Invited Paper

COUPLED FIELD PROBLEMS IN ELECTRICAL APPARATUS
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Abstract. *The paper gives brief overview and classification of the coupled problems encountered in electrical apparatus. Three particular problems are considered in more details. These are the problems for coupling electromagnetic field with electric circuit and thermal field, with electric circuit and mechanical motion, and with mechanical motion and electric arc rotation. Results are given for different parameters.*

INTRODUCTION

In order to model and design real devices it is very often necessary to solve coupled field problems. In electrical apparatus, the electromagnetic field is the main one and it always exists. The coupled problems encountered in electrical apparatus are most often coupling the electromagnetic field with:
- thermal field;
- electric circuit;
- mechanical motion;
- stress analysis problems;
- fluid dynamics problems.

Very often more than one problem has to be considered, e.g., the electric circuit is existing in almost all apparatus.

In the present paper, some examples of these problems are considered in more details and some results are presented.

COUPLED PROBLEM ELECTROMAGNETIC FIELD – ELECTRIC CIRCUIT – THERMAL FIELD

Coupled electromagnetic-thermal problems are of particular interest for induction heating systems [1-3] but usually most electromagnetic systems involve coupled

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electromagnetic and thermal processes as well. Induction motors are also subject of interest in this field [4-6].

The steady state of electromagnetic contactors is also a case where electromagnetic field, electrical circuit and thermal field problems have to be solved as a coupled problem as they influence each other. As usually their geometry is typically three-dimensional, the corresponding analysis should be carried out in three dimensions. Contactors have been studied mainly from electromagnetic and mechanical viewpoints [8-12].

An approach for solving the electromagnetic-thermal coupled problem of an AC contactor with the help of geometrical CAD system for data input is presented in [13].

The electromagnetic field of the contactor electromagnet (Fig. 1) is governed by the equation

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) + \sigma(T) \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right) = \mathbf{J}_e \ , \tag{1}$$

where μ is magnetic permeability;

$\sigma(T)$ is temperature dependent electrical conductivity, denoted below as σ for brevity;

$\mathbf{J}_e$ is the current density in the coil;

V is the electric scalar potential;
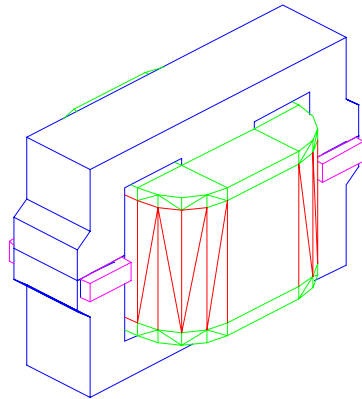
$\mathbf{A}$ is the magnetic vector potential.

Fig. 1. General view of the electromagnet

The potentials $\mathbf{A}$ and V are connected with the gauge

$$\nabla.\mathbf{A} + \mu\sigma V = 0 . \tag{2}$$

If the problem is treated as a linear one and after taking (2) into account, (1) could be written as

$$\nabla^2 \mathbf{A} - \mu\sigma \frac{\partial \mathbf{A}}{\partial t} + \mu \mathbf{J}_e = \mathbf{0} \tag{3}$$

Equation (3) has to be solved together with the electric circuit equation for the coil:

$$u = R(T)\, i_c + \frac{d\Psi}{dt} \tag{4}$$

where u is the supplied voltage;

$R(T)$ is the resistance of the coil;

i_c is the coil current;

ψ is the flux linkage.

Time-harmonic approximation of the problem is adopted. After assuming constant current density over the cross-sectional area of the coil, the following equations for the electromagnetic field and the circuit are obtained:

$$\frac{1}{\mu}\nabla^2 \mathbf{A} - j\omega\sigma\mathbf{A} + \frac{IN}{S_c}\mathbf{l}_c = \mathbf{0} \tag{5}$$

$$U = RI + j\omega \frac{N}{S_c} \int\limits_{V_c} \mathbf{A}\mathbf{l}_c dV \; , \tag{6}$$

where N is the number of coil turns;

S_c, V_c - cross section and volume of the coil;

$\mathbf{l}_c$ - unit vector in the direction of the winding up of the coil.

