SYNTHESIS OF NONLINEAR ELASTIC CHARACTERISTICS OF
A MULTI-DEGREE-OF-FREEDOM SYSTEM INDEPENDENCY
IN CASE OF MULTI-HARMONIC OSCILLATIONS

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Abstract In the phase of design of different machines, whose certain parts perform
oscillation movements, the synthesis of a certain number of mechanic characteristics is
carried out [1-5].

In the paper [5] using the Chebishevlev`s polynomials, the synthesis of the nonlinear elastic
characteristics was carried out for a one-side bonded chain system in the case of synphase
harmonic oscillations of a mass system and for given simple periodic disturbances.

In this paper, using the trigonometric final series, the synthesis of the nonlinear elastic
characteristics is considered for a one-side bonded chain system in the case of multi-
harmonic oscillations of a mass system in the presence of complex periodic disturbance
forces.

Key words: trigonometric final series, synthesis of the nonlinear elastic characteristics,
one-side bonded chain material systems, bi-harmonic oscillations, complex
periodic disturbance forces.

1. DIFFERENTIAL EQUAZIONS OF SYSTEM MOVEMENT

One-side bonded chain system with s degrees of freedom is being observed, where s – is
a final number. Differential equations of motion of this system are:

\[ m_1 x_1 = -f_1(x_1) + f_2(x_2) + F_1(t), \]
\[ m_2 (x_1 + x_2) = -f_2(x_2) + f_3(x_3) + F_2(t), \]
\[ \vdots \]
\[ m_s (x_1 + x_2 + \ldots + x_s) = -f_s(x_s) + F_s(t), \]

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where: \( m_1, m_2, ..., m_s \) – is the mass the system; \( x_1 \) – the absolute coordinate of the rectilinear displacement; \( x_2, x_3, ..., x_s \) – the relative coordinates; \( f_1(x_1), f_2(x_2), ..., f_s(x_s) \) - the elastic characteristics (force); and \( F_1(t), F_2(t), ..., F_s(t) \) – the disturbance forces.

Elastic characteristics \( f_r(x_r), (r = 1, 2, ..., s) \) will be functions with a single meaning [2] if during the period of oscillations the following conditions are fulfilled:
\[
\dot{x}_r(t^*_i) = 0; \quad \ddot{x}_r(t^*_i) = 0, \quad (i = 1, 2, ..., n),
\]
where \( n \) – is the number of the extremums of functions \( x_r(t) \); \( t^*_i \) – is the point of an extreme of a function.

2. DISTURBANCE FORCES AND THE OSCILLATION LAWS. ELASTIC CHARACTERISTICS IN A PARAMETRIC FORM FOR A SYSTEM OF THREE AND TWO DEGREES OF FREEDOM

Let the complex periodic disturbance forces have an influence on a mass particle system
\[
F_r(t) = \sum_k F_{rk} \sin kt, \quad (r = 1, 2, ..., s; \quad k = 1, 3, 5, ...),
\]
and the system should move according to the following law
\[
x_r(t) = \sum_k A_{rk} \sin kt, \quad (r = 1, 2, ..., s; \quad k = 1, 3, 5, ...).
\]

In the case of a one-side bonded chain system with three degrees of freedom, from equations (1), (2) and (3) the elastic characteristics in parametric form are derived.
\[
\begin{align*}
f_1(t) &= F_1(t) + F_2(t) + F_3(t) - m_1\ddot{x}_1 - m_2(\ddot{x}_1 + \ddot{x}_2) - m_3(\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3), \\
f_2(t) &= F_2(t) + F_3(t) - m_2(\ddot{x}_1 + \ddot{x}_2) - m_3(\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3), \\
f_3(t) &= F_3(t) - m_3(\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3). 
\end{align*}
\]
These characteristics for a system with two degrees of freedom are:
\[
\begin{align*}
f_1(t) &= F_1(t) + F_2(t) - m_2\ddot{x}_1 - m_3(\ddot{x}_1 + \ddot{x}_2), \\
f_2(t) &= F_2(t) - m_3(\ddot{x}_1 + \ddot{x}_2). 
\end{align*}
\]
3. ELASTICS CHARACTERISTICS OF A ONE-SIDE BONDED CHAIN SYSTEM
WITH TWO DEGREES OF FREEDOM. NUMERIC EXAMPLES

