

**THE INFLUENCE OF NON-LINEAR EXCITATIONS  
OF ASYNCHRONOUS ELECTRIC MOTORS  
ON THE WORK OF DRIVING MECHANISMS OF CRANES**

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**Abstract.** *The paper considers two laboratory apparatuses driven by electric motors, used to analyze the work of driving mechanisms of cranes in the period of acceleration. In one case, a cage electric motor, while in the other a slide-ring three-phase asynchronous electric motor with non-linear characteristics of starting, is used. The movement of these mechanisms in the period of acceleration was analytically solved and simulated by means of program package MATLAB/Simulink on a PC, based on an elastic-kinetic model with two revolving masses. The results of these simulations are in high accordance with experimental notes, primarily due to non-linear modeling of the curves of starting the electric motors. This justifies the procedures of analytic modeling and simulating the work of driving mechanisms of cranes, with the goal of obtaining a number of relevant data in the process of their design, whereby expensive experimental measurements are evaded.*

1. INTRODUCTION

Three-phase asynchronous electric motors are the most frequently used as driving mechanisms of cranes. Distinction can be made between cage and slide-disc asynchronous electric motors (in further text, motors). The former are started directly, while the latter are started by switching on resistors in the rotor's circuit. Analysis of the work of driving mechanisms of cranes requires that the changes of curves of starting these motors be known. In the first case, with direct starting of cage motors, it is a natural characteristic, whereas in the second case, with slide-disc motors, it is the so-called 'saw diagram', whose changes are given as functions of the number of revolutions or of angular velocity  $M_M = f(n) = f(\dot{\phi} = \pi \cdot n/30)$ . Most frequently, these changes are non-linear functions. Earlier, for solving certain problems in the dynamics of driving mechanisms of cranes, linearization of these changes used to be performed [1,2,3].

Nowadays, however, when there are adequate program packages (software) for non-linear analysis of dynamic problems, they are taken in their non-linear form [4].

The paper analyses two laboratory driving mechanisms driven by electric motors. For selected shafts of the above-mentioned mechanisms, changes of torsion moments  $M_t = T(t)$  for the period of acceleration were determined analytically and experimentally. Analytic simulations were performed in MATLAB/Simulink with the help of a PC, while experiments were realized by tensiometric measurements. A simple equivalent elastic-kinetic model with two revolving masses, and with an exact non-linear description of the moment of starting the motors, was used in the first case. This is the reason why the results of analytic simulations are in high accordance with experimental notes.

## 2. METHODOLOGY OF MODELING THE ACCELERATION OF CRANE MECHANISMS

The first example of modeling crane mechanisms with non-linear excitation of motors is a driving mechanism of a laboratory apparatus, mass 4.0 t and velocity  $v = 1.08$  m/s, by means of which the movement of the bridge crane or of its trundle is simulated [1, 3]. Figure 1 shows this mechanism, which consists of a brake cage motor, type KBF-90 LA 2, power  $P_M = 1.0$  kW, revolutions per minute  $n_1 = 2510$  min<sup>-1</sup> and nominal moment  $M_n = 3.8$  Nm, a vertical three-step reducer with gear ratio  $i_R = 35.5$ , and a driving wheel with diameter  $D_T = 250$  mm.

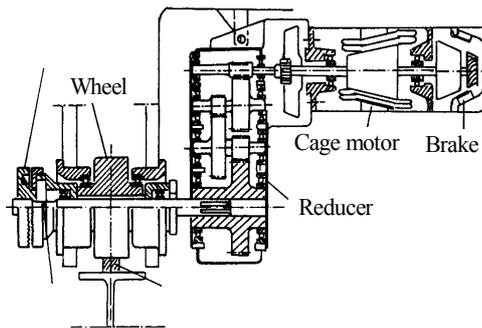


Fig. 1. Mechanism for moving the laboratory apparatus

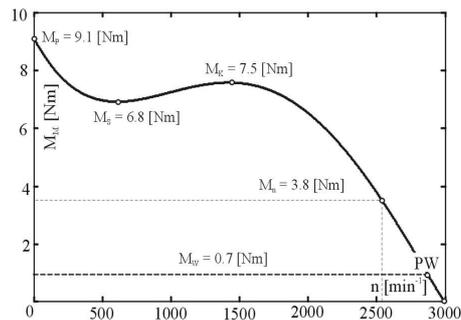


Fig. 2. Natural characteristic of the cage motor

In this example, the change of torsion moment  $M_t = T(t)$  at the wheel's shaft is analytically simulated and examined, for the period of acceleration during direct starting of the cage motor. The natural characteristic of this motor  $M_M = f(n) = f(\dot{\varphi} = \pi \cdot n/30)$  was modeled in MATLAB as a curve of the fourth order with defined characteristic points  $M_p$ ,  $M_s$ ,  $M_k$  and  $M_n$  (Fig. 2).