The thermal field at steady-state is governed by the Poisson's equation with respect to the temperature T

$$\nabla^2 T = -\frac{Q}{\lambda} \; , \tag{7}$$

where Q is the heat source defined by the electromagnetic losses in the electromagnet, i.e., by the solution to the field-circuit problem (5)-(6); λ is the thermal conductivity.

The coupling of the two fields is in both directions. The heat source is dependent on the results of the electromagnetic field analysis. On the other hand, as the electrical conductivity is temperature dependent, its value depends on the results of the solution to the thermal problem. Indirect coupling between the two problems (field-circuit and thermal one) has been utilized. The two problems are solved successively and after each solution of the thermal field the values of the conductivity in (5) and the resistance in (6) are updated. The flow-chart of the approach is shown in Fig. 2.
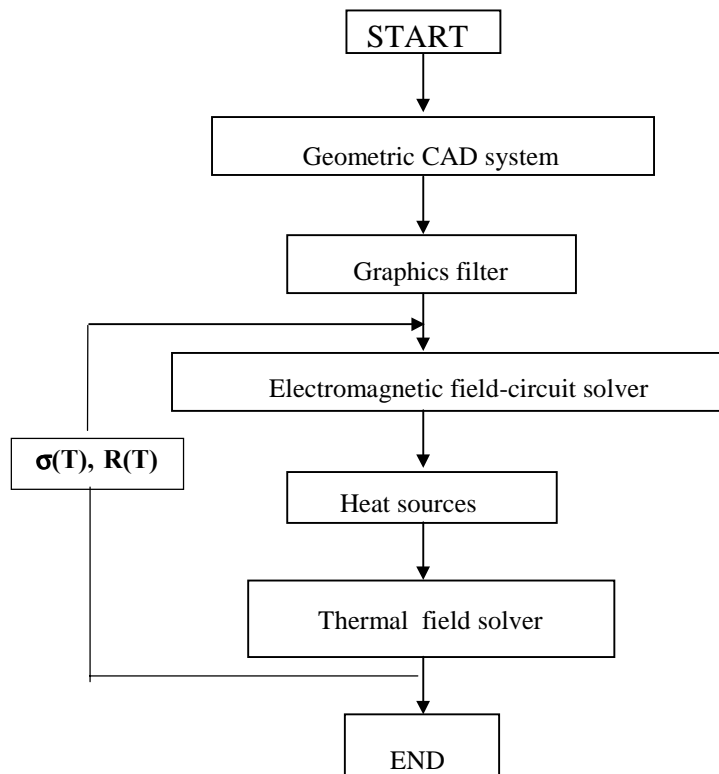

Fig. 2. General flow-chart of the approach

   Finite element method is used to solve both electromagnetic and thermal problem. A quarter of the electromagnet (Fig. 3) has been modelled in order to take advantage of the symmetry.
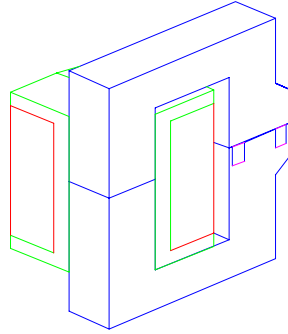

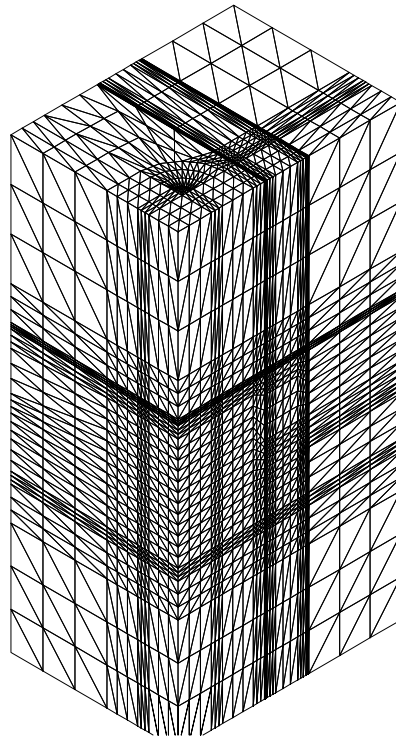Fig. 3. A quarter of the electromagnet

Fig. 4. FE mesh for the electromagnetic problem

   For the electromagnetic problem surrounding buffer zone is introduced and homogeneous Dirichlet's boundary conditions are imposed on its boundary. On the planes of symmetry either homogeneous Dirichlet's or natural boundary conditions are imposed. The mesh for this problem is shown in Fig. 4.

