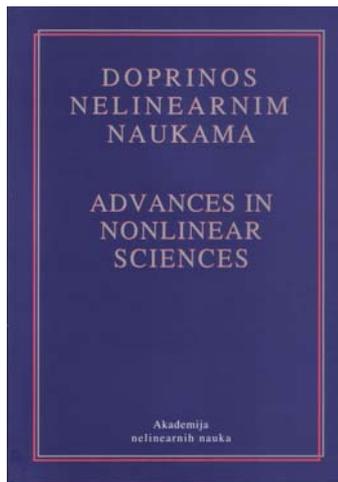


## ADVANCES IN NONLINEAR SCIENCES

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Serbian National Library classification of this monograph contains the following key words: Mathematical Physics and Nonlinear Theories.

Introductory part of the monograph contains the chapter dedicated to the establishment of the Yugoslav branch of nonlinear sciences and Vladimir Mefodievich Matrosov, the president of the International Academy of Nonlinear Sciences – Moscow.

At the beginning some words dedicated to the establishment of the Yugoslav branch of nonlinear sciences.

The name of Vladimir Mefodievich Matrosov cannot be exempted from the key words of the title of this book. He is the founder and the president of the international Academy of Nonlinear Sciences (ANS), and he is also the initiator for the establishment of the Yugoslavian Branch of the Academy of Nonlinear

Sciences. Veljko A. Vujičić and Katica (Stevanović) Hedrih were participants in the work of the International Conference on Stability, Control and Rigid Bodies Dynamics (2-6, September, 1996, Donetsk, Ukraine) with the invited lectures. During this Conference academician V. M. Matrosov, the president of ANS Moscow, introduced to the participants with structure of the ANS, and the procedure for the election of its members. He also invited scientists to take part at the founding assembly of the Ukrainian Branch of ANS. This act put in motion the idea of the establishing the Yugoslavian Branch of ANS. After returning to Belgrade Veljko A. Vujičić filled a form of application to become a member of ANS, prescribed by the statute of ANS, and enclosed the data concerned with his scientific achievements and his work on building the scientific youth in the country, together with the recommendation letters of two ANS academicians (V. V. Rumyantsev and F. Chernousko), and sent it to Moscow. On a meeting of the Presidential Board of ANS held on October 22, 1996, D35, Prof. Veljko Vujičić elected the member of ANS.

On a Symposium December 20, 1996 organized on the occasion of 50 years of the Mathematical Institute of Serbian Academy of Sciences and Arts, Vladimir M. Matrosov delivered a lecture under the title "Methods of nonlinear dynamical analysis and their trends". After the lecture he was talking about the ANS and used this opportunity to provide the membership diploma to the academician of ANS Prof. Veljko A. Vujičić. Academician V. V. Rumyantsev was also a guest of the Mathematical Institute. After the lecture academicians Vladimir M. Matrosov and V. V. Rumyantsev invited Katica (Stevanović) Hedrih to initiate the procedure of her election for the member of ANS.

The Presidential Board of ANS elected two new members of ANS Miomir Vukobratović, the member of Serbian Academy of Sciences and Arts and the foreign member of Russian Academy of Science, and Katica (Stevanović) Hedrih, the member of Ukrainian Higher Schools and Universities Academy of Sciences (May 13, 1997, D53 and D54).

In a new procedure two new members were elected: Vladan Djordjević, the member of Serbian Academy of Sciences and Arts, and Ranislav Bulatović, the member of Montenegrinian Academy of Sciences and Arts (October 22, 1997, D84 and D85).

On May 20, 1998 Vladimir M. Matrosov delivered a notable lecture in the Department of Mechanics of the Mathematical Institute of Serbian Academy of Sciences and Arts under the title "Method of Lyapunov functions at the end of the century". After the lecture the aforementioned members of ANS, together with the President of ANS, Vladimir M. Matrosov participated in a meeting on which (a) the decision about the establishment of the Yugoslavian Branch of ANS was drawn, (b) the Statute of the Branch was accepted, and (c) the President of the Branch - Veljko Vujičić was elected, becoming the representative of the Yugoslavian Branch of ANS.

Soon after that the Presidential Board of ANS confirmed the decision concerned with the establishment of the Yugoslavian Branch.

