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SYNTHESIS OF NONLINEAR ELASTIC CHARACTERISTICS OF A MULTI-DEGREE-OF-FREEDOM SYSTEM INDEPENDENCY IN CASE OF MULTI-HARMONIC OSCILLATIONS

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Abstract *In the phase of design of different machines, whose certain parts perform oscillation movements, the synthesis of a certain number of mechanic characteristics is carried out [1-5].*
In the paper [5] using the Chebishevliev`s polynomials, the synthesis of the nonlinear elastic characteristics was carried out for a one-side bonded chain system in the case of synphase harmonic oscillations of a mass system and for given simple periodic disturbances.
In this paper, using the trigonometric final series, the synthesis of the nonlinear elastic characteristics is considered for a one-side bonded chain system in the case of multi-harmonic oscillations of a mass system in the presence of complex periodic disturbance forces.

Key words: *trigonometric final series, synthesis of the nonlinear elastic characteristics, one-side bonded chain material systems, bi-harmonic oscillations, complex periodic disturbance forces.*

1. DIFFERENTIAL EQUAZIONS OF SYSTEM MOVEMENT

One-side bonded chain system with s degrees of freedom is being observed, where s – is a final number. Differential equations of motion of this system are:

$$\begin{aligned} &m_1 \ddot{x}_1 = -f_1(x_1) + f_2(x_2) + F_1(t), \\ &m_2(\ddot{x}_1 + \ddot{x}_2) = -f_2(x_2) + f_3(x_3) + F_2(t), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &m_s(\ddot{x}_1 + \ddot{x}_2 + \ldots + \ddot{x}_s) = -f_s(x_s) + F_s(t), \end{aligned} \tag{1}$$

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where: $m_1,m_2,...,m_s$ – is the mass the system; x_1 – the absolute coordinate of the rectilinear displacement; $x_2,x_3,...,x_s$ – the relative coordinates; $f_1(x_1),f_2(x_2),...,f_s(x_s)$ - the elastic characteristics (force); and $F_1(t),F_2(t),...,F_s(t)$ – the disturbance forces.

Elastic characteristics $f_r(x_r)$, $(r = 1,2,...,s)$ will be functions with a single meaning [2] if during the period of oscillations the folowing conditions are fulfilled $\dot{x}_r(t_i^*) = 0$; $\dot{f}_r\ (t_i^*) = 0,\ (i = 1,2,...,n)$, where n – is the number of the extremums of functions $x_r(t)$; t_i^* - is the point of an extreme of a function.

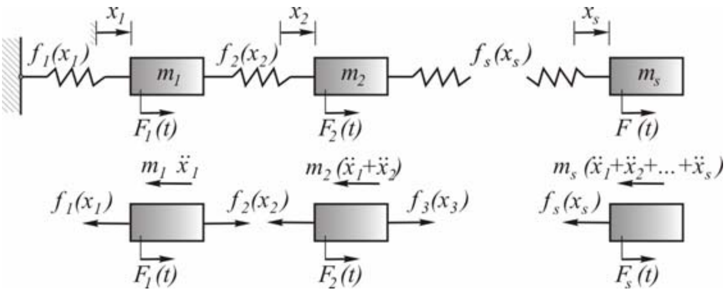

Fig. 1.

2. DISTURBANCE FORCES AND THE OSCILLATION LAWS. ELASTIC CHARACTERISTICS IN A PARAMETRIC FORM FOR A SYSTEM OF THREE AND TWO DEGREES OF FREEDOM

Let the complex periodic disturbance forces have an influence on a mass larticle system

$$F_r(t) = \sum_k F_{rk} \sin kt, \quad (r = 1,2,...,s;\ k = 1,3,5,...)\,, \tag{2}$$

and the system should move according to the following law

$$x_r(t) = \sum_k A_{rk} \sin kt, \quad (r = 1,2,...,s;\ k = 1,3,5,...)\,. \tag{3}$$

In the case of a one-side bonded chain system with three degrees of freedom, from equations (1), (2) and (3) the elastic characteristics in parametric form are derived.