For the thermal problem buffer zone is not introduced and convection boundary conditions are imposed on the outer surface of the electromagnet. On the planes of symmetry natural boundary conditions are used. The mesh for the thermal problem is shown in Fig. 5.
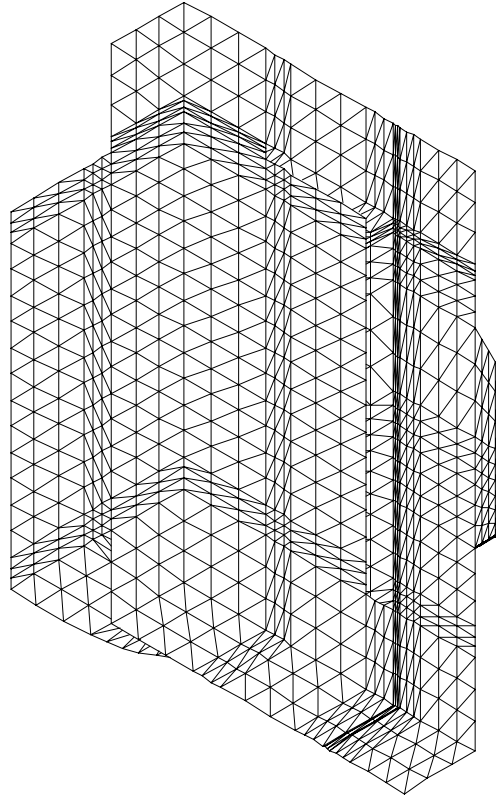

Fig. 5. FE mesh for the thermal problem

First order tetrahedral elements are used. The mesh generation is realized on the basis of hexahedral macroelements.

Having in mind that solving such problems could be an important part of CAD for contactors, the input to the FE programme is performed in the following way. Firstly, geometric CAD system is used for creating geometry of the electromagnet. The system employed for this purpose was AutoCAD. Then the geometrical model is transferred to the FE programme. To do this, DXF file format has been used and the corresponding filter has been developed and included into the FE programme. This approach gives the designer an opportunity of coupling the device design with FE analysis.

Finite element processor is realized in conventional way. For solving the systems of linear equations biconjugate gradient method with preconditioning for sparse systems is used.

Results are obtained for the electromagnet of a conventional contactor of type K6E fed by sinusoidal voltage of 220 V rms value. The values for the coil current Ic and the

coil temperature θc are computed and experiment has been carried out for verification. The computed and measured results are given in Table 1.

Table 1. Computed and measured values of the current and the temperature of the coil

	Computed value	Measured value
Coil current Ic, A	49.2	47.4
Coil temperature θc, °C	69.1	65.0

The difference is mainly due to the core nonlinearity and the residual air gap that are not taken into account in the computations.

COUPLED PROBLEM ELECTROMAGNETIC FIELD – ELECTRIC CIRCUIT – MECHANICAL MOTION

A coupled problem electromagnetic field - electric circuit - mechanical motion has been considered and solved for a solenoid actuator. Solenoid actuators are widely spread devices. Prediction of their dynamic characteristics is of major importance for their design and industrial applications.

Finite element method is mainly used for field modeling. As field - circuit coupling is well worked out during the past two decades, two major problems are significant - modelling of the motion in finite element analysis and coupling to the dynamic equations.

A technique of coupling two independent meshes (stationary and moving) based on Lagrange multipliers is presented in [14]. Re-meshing at each time step is thus avoided. The approach is suitable for constant air-gap problems and does not take into account eddy currents.

In [15] the problem of dynamics of a solenoid actuator for circuit breaker is studied using decoupling technique. First only electromagnetic field has been analysed for a wide range of values of the current and armature displacement. For each case the electromagnetic force is computed and stored in a database. Then the circuit and dynamic equations are solved employing electromagnetic force from the database. Eddy currents are not taken into account.