In accordance with the Statute of ANS and with the Statute of its Yugoslavian Branch, the following new members were unanimously proposed, and elected: corresponding member of European Academy of Sciences, Arts and Literature, Viktor Saljnikov (November 15, 1999, D126), scientific adviser and director of the Astronomical Observatory in Belgrade, Dr. Milan Dimitrijević (January 9, 2001, D151), member of the Serbian Academy of Sciences and Arts, Vojislav Marić (January 9, 2001, D153), professor of the University in Kragujevac, Milos Kojić (January 9, 2001, D152) and professor of the University in Valenciennes, Radomir Asković (June 19, 2001, D160).

Monograph contain ten papers in the area of nonlinear sciences as scientific contribution by members ANS.

The first paper under the title:

### **THE ROLE OF DIFFERENTIAL INEQUALITIES IN A WIDE RANGE OF NONLINEAR PROBLEMS,**

is written by V. Lakshmikantham, the president of IFNA (International Federation of Nonlinear Analysts), Head of Dept. of Mathematical Sciences, Florida Institute of Technology, Dept. of Mathematical Sciences, Melbourne, Organizer/WCNA, Chairman

**Abstract:** The author's scientific work in Nonlinear Sciences is sketched briefly. It starts in 1956 without guidance from anyone and with self study, and stretches up to the present time. It deals with a wide variety of nonlinear problems in the finite and infinite dimensions and with the classical and modern techniques. It is collected in several research monographs and survey papers, in addition to many research papers.

### **A CONTRIBUTION TO NONLINEAR MECHANICS**

written by Veljko A. Vujičić

Author starts from the preprinciples of: existence, Causal determinancy and Invariance, which is not called into doubt at any level of knowledge. Preprinciple of existence express that there are: bodies, distance and time. The existence of a body is manifested in

the theoretical mechanics as a body mass for which the denotation  $m$  and its dimension  $M$ , ( $dim\ m=M$ ) are accepted. Consequently, every existing body has its mass.

The existence of distance is identified everywhere: among particles, celestial bodies or between various points on the pathway that the body moves along, as well as between the place of the body and the place of observation. It is denoted by the letter  $l$  and is measured in units of dimension of length  $L$ , ( $dim\ l=L$ ).

Time is denoted by the letter  $t$ , and its dimension with  $T$ , ( $dim\ t=T$ ). It is continuous and irrevocable. In the mathematical description it can be represented by a numerical straight line or an ordered multitude of concrete named real numbers. Once the existence of time is accepted, the existence of motion, change, duration, the past, the present and the future is also accepted.

Preprinciples of Casual Determinacy demand that theory of the body motion is explicitly determined throughout the whole of time, in the future as in the past, and with as much accuracy as determinates of motion are known at any particular moment of time. Mechanics as a theory of the body motion is an accurate science in the mathematical sense, while as an applied science, it is so accurate as long as the data which are of importance for motion are accurately measured at one particular moment of time.

Preprinciples of Invariance advance that neither the motion nor the properties of the body motion depend upon the form of statement: the determined truth about motion, once it is written in some linguistic form, is equally contained in the written output of some other form or some other alphabet.

Principles of mechanics can differ from preprinciples. The concept of the principle of mechanics implies here an expression of general significance, based on the introduced concepts and definitions of mechanics whose truthfulness is not liable to verification; principle of mechanics represent the basis for developing a whole theory of mechanics. The principles of mechanics must be concordant with the preprinciples.

### **EFFECT OF THE VISCOSITY ON THE TRANSIENT HEAT TRANSFER TO AN IMPULSIVELY STARTING DROPLET**

written by Radomir V. Ašković

**Abstract.** An analysis is made for the transient response behaviour of the both, outer and inner, thermal boundary layers of an impulsively started fluid sphere at constant velocity with internal circulation in another viscous fluid of large extent under the condition of large Reynolds and Peclet numbers. The disturbance is initiated by a step change in temperature of either the continuous or disperse region fluids. An exact solution of the governing energy equation is found by using the solutions of the viscous flow fields. From algebraic and numerical considerations, we show that the viscous effects on the heat transfer at the interface are negligible comparing to the inviscid approximation. On the contrary, they predominantly occur in the both adjacent boundary layers on the interface, with a spreading effect in time.