The first phase of modeling includes the selection of the equivalent dynamic model, which is a reduced elastic-kinetic model with only two revolving masses and one elastic constraint between these masses (Fig. 3). In this particular example, reduction was performed on the wheel's shaft [1, 3]. The first mass is the driving one with moment of inertia  $J_1 = 27.35$  kgm<sup>2</sup>, and which consists of all masses up to the mechanism's outlet shaft, while the other is the followed mass with  $J_2 = 62.50$  kgm<sup>2</sup>, composed of masses beyond the shaft. The elastic constraint is defined by coefficients of: rigidity  $c_1 = 24206$  Nm, damping  $b_1 = 14.0$  Nms, and gap  $\varphi_z = 0$ . The first driving mass is influenced by the

non-linear change of the motor's moment  $M_1 = M_M(\phi)/i_R$  (Fig. 2), whereas the second followed mass is influenced by constant resistance to motion  $M_2 = M_w = - 22.4$  Nm. Within this model, values  $\phi_i, \dot{\phi}_i, \ddot{\phi}_i$  represent generalized coordinates of the trajectory, velocity, and acceleration of the driving ( $i = 1$ ) and followed ( $i = 2$ ) masses.

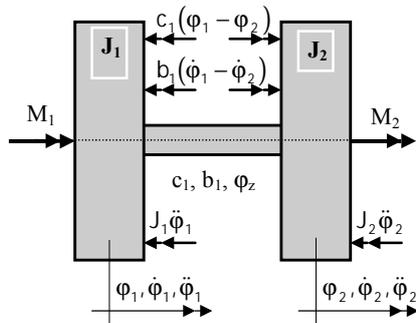


Fig. 3. Elastic-kinetic model with two revolving masses

etc.) is realized, on condition that basic features of real crane mechanisms are not lost. It is clear that two revolving masses are dominant in the case of this test board, which corresponds to the model with two degrees of freedom of motion (Fig. 3). The driving mass with moment of inertia  $J_1$  consists of all effective revolving masses in front of (to the left of) the measuring shaft (8), whereas the followed mass with moment of inertia  $J_2$  consists of all masses behind (to the right of) the measuring shaft. Elastic deformations of the measuring shaft are registered by means of measuring bands tied into Wheatston's bridge (8), i.e. changes in its torsion moment,  $M_t = T(t)$  [3], are recorded experimentally.

In the concrete example considered here, the adopted drive is a slide-ring motor, type IZMB 160 M2-4, power  $P_M = 7.5$  kW, number of revolutions  $n_1 = 1450 \text{ min}^{-1}$ , and nominal moment  $M_n = 49.40$  Nm. Its starting is performed by means of resistors in the rotor's circuit, so that five lines of change of moment  $M_M = f(n) = f(\phi)$  are obtained –the so-called 'saw' diagram (Fig. 5) [3]. This diagram was modelled on the basis of known data in MATLAB. It can be noticed that the change of the motor's moment along the first four lines is linear, whereas in the case of the final natural characteristic it is highly non-linear, described by a curve of the fourth order.

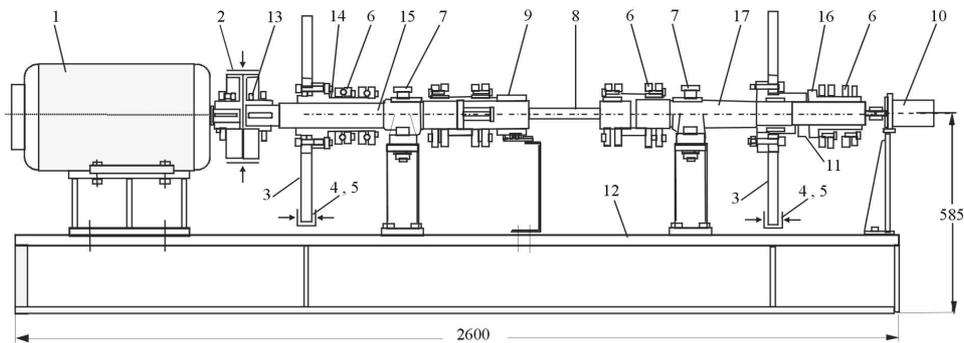


Fig. 4. Universal testing apparatus (board) for simulating the work of crane mechanisms