A strong coupled model solving simultaneously electric circuit, magnetic field and mechanical motion equations is presented in [16]. Edge element formulation in terms of magnetic vector potential is used for field modelling. The electromagnetic force is calculated by the virtual work method. Numerical example for an electromagnetic relay is given together with comparison to sequential coupling model.

Dynamics of two electromagnetic actuators is simulated in [17] and [18]. The field is modelled as axisymmetrical. Crank-Nicholson method is used for time discretization of field equation. An iterative procedure for evaluation of the electromagnetic force and the displacement at each time step is proposed in [18].

In [19], after summarising different approaches for motion modelling, a new one is proposed. It is based on incomplete shape functions and dividing the air gap on integration sectors. Thus the problem of coupling the two stationary and the moving mesh is solved. The approach could be used in problems with constant air gap.
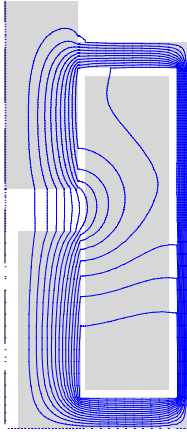
Fig. 6. Geometry of the actuator with field plot

An approach for solving the problem of dynamics of a solenoid actuator based on the backward Euler method for time discretization of all coupled equations is presented in [20]. The geometry of the actuator is shown in Fig. 6

The problem of dynamics of the actuator is a coupled problem electromagnetic field - electric circuit - mechanical motion. Therefore the governing equations for the three component problems should be included in the mathematical model:

Due to the axial symmetry of the actuator, the field is assumed to be axisymmetrical as well. Cylindrical coordinate system (r, φ, z) is used. The motion is in axial, i.e., z-direction.

The governing equations for each of these three problems are:

• Electromagnetic field equation:

$$\nabla \times (\nu \nabla \times \vec{A}) - \vec{J}_e + \sigma \frac{\partial \vec{A}}{\partial t} - \sigma \vec{v} \times \vec{B} = 0 , \tag{8}$$

where $\vec{A}$ is magnetic vector potential having only one nonzero component (A_φ);

ν is reluctivity;

$\vec{J}_e$ is current density of the external sources (i.e. in the coil);

σ is electrical conductivity;

t is time;

$\vec{v}$ is velocity of the moving part;

$\vec{B}$ is flux density;

μ_0 is magnetic permeability of the free space;

• Electric circuit equation:

$$u = Ri + \frac{d\Psi}{dt} , \tag{9}$$

where u is the supplied voltage;

R is the resistance of the coil;

Ψ is the flux linkage of the coil;

i is the current in the coil.

• Mechanical motion equation (force equation):

$$m\frac{d^2\zeta}{dt^2} + \beta\frac{d\zeta}{dt} = F_{em} - F_{load} , \tag{10}$$

where m is the mass of the moving part;

ζ is the displacement of the moving part (relative to the initial position);

β is damping coefficient;

F_{em} is the electromagnetic force;

F_{load} is the load force

(spring force ± weight of the mover – the actuator is in vertical position).

As the motion is only in *z*-direction, only *z*-component of the velocity is non-zero. After taking into account that $\vec{B} = \nabla \times \vec{A}$, carrying out vector operations in cylindrical coordinate system, and expressing the current density in terms of the coil current, Eqn. (1) could be presented in the form (vector notation is omitted as all resulting vectors have only one component)

$$\frac{\partial}{\partial r}\left(\frac{\nu}{r}\frac{\partial A^{\psi}}{\partial r} \right) + \frac{\partial}{\partial z}\left(\frac{\nu}{r}\frac{\partial A^{\psi}}{\partial z} \right) + \eta_j \frac{N_c i}{S_c} - \frac{\sigma}{r}\frac{\partial A^{\psi}}{\partial t} + \frac{\sigma}{r}\nu\frac{\partial A^{\psi}}{\partial z} = 0 ,\qquad (11)$$

where A^{ψ} is modified magnetic vector potential ($A^{\psi} = rA$);

N_c is the number of turns of the coil;

S_c is the c.s.a. of the coil;

η_j is parameter for correspondence between the point the equation is applied to and the cross section of the coil:

$$\eta_j = \begin{cases} 1 & \text{if the point belongs to the coil cross section} \\ 0 & \text{otherwise} \end{cases}$$