### **ON THE HAMILTON-JACOBI EQUATION OF NATURAL SYSTEMS**

written by Ranislav Bulatović

**Abstract.** The study of solutions of the abbreviated Hamilton-Jacobi equation of natural systems published previously is presented. In the Section 1 the problem of the

existence of solutions of the Hamilton-Jacobi equation in the whole domain of possible motions with a boundary is investigated. Constraints on the topology of the domains of possible motions, in which the Hamilton-Jacobi equation is solvable in the large, are pointed out. In particular, the boundary cannot be connected. The existence of solutions in the whole domain of possible motions is obstructed by focal points at which infinitely close trajectories leaving the boundary intersect. A connection between the complete integral of the Hamilton-Jacobi equation and the particular solutions in the neighborhood of the boundary is indicated.

In the Section 2 the Hamilton-Jacobi equation is considered in the neighborhood of a position of equilibrium, which is not a local minimum of the potential energy. It is shown that, when in the equilibrium position the analytic potential energy is non-degenerate, then there exist an analytic solution of the Hamilton-Jacobi equation at the zero level of the total energy. In the case of a saddle point a solution is sought in the complex form. The problem of the presence of smooth solutions in cases of degeneracy is investigated. These solutions define manifolds in the phase space, which are filled with trajectories which asymptotically approach an equilibrium state.

#### **INFLUENCE OF ASTROPHYSICAL PLASMA ON SPECTRAL LINESHAPES**

written by Milan S. Dimitrijević

**Abstract.** A review of astrophysical problems where Stark broadening data are of interest is given. Also, the results of Stark broadening study relevant to astrophysical problems and to the laboratory plasma research have been reviewed and discussed. Particular attention has been paid to the semiclassical method and the modified semiempirical method as well as to the use in astrophysics of results and achievements in Stark broadening research of the Belgrade school.

#### **STABILITY OF LAMINAR FLOWS AND TRANSITION TO TURBULENCE**

written by Vladan Djordjević

Due to their great theoretical and practical importance the problems of hydrodynamic stability theory have occupied the attention of scientists involved in fluid mechanics research for almost hundred years. Since the basic task of the hydrodynamic stability theory is the explanation of all possible phases of the transition of laminar flow regime into the turbulent one in various dynamic systems of fluid mechanics, and since the turbulent flows belong to relatively small number of problems which are, from the point of view of theoretical physics, still considered as unresolved, it is not surprising that many famous physicists in the past have dealt with this theory (Bohr, Rayleigh, Kelvin, Helmholtz, and others). It is difficult to define the origin and the basic task of this theory better than Landau and Lifshitz, and Chandrasekhar did in their books.

"Yet not every solution of the equations of motion, even if it is exact, can actually occur in Nature. The flows that occur in Nature must not only obey the equations of fluid dynamics, but also be stable", and: Different fluid flows, which represent the exact solutions of hydrodynamic equations, can be "realized only for certain ranges of the parameters characterizing them. Outside these ranges, they cannot be realized. The reason for this lies in their inherent instability, i.e. in their inability to sustain themselves against small perturbations to which any physical system is subject. It is in the differentiation of

the stable from the unstable patterns of permissible flows that the problems of hydrodynamic stability originate".

From the point of view of contemporary developments in the hydrodynamic stability theory these definitions could be supplemented by the following comments: Since the turbulence is a phenomenon which is a characteristic of the field of fluid mechanics only and since every route to turbulence of a system in fluid mechanics begins with the analysis of the evolution of small disturbances to which this system is subjected, it does not mean that the corresponding theory dealing with the evolution of such disturbances - linear theory, can answer all questions related to the laminar-turbulent transition. Theories called weakly nonlinear and fully nonlinear represent also necessary links in the series of complex flow phenomena arising on the route to fully developed turbulence.

Differentiation of stable and unstable regimes of flow based on the values of parameters characterizing them is only one of the aims of the theory. Very important theoretical and practical task is also the detailed evaluation of all new flow states occurring as a consequence of the stability loss of some previous states.