$$\begin{aligned} f_1(t) &= F_1(t) + F_2(t) + F_3(t) - m_1\ddot{x}_1 - m_2(\ddot{x}_1 + \ddot{x}_2) - m_3(\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3), \\ f_2(t) &= F_2(t) + F_3(t) - m_2(\ddot{x}_1 + \ddot{x}_2) - m_3(\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3), \\ f_3(t) &= F_3(t) - m_3(\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3). \end{aligned} \tag{4}$$

These characteristics for a system with two degrees of freedom are:

$$\begin{aligned} f_1(t) &= F_1(t) + F_2(t) - m_1\ddot{x}_1 - m_2(\ddot{x}_1 + \ddot{x}_2), \\ f_2(t) &= F_2(t) - m_2(\ddot{x}_1 + \ddot{x}_2)\,. \end{aligned} \tag{4'}$$

3. ELASTICS CHARACTERISTICS OF A ONE-SIDE BONDED CHAIN SYSTEM WITH TWO DEGREES OF FREEDOM. NUMERIC EXAMPLES

Let us asume that the oscillation of the first mass system is given in the form

$$x_1(t) = A_{11}\sin t + A_{13}\sin 3t, \tag{5}$$

In the case of all values of the coefficients A_{11} and A_{13} it is needed to determine the nonlinear elastic characteristics $f_1(x_1)$ and $f_2(x_2)$ which ensure a multi-harmonic oscillation system.

We assume the function of a non-linear characteristic $f_1(x_1)$ in the form

$$f_1(x_1) = B_1 x_1 + B_3 x_1^3. \tag{6}$$

Substituting (5) into (6) the expression for the elastic characteristic of the first spring is obtained in the form of trigonometrical final series

$$f_1(t) = B_{11}\sin t + B_{13}\sin 3t + ... + B_{19}\sin 9t. \tag{7}$$

The oscillation law of a second mass system is taken in the same form

$$x_2(t) = A_{21}\sin t + A_{23}\sin 3t + ... + A_{29}\sin 9t. \tag{8}$$

Substituting (6-8) and (5) in a system of differential equations ($4'$), after certain mathematical operations we obtain

$$f_2(t) = B_{21}\sin t + B_{23}\sin 3t + ... + B_{29}\sin 9t. \tag{9}$$

From the equations ($4'$) we determine the formulae for the unknown coefficient – amplitude of the trigonometric final series B_{2k} and A_{2k} (k = 1,3,5,...,9):
for B_{2k} –

$$\begin{aligned}
B_{21} &= B_1 A_{11} + \frac{3}{4}B_3 A_{11}(A_{11}^2 - A_{11}A_{13} + 2A_{13}^2) - F_{11} - m_1 A_{11}\ , \\
B_{23} &= B_1 A_{13} + \frac{1}{4}B_3(-A_{11}^3 + 6A_{11}^2 A_{13} + 3A_{13}^3) - F_{13} - 9m_1 A_{13}\ , \\
B_{25} &= \frac{3}{4}B_3 A_{11}A_{13}(A_{13} - A_{11}) - F_{15}\ , \\
B_{27} &= -\frac{3}{4}B_3 A_{11}A_{13}^2 - F_{17}\ , \\
B_{29} &= -\frac{1}{4}B_3 A_{13}^3 - F_{19}\ ;
\end{aligned} \tag{10}$$

for A_{2k} –

$$A_{21} = \frac{1}{m_2}[B_1 A_{11} + \frac{3}{4}B_3 A_{11}(A_{11}^2 - A_{11}A_{13} + 2A_{13}^2) - (F_{11} + F_{21}) - (m_1 + m_2)A_{11}]\ ,$$

$$A_{23} = \frac{1}{m_2}[(B_1 A_{13} + \frac{1}{4}B_3(-A_{11}^3 + 6A_{11}^2 A_{13} + 3A_{13}^3))\frac{1}{9} - \frac{1}{9}(F_{13} + F_{23}) - (m_1 + m_2)A_{13}]\ ,$$

$$A_{25} = \frac{1}{m_2}[\frac{3}{4}B_3 A_{11} A_{13}(A_{13} - A_{11})\frac{1}{25} - (F_{15} + F_{25})\frac{1}{25}] \,, \tag{11}$$

$$A_{27} = \frac{1}{m_2}[-\frac{3}{4}B_3 A_{11} A_{13}^2 - (F_{17} + F_{27})]\frac{1}{49} \,,$$

$$A_{29} = \frac{1}{m_2}[-\frac{1}{4}B_3 A_{13}^3 - (F_{19} + F_{29})]\frac{1}{81} \,.$$