The flux linkage can be written using the modified vector potential. Then Eqn. (9) becomes

$$u - Ri - \frac{2\pi N_c}{S_c}\int_{S_c}\frac{\partial A^{\psi}}{\partial t}dS_c = 0 .\qquad (12)$$

To reduce the order of derivatives in the force equation (10), it is split into two equations:

$$m\frac{d\nu}{dt} + \beta\nu - F_{em} + F_{load} = 0 \qquad (13)$$

$$\frac{d\zeta}{dt} - \nu = 0 . \qquad (14)$$

The system of equations to be solved is (11)-(14), unknown functions are A^{ψ} *i*, v and ζ. This system should be solved under the corresponding initial and boundary conditions. Homogeneous Dirichlet's boundary conditions are imposed on the boundary of a buffer zone around the actuator, as well as to the axis of symmetry. All initial conditions are taken to be zero – this means that there hasn't been voltage supply and movement before the initial moment.

The motion is modelled using re-meshing of the zone between the stationary and the moving part. The mesh of the stationary part is fixed and is not changed during the time. The mesh of the moving part also remains unchanged in time with respect to a reference frame fixed to the moving part. Thus the velocity term in the field equation can be omitted. Mesh regeneration is carried out at each value of the displacement in a way

ensuring minimal distortion of the elements in the deformed air region. The data for the initial mesh are transferred from a general purpose CAD system.

Finite element method and authors' software has been employed for field analysis. Nodal formulation in terms of modified magnetic vector potential is used. Circuit equation is incorporated into the FEM system. Non-linearity is handled by the Newton-Raphson method. The resulting linear system of equations at each non-linear iteration is solved by bi-conjugate gradient method with preconditioning. The electromagnetic force is computed using Maxwell stress tensor and three integration paths.

For integration in time of the whole system (11)-(14), backward Euler method is used. Thus stability of the solution is obtained which is not always achievable in the case of forward schemes.

Due to the implicit formulation, at each time step successive solutions of field-circuit and mechanical motion problems are carried out until the desired accuracy is obtained.

Automatic control of the time step is ensured in order to fulfil the geometrical constraint for the displacement.

The actuator has been studied during the transient process of switching. The supplied voltage was 56 V, the resistance of the coil – 255 Ω, number of turns – 5800, initial air gap – 8.2 mm, final air gap – 2.5 mm. The spring force value at the initial air gap is 2.6 N; the spring coefficient – 670 N/m, the mass of the mover - 0.16 kg. Total number of FE nodes has varied between 5515 at the initial gap and 5053 at the final gap.