### **CONTRIBUTION TO THE COUPLED ROTOR NONLINEAR DYNAMICS**

written by Katica (Stevanović) Hedrih

A brief review of the current research. Dynamics of coupled rotors is a very old engineering problem with many different search results and discoveries of new nonlinear phenomena, and of the stationary and nonstationary vibration regimes with different kinetic parameters of the dynamical system. However, even nowadays many researchers pay attention to this problem again, and again arise the interest in researching the nonlinear dynamics of coupled rotors by using new analytical, numerical and experimental methods to discover the properties of nonlinear dynamics and for finer possibilities for controlling nonlinear phenomena, instabilities and non stationary regimes and the appearance of the chaotic-like and the stochastic-like processes. One of the reasons for the research of this problem by using the analytical and numerical methods and MathCad on one of the not complex models is that it is a very good model for pointing out the complexity of the multi frequency regimes in the nonlinear system, which introduces many different regimes with sensitive dependence dynamical system properties and nonstationary processes by changing system kinetic parameters. On the basis of the changing distribution of the deviation material particles on the coupled rotors, we build a nonlinear model of the dynamical system with many varieties of the kinetic state of the coupled rotor dynamics.

For examine natural clocks of reductor, as well as source of noise in its dynamics, it is necessary to investigate the properties of nonlinear dynamics, and phase portraits, as well as structures of homoclinic orbits, layering and sensitivity of this layering of homoclinic orbits and bifurcation of homoclinic points.

A part of this paper was presented as Invited Lecture under the title: Vector Method and Phase Plane Method, Approach to the Analysis of Kinetic Parameters and Homoclinic Orbits of Coupled Rotor Dynamics at The International Symposium on Analysis, Manifolds and Mechanics, 2003., organized by M. C. Chaki Centre for Mathematics and Mathematical Sciences, Calcutta, India. Also, the content of this paper was presented as Invited Lecture at Institut für Mechanik, Technische Universität Wien 2003.

**THE GOVERNING PARAMETER METHOD AS A GENERAL PROCEDURE  
FOR STRESS INTEGRATION OF INELASTIC CONSTITUTIVE RELATIONS  
WITHIN INCREMENTAL NONLINEAR ANALYSIS**

written by Miloš Kojić

**Abstract.** The governing parameter method (GPM), as a general implicit stress integration procedure of the constitutive relations in inelasticity, is presented. A short review of various computational algorithms is given, followed by the formulation of the GPM and application of the GPM to a general isothermal plasticity material model. A basic relation showing the extension of the GPM to large strains is also presented, including two solved numerical examples. In the concluding remarks are summarized the basic features of the GPM and inelastic material models to which the GPM is applicable.

**ASYMPTOTIC BEHAVIOR OF SOLUTIONS  
OF NONLINEAR DIFFERENTIAL EQUATION OF THE FIRST ORDER**

written by Vojislav Marić

**Dedicated to the memory of J. Karamata**

**Abstract.** The asymptotic behavior at infinity of solutions of the equation  $u' = P(u, t) / Q(u, t)$  is studied.  $P, Q$  are polynomials in  $u$  whose coefficients are functions of  $t$ , and belong to the Hardy class  $H$ , (i.e., to the set of all real-valued functions defined by finite many ordinary algebraic, *exp* and *log* operations.) It is proved that, for any continuously differentiable solution  $u(t)$  there exists one or the other of the asymptotic formulae  $u(t) \sim h(t)$ ,  $\ln u(t) \sim h(t)$ , within the class  $H$ , i.e.,  $h(t)$  elements of  $H$ .

As the main tool for the proof it is first shown that any (real) solution  $y(t)$  of an algebraic equation whose coefficients are elements of  $H$  behaves at infinity again as an element of  $H$ .

**THE THEORY OF THE BOUNDARY LAYER AND THE GENERALIZED  
SIMILARITY METHOD IN THE LIGHT OF THE CONTEMPORARY  
TENDENCIES IN FLUID MECHANICS**

written by Viktor N. Saljnikov

**Abstract.** In this reviewing paper the application of generalized similarity for the calculation of laminar boundary layer on airfoils is presented. This method in its nature is analytical and numerical, and it fits fully into the current tendencies in fluid mechanics. The results of the theoretical investigation performed in the DLR Intitute in Göttingen, aimed at further "laminarisation" of a transsonic wing, designated as DFVLR-5, have led to the shifting of the transition line downstream to 70% of the chord. The method of generalized similarity gives very good results which can be used as self-contained, or can serve for testing of DNS results of full Navier-Stokes equations.