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THE MATHEMATICAL MODEL FOR SPIDER DRAGLINE

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Abstract. *The spider dragline is of intelligent character, different from that of traditional smart materials. A constitutive relation is established by taking into account the effect of vibration on the stress and electric field. The effect of oscillation frequency on its mechanical property is illustrated.*

Key words: *spider dragline, intelligent fiber, constructive equation, vibration technology, oscillation frequency*

1. INTRODUCTION

The spider-spun fiber [1-8] is of extraordinary strength and toughness comparable to that of electrospun fiber [9-13], the latter needs a very high voltage(from several thousands voltage to several ten thousands voltages) applied to water-soluble protein "soup" that was produced by spider; furthermore, its mechanical strength dramatically decreases comparable to spider silk [10]. Even in the modern times, it is difficult to synthesize a material having advantages of strength and toughness except carbon nanotube fibers [14], which is spun from solution in very hot temperature or pressure, while the spider silks are produced at room temperature and from aqueous solutions. Spider silk is the only nature material that combines the properties unmatched by any known synthetic high-performance fibers.

Spider silks are protein-based "biopolymer" filaments or threads secreted by specialized epithelial cells as concentrated soluble precursors of highly repetitive primary sequences. Though many experiments have been conducted, and much research is focused on gene sequencing of spider [3,15], and a bio-mimicry technology is developed; however, theoretical analysis of its intelligent dragline is not yet dealt with, and our understanding of the mechanism of spider-spinning and the mechanical character of its dragline silks is rare and primitive. If the mystery in the process of spider-spinning can be solved, then we can apply the mechanism to synthetic high-performance fibers such as electrospun fibers [10].

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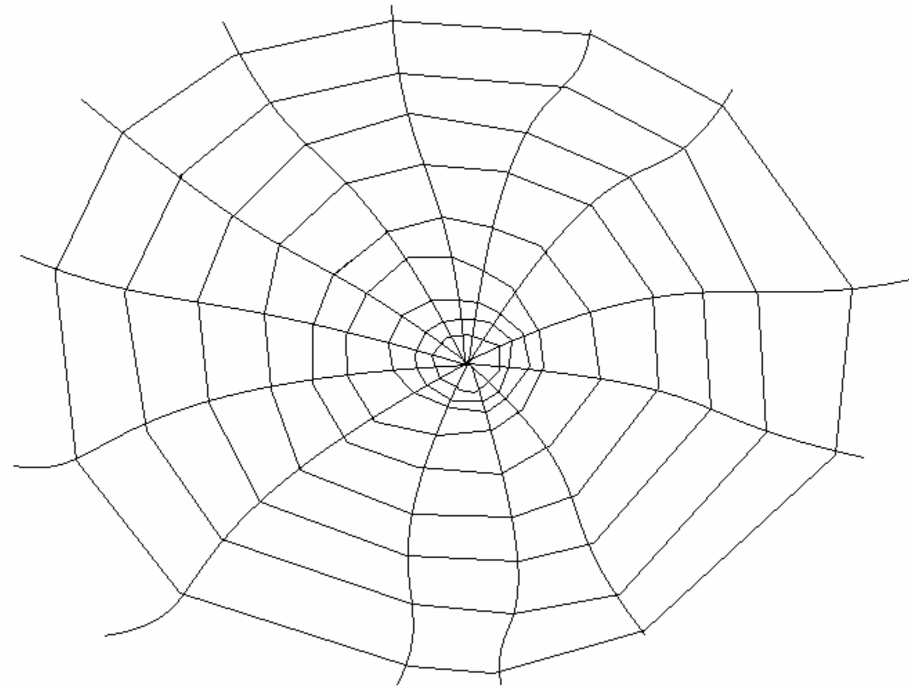

Fig. 1. Spider silks have the maximal strength and have a special smart character different from the traditional piezoelectricity.