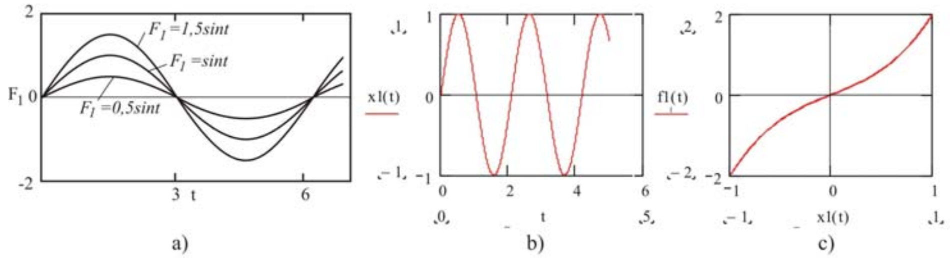

Fig. 2.

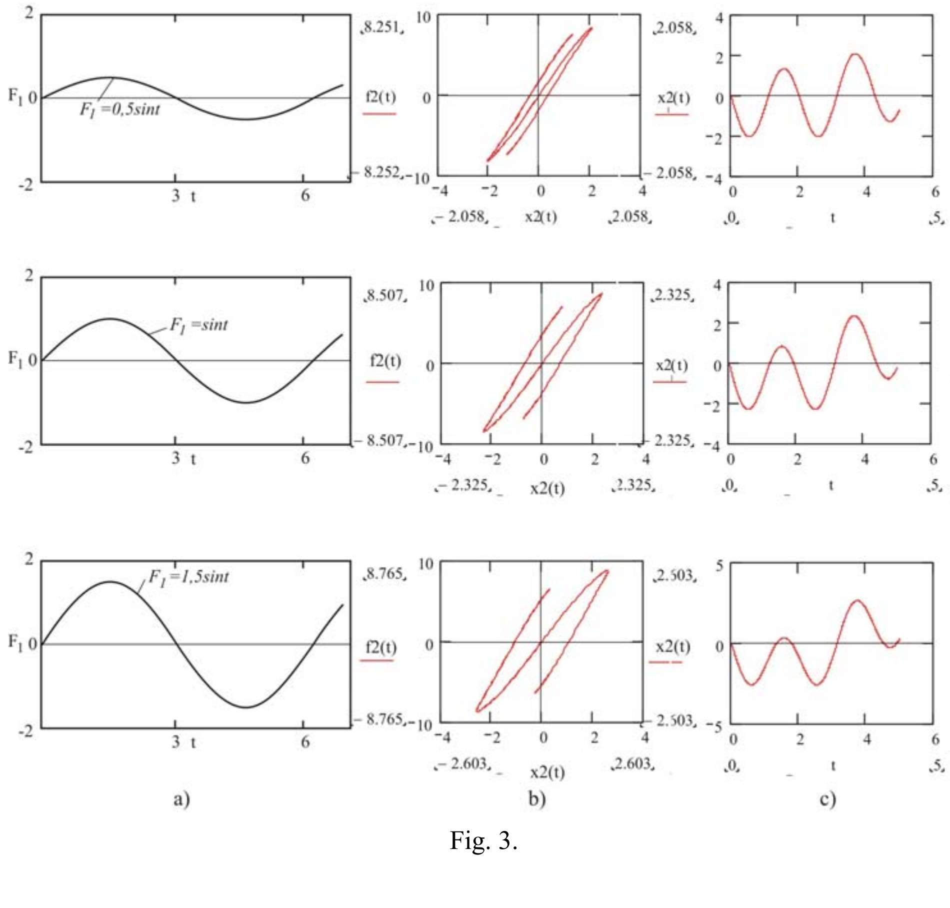

Fig. 3.