In this example, the characteristics of the two-mass elastic-kinetic model (Fig. 3), apart from the known change of the motor's moment  $M_1 = M_M = f(n) = f(\dot{\varphi})$  (Fig. 5), are: moments of inertia  $J_1 = 0.143 \text{ kgm}^2$ ,  $J_2 = 1.104 \text{ kgm}^2$ , coefficients of rigidity  $c_1 = 27700 \text{ Nm}$  and damping  $b_1 = 2.0 \text{ Nms}$ , gap  $\varphi_z = 0$ , work and resistance to motion on the followed mass  $M_2 = M_w = 8.0 \text{ Nm}$  [3].

Apart from the selection of the equivalent dynamic model, modelling of acceleration also involves setting an abstract mathematical model. For the elastic-kinetic model (Fig. 3) these are common non-homogenous differential equations of the second order. In the most general case, however, when there exists gap and the elastic constraint is balanced, acceleration of driving mechanisms can be divided into three stages of movement [3]:

- passing of the first driving mass through the gap,
- continuation of movement of the first driving mass and deformation of the elastic constraint until starting the second followed mass,
- asynchronous movement of both masses until the end of the period of acceleration, i.e. the achievement of the stationary work regime.

For the first period, i.e. passing of the driving mass as a rigid body through the gap, the differential equation of motion is:

$$J_1 \cdot \ddot{\varphi}_1 = M_1 = M_M = f(\dot{\varphi}) . \quad (1)$$

The solution of this differential equation, for known change of the motor's moment of starting  $M_M = f(\dot{\varphi})$  (Fig. 2 or Fig. 5) and initial conditions  $t_{po} = t_0$ ,  $\varphi_{10}(t_0) = -\varphi_z$ ,  $\dot{\varphi}_{10}(t_0) = 0$ , yields the laws of motion for the first driving mass  $\varphi_1(t)$ ,  $\dot{\varphi}_1(t)$  and  $\ddot{\varphi}_1(t)$ . This stage lasts a relatively short period of time ( $t_z$ ), culminating in elimination of the gap. The final values for  $t_{kr} = t_z$  are:  $\varphi_1(t_z) = 0$  and  $\dot{\varphi}_1(t_z) = \dot{\varphi}_{1z}$ ; these are the initial conditions for the next stage of motion.

When analyzing the second stage of motion, i.e. deformation of the elastic constraint, the elastic-kinetic model from Fig. 3 is transformed into a torsional oscillator, since the first mass, under the influence of excitation of the motor  $M_1 = M_M$  continues with motion and twists the elastic constraint, whereas the second mass stands under the influence of moment  $M_2 = -M_w$ . Thus the differential equation in this case is:

$$J_1 \cdot \ddot{\varphi}_1 + b_1 \cdot \dot{\varphi}_1 + c_1 \cdot \varphi_1 = M_1 = M_M = f(\dot{\varphi}) . \quad (2)$$

As in the first case, the solution of this equation for initial conditions  $t_{po} = t_z$ ,  $\varphi_{10}(t_z) = 0$ ,  $\dot{\varphi}_{10}(t_z) = \dot{\varphi}_{1z}$ , yields the laws of motion for the first driving mass  $\varphi_1(t_1)$ ,

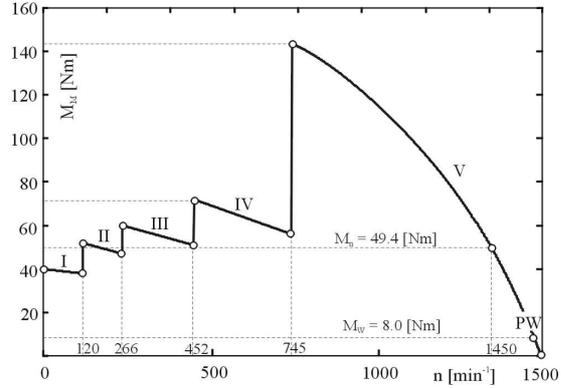


Fig. 5. "Saw" diagram of the slide-ring motor

$\dot{\varphi}_1(t_1)$  and  $\ddot{\varphi}_1(t_1)$ . It is now possible to define the change of torsion moment  $M_t = T(t)$  in the elastic constraint, i.e. the referent shaft where reduction was performed, as:

$$M_t = T(t) = c_1 \cdot \varphi_1. \quad (3)$$

This stage also last a short period of time ( $t_d$ ), and ends at the moment when the torsion moment in the elastic constraint  $M_t = T(t)$  (3) becomes equal to the moment of resistance to motion on the followed mass  $M_1 = M_w$ . Then the final values for  $t_{kr} = t_z + t_d$  are:  $\varphi_1(t_z + t_d) = \varphi_{1d} = M_w/c_1$  and  $\dot{\varphi}_1(t_z + t_d) = \dot{\varphi}_{1d}$ . These values are initial conditions for the next stage of motion, i.e. asynchronous movement of both masses in the period of acceleration. In practical analyses, gap  $\varphi_z = 0$  is frequently eliminated, which was done in the two previous examples; then the initial conditions for this stage are  $t_{po} = 0$  and  $\varphi_{10}(0) = \dot{\varphi}_{10}(0) = 0$ .

In the third and final stage of the system's acceleration, both masses move asynchronously, i.e. they perform oscillatory motion in relation to each other. According to Fig. 3, this motion is described by the following system of differential equations:

$$\begin{aligned} J_1 \cdot \ddot{\varphi}_1 + b_1 \cdot (\dot{\varphi}_1 - \dot{\varphi}_2) + c_1 \cdot (\varphi_1 - \varphi_2) &= M_1 = M_M(\dot{\varphi}_1) \\ J_2 \cdot \ddot{\varphi}_2 - b_1 \cdot (\dot{\varphi}_1 - \dot{\varphi}_2) - c_1 \cdot (\varphi_1 - \varphi_2) &= M_2 = -M_W = \text{const.} \end{aligned} \quad (4)$$

As in the previous case, the solution of the system of differential equations (4), for initial conditions  $t_{po} = t_z + t_d$ ,  $\varphi_1(t_z + t_d) = \varphi_{1d} = M_w/c_1$ ,  $\dot{\varphi}_1(t_z + t_d) = \dot{\varphi}_{1d}$ ,  $\varphi_2(t_z + t_d) = 0$  and  $\dot{\varphi}_2(t_z + t_d) = 0$ , yields the laws of motion for each particular mass  $\varphi_1(t)$ ,  $\dot{\varphi}_1(t)$ ,  $\ddot{\varphi}_1(t)$  - driving, and  $\varphi_2(t)$ ,  $\dot{\varphi}_2(t)$ ,  $\ddot{\varphi}_2(t)$  - followed. Then the change of the torsion moment of the elastic constraint, i.e. of the referent shaft  $M_t = T(t)$ , will be:

$$M_t = T(t) = c_1 \cdot (\varphi_1 - \varphi_2) = c_1 \cdot \Delta\varphi, \quad (5)$$

where  $\Delta\varphi = (\varphi_1 - \varphi_2)$  - is deformation of the elastic constraint.

This stage ends when angular velocities of masses reach the value of stationary velocity  $\dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}_w$ , i.e. when the line of the motor's curve of starting  $M_M = f(\dot{\varphi})$  intersects with the moment of resistance to motion  $M_w$  (workpoint WP in Fig. 2 i Fig. 5). If the time of this stage is marked with  $t_a$ , then the complete time of the system's acceleration is  $t_u = t_z + t_d + t_a$ . Throughout stationary motion, the deformation of the constraint  $\Delta\varphi = (\varphi_1 - \varphi_2)$  remains unchangeable, i.e. the shaft's torsion moment  $M_t = T(t) = M_w = \text{const}$ . The case where at the beginning of movement the constraint is already deformed can frequently be met in practice, during the analysis of the work of crane mechanisms. In this case, the first two stages do not exist, and the third stage of motion begins immediately. Deformation of constraint is then  $\Delta\varphi = (\varphi_1 - \varphi_2) = -M_2/c_1 = M_w/c_1$ , so that initial conditions are:  $t_{po} = t_0 = 0$ ,  $\varphi_1(0) = 0$ ,  $\dot{\varphi}_1(0) = 0$ ,  $\varphi_2(0) = M_w/c_1$  and  $\dot{\varphi}_2(0) = 0$ .