2. THE MATHEMATICAL MODEL FOR INTELLIGENT SPIDER DRAGLINE

At present and despite decades of concentrated effort, we do not have a rational theory which could explain the intelligent characters of spider dragline, let alone the mystery in spider-spinning process.

The spider will not respond when the web is under the nature forces, such as wind, rain. But when a mosquito struggles to get free from its web, the spider finds its prey immediately no matter whether it is in the center of the web, or on the fringe of the web. The spider can distinguish various preys with ease by their fluttering frequency.

The spider can feel oscillation just like man can hear voice, which is also a kind of oscillation. The eardrum is oscillated when the ear is subjected to oscillating sound, and information is sent to brain through nerves. But spider silks in the web have no life, so the oscillation information can not be sent in as similar way as that in ion pumps in biology. The oscillation information is transferred through electrical behavior of the web caused by vibration, and the spider can only "hear" some oscillations with defined frequency. It can not "hear" the oscillation caused by the wind or rain. Some species of orb builders do not stay on the web itself, but attach a "telegraph" line to the sheet of webbing, then sit in a nearby bush holding the line taut, the web diaphragm vibrates, the message travels down the non-sticky radial threads, and the spider instantly "hears" the vibration and knows in which direction to head for its dinner.

It seems that spider dragline is of intelligent character. When acted by a vibrating force with definite frequency, the spider dragline may produce an electric field. Or may it oscillate when subjected to an electric field? It needs experimental verification. This intelligent character differs from that of piezoelectric materials, which have been widely used in engineering, and it was first discovered by Pierre and Jacques Curie in 1880 that certain crystals may, when stressed, produce an electric field, or when subjected to an electric field, deform. Piezoelectric material is widely applied in engineering, extensive research has been done recently [16~24].The different approach to piezoelectricity can be found in Refs. [25~28], one-dimensional ionic polymer-metal composites are studied in Ref. [29].

Hereby we consider the linear spider dragline silk. The governing equations are:

1) *Equilibrium equations*

$$\sigma_{ij,j} = 0\,, \tag{1}$$

in which σ_{ij} is the symmetric stress tensor, $\sigma_{ij,j} = \partial\sigma_{ij} / \partial x_j$.

2) *Constitutive equations*

$$\sigma_{ij} = a_{ijkl}\gamma_{kl} - e_{mij}E_m\,, \tag{2}$$

$$D_i = \varepsilon_{ij}E_j + e_{ijk}\gamma_{jk} + k\omega\,, \tag{3}$$

in which γ_{ij} is the symmetric strain tensor, D_i is the vector of the electric displacement, E_i is the vector of the electric field, ω is the frequency, k is a constant. The elastic moduli a_{ijkl} measured at constant (zero) electric field, and the piezoelectric moduli e_{mij}, and the dielectric permittivity ε_{ij} have the following symmetry properties, respectively

$$a_{ijkl} = a_{jikl} = a_{ijlk} = a_{klij}\,,$$
$$e_{mij} = e_{mji}\,, \text{ and } \varepsilon_{ij} = \varepsilon_{ji}\,.$$

When $k = 0$ or $\omega = 0$, our model turns out to be the traditional one in piezoelectricity [25~28].

3) *Strain-displacement relations*

$$\gamma_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})\,, \tag{4}$$

where u_i is the vector of the elastic displacement.

4) *Quasi-static Maxwell's equations*

$$D_{i,i} = 0\,. \tag{5}$$

3. THE EFFECT OF OSCILLATION ON TRAVELING WAVE

Oscillation plays an import role in spider-spinning [10] and in the detection of preys. Vibration technology [30~35] was introduced into polymer processing many years ago, including injection molding, extrusion and compression molding/thermoforming for reduction of viscosity to lowering processing temperature and pressure to the elimination of melt defects and weld lines, and enhancement of mechanical properties by modification of the amorphous and semicrystalline texture and orientational state.

Ibar [35] observed the effect of low-frequency vibration during processing for PMMA. The result shows that increasing frequency makes the solution viscosity decreased dramatically, see Fig. 2. The application of vibration technology to electrospinning was first proposed by He, Wan, and Yu [10].