Let us assume that the oscillation of the first mass system is given in the form

$$x_1(t) = A_{11} \sin t + A_{13} \sin 3t,$$

(5)

In the case of all values of the coefficients $A_{11}$ and $A_{13}$ it is needed to determine the nonlinear elastic characteristics $f_1(x_1)$ and $f_2(x_2)$ which ensure a multi-harmonic oscillation system.

We assume the function of a non-linear characteristic $f_1(x_1)$ in the form

$$f_1(x_1) = B_1 x_1^1 + B_3 x_1^3.$$

(6)

Substituting (5) into (6) the expression for the elastic characteristic of the first spring is obtained in the form of trigonometrical final series

$$f_1(t) = B_{11} \sin t + B_{13} \sin 3t + ... + B_{19} \sin 9t.$$

(7)

The oscillation law of a second mass system is taken in the same form

$$x_2(t) = A_{21} \sin t + A_{23} \sin 3t + ... + A_{29} \sin 9t.$$

(8)

Substituting (6-8) and (5) in a system of differential equations (4'), after certain mathematical operations we obtain

$$f_2(t) = B_{21} \sin t + B_{23} \sin 3t + ... + B_{29} \sin 9t.$$

(9)

From the equations (4) we determine the formulae for the unknown coefficient – amplitude of the trigonometric final series $B_{2k}$ and $A_{2k}$ ($k = 1,3,5,...,9)$:

for $B_{2k}$ –

$$B_{21} = B_1 A_{11} + 3 \frac{3}{4} B_1 A_1 (A_{11}^1 - A_{11} A_{13} + 2 A_{13}^3) - F_{11} - m_1 A_{11},$$

$$B_{23} = B_1 A_{13} + 1 \frac{3}{4} B_1 (-A_{11}^1 + 6 A_{11}^2 A_{13} + 3 A_{13}^3) - F_{13} - 9m_1 A_{13},$$

$$B_{25} = 3 \frac{3}{4} B_1 A_1 A_{13} (A_{13} - A_{11}) - F_{15},$$

$$B_{27} = -3 \frac{3}{4} B_1 A_1 A_{13}^3 - F_{17},$$

$$B_{29} = -1 \frac{3}{4} B_1 A_{13}^3 - F_{19};$$

(10)

for $A_{2k}$ –

$$A_{21} = \frac{1}{m_2} [B_1 A_{11} + 3 \frac{3}{4} B_1 A_1 (A_{11}^1 - A_{11} A_{13} + 2 A_{13}^3) -(F_{11} + F_{21})-(m_1 + m_2) A_{11}],$$

$$A_{23} = \frac{1}{m_2} [(B_1 A_{13} + 1 \frac{3}{4} B_1 (-A_{11}^1 + 6 A_{11}^2 A_{13} + 3 A_{13}^3)) \frac{1}{9} - \frac{1}{9} (F_{13} + F_{23})-(m_1 + m_2) A_{13}],$$

$$A_{25} = \frac{3}{4} B_1 A_1 A_{13} (A_{13} - A_{11}) (F_{15}),$$

$$A_{27} = \frac{3}{4} B_1 A_1 A_{13}^3 (F_{17}),$$

$$A_{29} = \frac{1}{4} B_1 A_{13}^3 (F_{19});$$
\[ A_{25} = \frac{1}{m_2} \left[ \frac{3}{4} B_2 A_{11} A_{13} (A_{13} - A_{11}) - \frac{1}{25} (F_{15} + F_{25}) \right], \]
\[ A_{27} = \frac{1}{m_2} \left[ \frac{3}{4} B_2 A_{11} A_{13}^2 - (F_{17} + F_{27}) \right] \frac{1}{49}, \]
\[ A_{29} = \frac{1}{m_2} \left[ \frac{1}{4} B_2 A_{13}^3 - (F_{19} + F_{29}) \right] \frac{1}{81}. \]
Let a disturbance force \( F_1(t) = F_{11} \sin t \) have an impact on the mass \( m_1 \) and, in addition, this mass should perform the motion in accordance with the low \( x_1(t) = A_1 \sin 3t \). It is needed to determine laws of a change of the elastic characteristics \( f_2(t) \) and multi-harmonic oscillations \( x_2(t) \) of a mass system \( m_2 \).