The previously postulated non-homogenous differential equations of the second order (1), (2) and (4) were solved in the closed form when the motor's excitations

$M_1 = M_M = f(n) = f(\dot{\varphi})$  were linearized, by substituting the non-linear parts with rectilinear sectors, as had already been done in papers [1, 2, 3]. Nowadays, however, this type of problem can be solved quickly and efficiently in the non-linear domain by applying MATLAB/Simulink software [4].

### 3. SIMULATIONS, EXPERIMENTS, AND ANALYSIS OF OBTAINED RESULTS

A simulation is a procedure for imitating and analyzing the work of real systems, with the aid of its equivalent and mathematical models, so as to create real systems with the most favourable technical, technological, and economic indexes. This paper simulates changes of the torsion moment  $M_t = T(t) = c_1 \cdot \Delta\varphi$  (3, 5) of referent shafts on driving mechanisms (Fig. 1 and Fig. 4) in the period of acceleration under the influence of two types of asynchronous motors (Fig. 2 and Fig. 5). The simulations were made in the non-linear domain by solving differential equations (1, 2, 4), with the aid of a PC and MATLAB/Simulink software, with graphic interpretation of those changes. For these two driving mechanisms, experimental records of the changes were also made, in order to compare and confirm the adequacy of the analytic modelling procedure [3]. The results of these researches, without the participation of gap in the system, are shown in the following figures in parallel.

Figure 6 shows the simulation and experimental record of the change of the torsion moment  $M_t = T(t)$  of the output shaft on a driving mechanism (Fig. 1) with a cage motor (Fig. 2). Figure 7 shows this type of change on the driving shaft of the laboratory apparatus (Fig. 4) in the period of acceleration under the influence of a slide-ring motor with five starting lines (Fig. 5). In both cases, comparison of simulations and experimental records confirms their high accordance, which is sufficient for analysis and practical application. It must be pointed out that, for these two examples, the same simulations were performed earlier in papers [1, 2, 3], but with linearized motor excitations; in these cases, a high degree of accordance with experiments was also met.

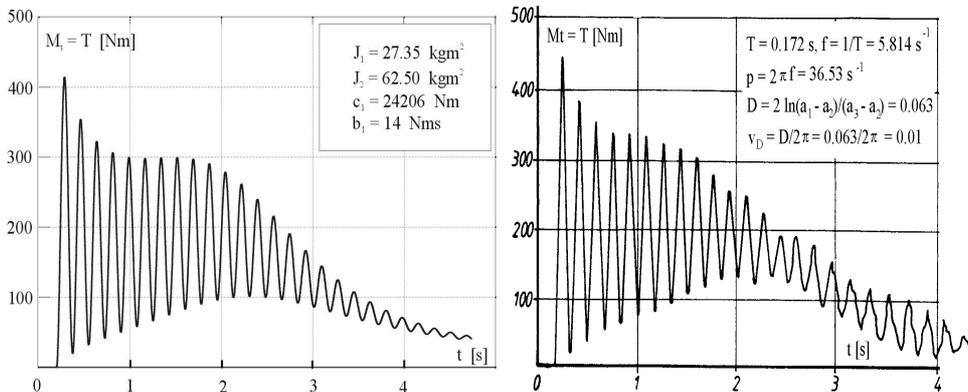


Fig. 6. Simulation and experimental record of change  $M_t = T(t)$  of the outlet shaft of the mechanism for moving with a cage motor in the period of acceleration

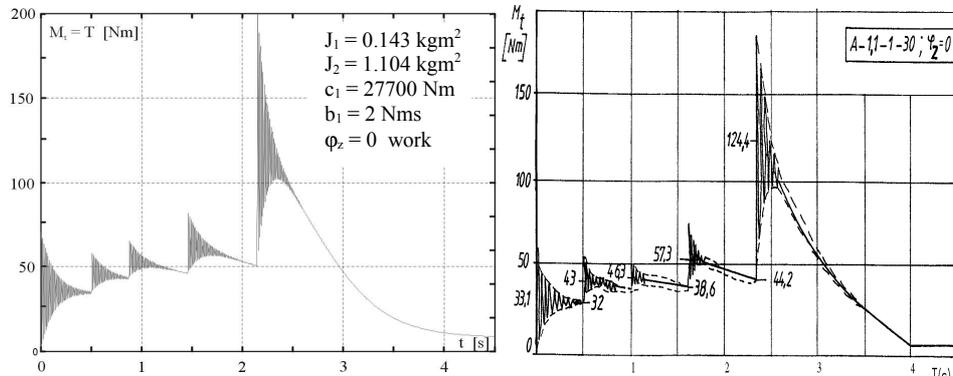


Fig. 7. Simulation and experimental record of change  $M_t = T(t)$  of the driving shaft of the testing apparatus with a slide-ring motor in the period of acceleration