The computed and experimental results for the current in the coil are shown in Fig. 7 and Fig. 8, respectively.
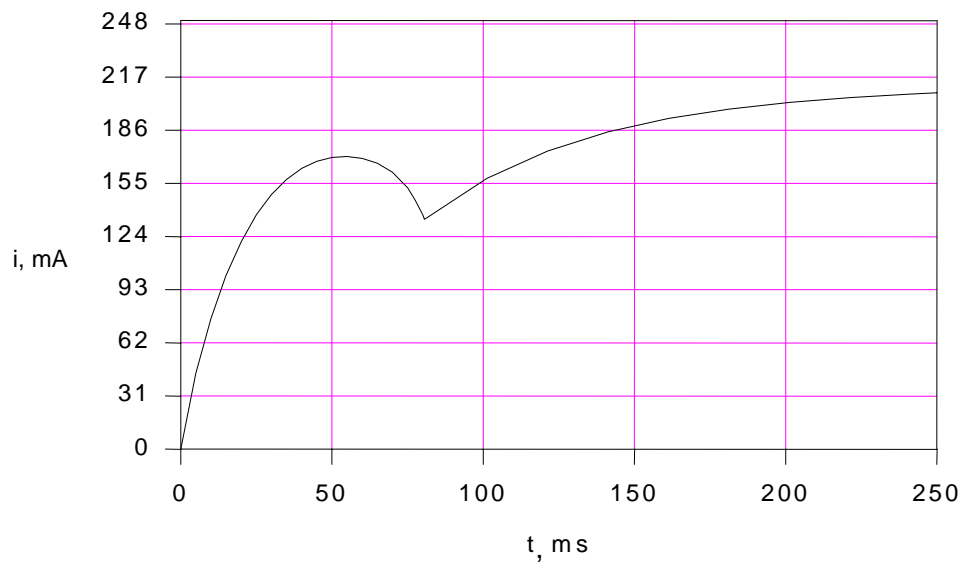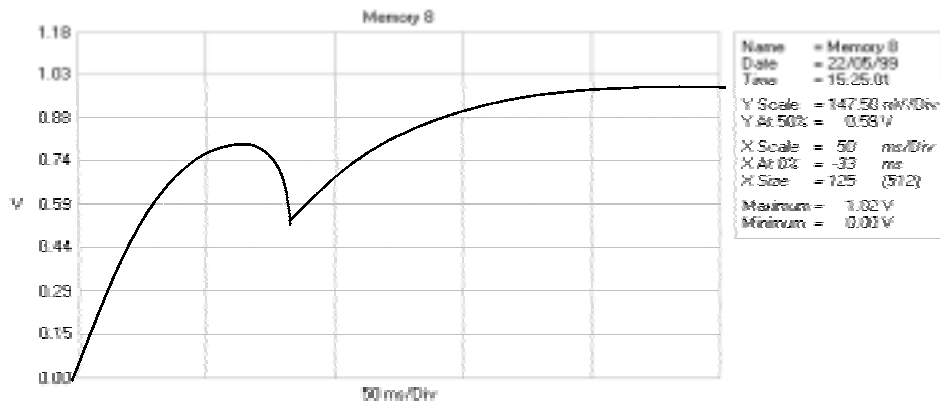

Fig. 7. Computed current waveform

Fig. 8. Measured current waveform (1V corresponds to 210 mA)

The computed and measured currents are close to each other and the time for reaching the final air gap is practically the same in the two curves.

More results have been obtained for other dynamic characteristics of the actuator. These are electromagnetic force, displacement and velocity as a function of time.

COUPLED PROBLEM ELECTROMAGNETIC FIELD – MECHANICAL MOTION – ELECTRIC ARC ROTATION

A particular coupled problem is considered for rotating arc SF6 circuit breaker in [21]. The process of interaction between the magnetic field and the electric arc rotating in fluid during the current interruption has been studied. The problem has been solved for a particular SF6 circuit breaker of geometry shown in Fig. 9.

The problem is solved taking into account the movement of the moving contact under the following conditions and assumptions:

- the magnetic field is axisymmetrical and it is excited by a series coil with a core of tube shape;
- the waveform of the current is known;
- the law of the movement of the moving contact is known;
- the axis of the arc is straight line and the arc is rotating in such a way that its axis lies on one plane with the axis of symmetry;
- the influence of the arc on the magnetic field is neglected.

The magnetic field is governed by Eqn. (8) in cylindrical co-ordinate system. It is solved under initial and boundary conditions. The initial conditions are zero conditions for the magnetic vector potentials as magnetic field is initiated after the start of the interrupting process. The current flowing before interruption excites magnetic field only in azimuthal direction. The magnetic field problem is solved under homogeneous Dirichlet boundary conditions on the boundary of a buffer zone around the extinguishing chamber of the circuit breaker.
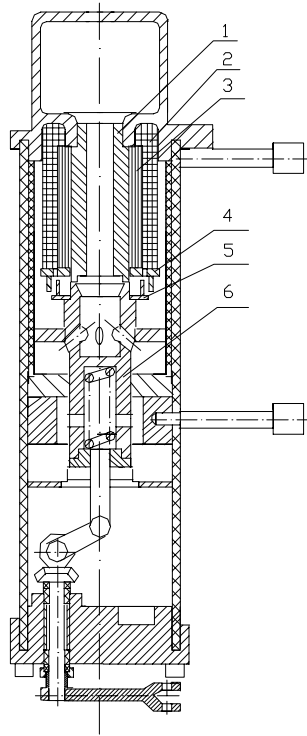
Fig. 9.   Principal geometry of the studied circuit breaker: 1 – fixed main contact;
          2 – coil; 3 – magnetic core; 4,5 – arcing rings; 6 – moving main contact

Time stepping and backward Euler method are used for solving the problem. The solution process starts from the moment of transferring the arc from the main to the arcing contacts. The time for this moment is obtained from the law of movement of the moving contact. At the same moment the current is transferred to the coil. The magnetic field analysis is carried out using the finite element method. As there is a movement, at each time step the mesh of the moving parts is moving and new elements introduced with the extension of the gap between contacts. The system of non-linear equations is solved using relaxation method with underrelaxation with respect to the magnetic permeability. The electromagnetic force acting on the arc is determined by the Laplace's law.