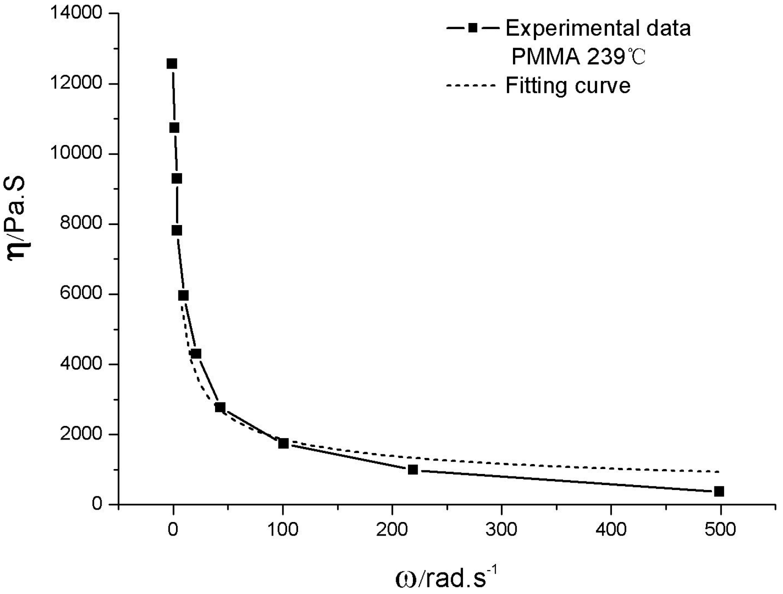

Fig. 2 Solution viscosity (η) of PMMA vs. the vibration frequency (ω) [35]

The experiment and theoretical analysis show that [10,35]

$$\eta \sim \omega^{-0.4}, \tag{6}$$

for PMMA solution.

The spider dragline can be considered as a one-dimensional problem. Considering the viscous effect of the dragline, the governing equations for one-dimensional spider dragline can be expressed as follows

$$\ddot{u} + \eta \dot{u} = \frac{d\sigma}{dz}, \tag{7}$$

$$\sigma = a\frac{du}{dz} - eE, \tag{8}$$

$$D = \varepsilon E + e\frac{du}{dz} + k\omega, \tag{9}$$

$$\frac{dD}{dz} = 0\,. \tag{10}$$

From (7)~(10), we can obtain the following wave equation

$$\ddot{u} + \eta\dot{u} = (a + \frac{e^2}{\varepsilon})\frac{d^2u}{dz^2} + \frac{ek}{\varepsilon}\frac{d\omega}{dz}\,. \tag{11}$$

We will discuss this equation in a forthcoming paper to discover mathematically why the spider can not "hear" the oscillations caused by wind or rain.

4. CONCLUSION

Despite the great success and undeniable brilliance of the biologic approach to spider dragline, it is fair to say that no mathematical model for spider dragline exists in open literature, and our model is by no means perfect. The model can be readily extended to nonlinear cases. Of course, the author understands that no matter how rigorous, some experimental verification is needed to validate the model. The straightforward verification is to measure electronic current in spider dragline when subjected to an oscillating force, or measure the oscillation frequency of the spider dragline when subjected to an electric field. Our university can not do such an experiment due to its small financial support of this project. I hope the experiment can verify partly or wholly my theory in future, or even deny it, so that a new theory can be proposed.

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MATEMATIČKI MODEL PAUKOVE MREŽE

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Materijal mreže pauka je inteligentnog karaktera i razlikuje se od ostalih pametnih materijala. Konstitutivna relacija materijala paukove mreže se postavlja uzimajući u obzir efekat vibracije na mehanički napon i električno polje. Efekat frekvencije oscilacije na mehanička svojstva materijala je ilustrovan primerom.

Ključne reči: *paukova mreža, inteligentno vlakno, konstruktivna relacija,*
vibraciona tehnologija, frekvencija oscilovanja