Let a disturbance force $F_1(t) = F_{11} \sin t$ have an impact on the mass m_1 and, in addition, this mass should perform the motion in acccordance with the low $x_1(t) = A_{13} \sin 3t$. It is needed to determine laws of a change of the elastic characteristics $f_2(t)$ and multi-harmonic oscillations $x_2(t)$ of a mass system m_2.

In this case the coefficients B_{2k} and A_{2k} ($k = 1,3,...,9$) will be

$$B_{21} = -F_{11}\,,\quad B_{23} = \frac{3}{4} B_3 A_{13}^3 - 9 m_1 A_{13}\,,\quad B_{25} = B_{27} = 0\,,\quad B_{29} = -\frac{1}{4} B_3 A_{13}\,;$$

$$A_{21} = -\frac{F_{11}}{m_2},\quad A_{23} = \frac{1}{9m_2}[B_1 A_{13} + \frac{3}{4} B_3 A_{13}^3 - 9(m_1 + m_2) A_{13}],\quad A_{25} = A_{27} = 0,\quad A_{29} = -\frac{B_3 A_{13}^3}{324 m_2}.$$

For $m_1 = m_2 = B_1 = B_3 = A_{13} = 1$ when numeric values of this variables are expressed in the units of the International system of measures, laws of the multi-harmonic oscillations and the matching elastic characteristics will be

$$x_1(t) = \sin 3t\,;$$

$$f_1(t) = x_1(t) + x_1^3(t) = \sin 3t + (\sin 3t)^3,$$

$$f_2(t) = -F_{11} \sin t - \frac{33}{4} \sin 3t - \frac{1}{4} \sin 9t;$$

$$x_2(t) = -F_{11} \sin t - \frac{65}{36} \sin 3t - \frac{1}{324} \sin 9t\ .$$

The graphic representations of this functions, for $F_{11} = 0{,}5$ N, $F_{11} = 1$ N and $F_{11} = 1{,}5$ N, are given in Fig. 2c and Fig. 3b-c.

Conclusion

In this paper the synthesis of the nonlinear elastic characteristics of a one-side bonded chain system with two degrees of freedom is considered. For given complex periodic disturbance force $F_1(t)$ and given biharmonic oscillation of the first mass system $x_1(t)$ the following was derived: 1) the analytic expressions for the elastic characteristics system in the parametric form $f_1(t)$ and $f_2(t)$; 2) analytic expression of the multi-harmonic oscillations of the second mass system $x_2(t)$. Graphic representation of the functions $F_1(t)$, $x_1(t)$, $f_1(x_1)$, $f_2(x_2)$ and $x_2(t)$ are given for $A_{11} = F_{13} = 0$ and three values of amplitude of disturbance force F_{11}.

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SINTEZA NELINEARNIH ELASTIČNIH KARAKTERISTIKA U SISTEMU SA VIŠE STEPENI SLOBODE PRI VIŠEHARMONIJSKOM OSCILOVANJU

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U fazi projektovanja raznih oscilatornih mašina, čiji organi pri vršenju funkcije treba da izvode određeno oscilatorno kretanje, vrši se sinteza jednog broja mehaničkih karakteristika [1-5].

U radu [5], korišćenjem Čebiševljevih polinoma, obrađena je sinteza nelinearnih elastičnih karakteristika u jednostrano vezanom lančanom sistemu pri sinfaznom harmonijskom oscilovanju masa sistema i za zadate proste periodične poremećaje.

U ovom radu, koričćenjem trigonometrijskih konačnih redova, razmatra se sinteza nelinearnih elastičnih karakteristika u jednostrano vezanom lančanom sistemu pri višeharmonijskom oscilovanju masa sistema i za složene periodične poremećajne sile.

Ključne reči: *trigonometrijski konačni redovi, sinteza nelinearnih elastičnih karakteristika, jednostrano vezani lančani materijalni sistem, biharmonijsko oscilovanje i složene periodične poremećajne sile.*