The results are obtained for 15 kA rms value of sinusoidal interrupting current. The same approach can be used for other waveforms of the current. In Fig. 10, the results for the flux density and the electromagnetic force acting on the arc are shown. The eddy currents in the conductive parts cause a phase shift between the current and the flux density components and a decrease of the magnitude of the electromagnetic force. Due to the movement of the moving contact the magnitudes of the components of the average flux density in the arc zone descend while in the case of fixed contacts they remain the same.
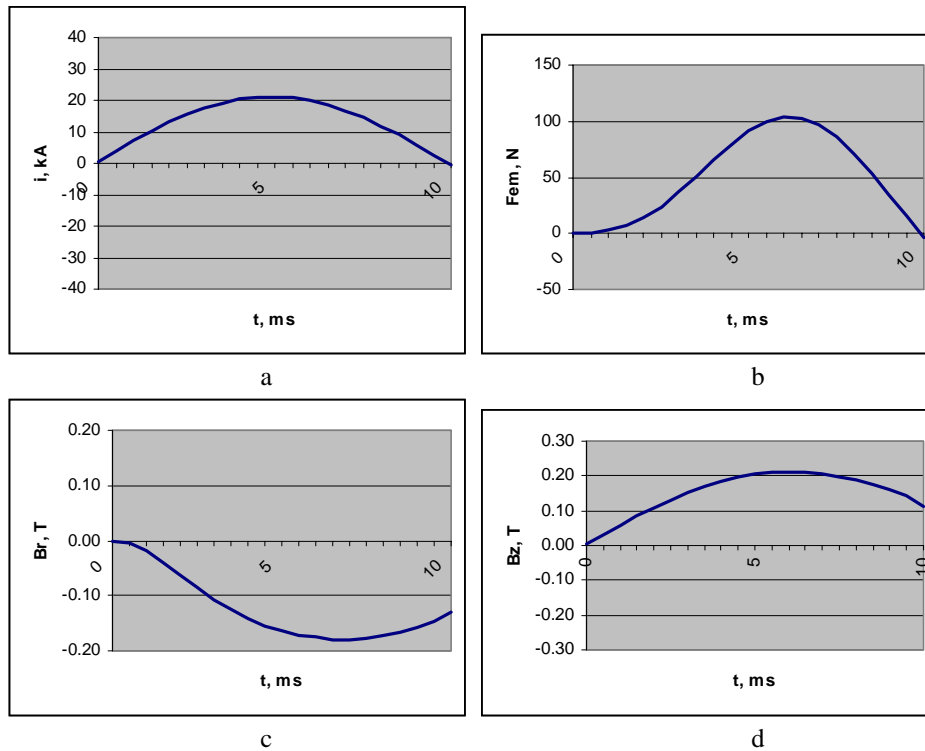
Fig. 10. Waveforms of: a) interrupting current; b) electromagnetic force acting on the arc;
c, d) flux density components in the arc zone

CONCLUSION

Several coupled problems encountered in electrical apparatus are presented, mainly of electromechanical nature. The solution of coupled problems is of major importance for the adequate modelling and design of electrical apparatus. Although a lot of research efforts have been employed in this area, there are still problems that are open to be solved.

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PROBLEMI SPREGNUTIH POLJA
KOD ELEKTRIČNIH APARATA

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Prikazan je kratak pregled i klasifikacija problema spregnutih polja kod elektricnih aparata. Tri posebna spregnuta polja su posmatrana detaljnije: elektromagnetno polje, polje električnih kola i temperaturno polje, kada se sistem kreće i kada je elektromagnetno polje obrtno. Prikazani su rezultati za razlicite vrednosti uticajnih parametara.