V(E)$ struktura $f_V(2k+1, 1)$. Tada se na prirodan način može dobiti na horizontalnom prostoru $T_H(E)$ struktura istog tipa $f_H(2k+1, 1)$, pa i na celom tangentnom prostoru $T(E)$ struktura $f(2k+1, 1)$. Dobijen je potreban i dovoljan uslov za redukciju strukturne grupe mnogostrukosti da bi se ona mogla snabdeti $f(2k+1, 1)$ – strukturom.

U ovakvom prostoru ispitivane su koneksije i izvršena je generalizacija d – koneksije

Ključne reči: *generalizovana koneksija, gauge koneksija, f – struktura*