In this case the coefficients \( B_{2k} \) and \( A_{2k} \) \((k = 1,3,\ldots,9)\) will be

\[
B_{23} = \frac{3}{4} B_1 A_{11} - 9 m_1 A_{13}, \quad B_{25} = B_{27} = 0, \quad B_{29} = -\frac{1}{4} B_1 A_{11};
\]

\[
A_{21} = -\frac{F_{11}}{m_2}, \quad A_{23} = \frac{1}{9 m_2} \left[ B_1 A_{13} + \frac{3}{4} B_1 A_{13}^3 - 9 (m_1 + m_2) A_{13} \right], \quad A_{25} = A_{27} = 0, \quad A_{29} = -\frac{B_1 A_{13}}{324 m_2}.
\]

For \( m_1 = m_2 = B_1 = B_3 = A_{13} = 1 \) when numeric values of this variables are expressed in the units of the International system of measures, laws of the multi-harmonic oscillations and the matching elastic characteristics will be

\[
x_1(t) = \sin 3t;
\]

\[
f_1(t) = x_1(t) + x_1^3(t) = \sin 3t + (\sin 3t)^3,
\]

\[
f_2(t) = -F_{11} \sin t - \frac{33}{4} \sin 3t - \frac{1}{4} \sin 9t;
\]

\[
x_2(t) = -F_{11} \sin t - \frac{65}{36} \sin 3t - \frac{1}{324} \sin 9t.
\]

The graphic representations of this functions, for \( F_{11} = 0.5 \) \( N \), \( F_{11} = 1 \) \( N \) and \( F_{11} = 1.5 \) \( N \), are given in Fig. 2c and Fig. 3b-c.

**CONCLUSION**

In this paper the synthesis of the nonlinear elastic characteristics of a one-side bonded chain system with two degrees of freedom is considered. For given complex periodic disturbance force \( F_1(t) \) and given biharmonic oscillation of the first mass system \( x_1(t) \) the following was derived: 1) the analytic expressions for the elastic characteristics system in the parametric form \( f_1(t) \) and \( f_2(t) \); 2) analytic expression of the multi-harmonic oscillations of the second mass system \( x_2(t) \). Graphic representation of the functions \( F(t), x_1(t), f_1(x_1), f_2(x_2) \) and \( x_2(t) \) are given for \( A_{11} = F_{11} = 0 \) and three values of amplitude of disturbance force \( F_{11} \).

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SINTEZA NELINEARNIH ELASTIČNIH KARAKTERISTIKA
U SISTEMU SA VIŠE STEPENI SLOBODE
PRI VIŠEHARMONIJSKOM OSCILOVANJU

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U fazi projektovanja raznih oscilatornih mašina, čiji organi pri vršenju funkcije treba da izvode određeno oscilatorno kretanje, vrši se sinteza jednog broja mehaničkih karakteristika [1-5].

U radu [5], korišćenjem Čebiševljevih polinoma, obrađena je sinteza nelinearnih elastičnih karakteristika u jednostrano vezanom lancanom sistemu pri sinfaznom harmonijskom oscilovanju masa sistema i za zadate proste periodične poremećaje.

U ovom radu, korišćenjem trigonometrijskih konačnih redova, razmatra se sinteza nelinearnih elastičnih karakteristika u jednostrano vezanom lancanom sistemu pri višeharmo

Ključne reči: trigonometrijski konačni redovi, sinteza nelinearnih elastičnih karakteristika, jednostrano vezani lancani materijalni sistem, biharmonijsko oscilovanje i složene periodične poremećajne sile.