Analytic simulations obtained in this way provide good grounds for a relatively cheap and quick analysis of the work and performance of driving mechanisms of cranes and of other driving systems driven by electric motors –naturally, without recourse to expensive experiments. Thus it can be noticed, on the basis of analysis of the given simulations, that the first change of the torsion moment  $M_t = T(t)$  of referent shafts is continual, while the other is abrupt, due to the application of different types of asynchronous motors. These changes consist of a so-called low-frequency component, which stems from the system's inertia, and an oscillatory high-frequency component, which stems from elastic characteristics of the system. The damped character of these changes is a consequence mainly of the negative inclination of the lines of starting the motors, rather than of system damping in the mechanisms. This can be noticed the most easily on the slide-ring motor (Fig. 7). Realization of a large number of simulations by varying the influential factors enables a highly qualitative analysis of the dynamic performance of driving mechanisms of cranes. There are two approaches to solving this type of problem. Pri rešavanju ovakvih problema ponudena su dva pristupa [3]:

- single-factor ("simple") simulation experiment, where only one influential factor with several concrete values varies, while the other values remain constant,
- multiple-factor ("complex") simulation experiment, where several influential factors vary simultaneously, by means of Box-Wilson's methodology.

It is manifest that nowadays, with the aid of such methodology and the resources available within computer and informational technology, various complex tasks in the field of non-linear dynamic analysis of driving mechanisms of cranes driven by electric motors can be solved successfully, quickly, qualitatively, and at relatively low costs.

#### 4. CONCLUSION

On the basis of what has been stated above, the following conclusions can be formulated:

- modelling of complex crane mechanisms is successfully realized by means of a simple elastic-kinetic model with two revolving masses and adequate non-homogenous differential equations of movement of the second order,

- the quality of modelling is achieved through a more accurate description of the model's characteristics, especially of non-linear excitations which stem from asynchronous motors, and through breaking down the mechanism's period of acceleration into several stages of motion (passing through the gap, deformation of the elastic constraint, and asynchronous movement of masses),
- modelling non-linear excitations of motors and solving differential equations, so as to obtain simulations of the laws of motion  $\varphi_i(t)$ ,  $\dot{\varphi}_i(t)$ ,  $\ddot{\varphi}_i(t)$  and the change of the torsion moment  $M_t = T(t)$  of crane mechanisms, were efficiently realized by MATLAB/Simulink software and are in high agreement with experimental records,
- the application of analytic simulation of the work of crane mechanisms is absolutely justified, since varying the influential values enables a more thorough analysis of these systems and the selection of optimal solutions without recourse to expensive experiments..

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### UTICAJ NELINEARNIH POBUDA ASINHRONIH ELEKTROMOTORA NA RAD POGONSKIH MEHANIZAMA DIZALICA

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*U radu su razmatrana dva laboratorijska uređaja na elektromotorni pogon, kojima se analizira rad pogonskih mehanizama dizalica u periodu ubrzanja. U jednom slučaju koristi se kavezni a u drugom kliznokolutni trofazni asinhroni elektromotor sa nelinearnim karakteristikama puštanja u rad. Kretanje ovih mehanizama u periodu ubrzanja je na bazi elasto-kinetičkog modela sa dve obrtne mase analitički rešavano i simulirano programskim paketom MATLAB/Simulink na PC računaru. Rezultati ovih simulacija su veoma saglasni sa eksperimentalnim zapisima, pre svega zahvaljujući nelinearnom modeliranju krivih puštanja u rad elektromotora. Ovim se opravdavaju postupci analitičkog modeliranja i simuliranja rada pogonskih mehanizama dizalica u cilju dobijanja niza relevantnih podataka u procesu njihovog projektovanja, čime se zaobilaze skupa eksperimentalna merenja.*