FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol.3, N° 15, 2003, pp. 1073 - 1076

DYNAMIC RESPONSE OF NONLINEAR SYSTEMS UNDER STATIONARY HARMONIC EXCITATION

UDC 534.01

Polidor Bratu

Member of the Romanian Academy for Technical Sciences
Research Institute for Construction Equipment and Technology-ICECON,
266 Pantelimon, PB 3-33, tel/fax +40212551420, Bucharest, Romania
e-mail: icecon@icecon.ro

Abstract. *The paper focus on elastic and dissipative nonlinear dynamic systems subjected to external harmonic disturbing forces. Thus, the correlation between the nonlinearity degree and time harmonic response in terms of frequency with high-order harmonics as related to the excitation harmonic is pointed out.*

1. INTRODUCTION

The dynamic behavior of nonlinear viscoelastic systems can be put into evidence by the time and frequency analyze of response signal for instantaneous displacement. Thus, depending on the viscous, elastic or both simultaneous nonlinearity degree it has been theoretical and experimentally demonstrated the displacement signal is a periodical anharmonic function when the system is excited by an external harmonic force. For example, we consider only the nonlinear viscosity or only the nonlinear elasticity, determining the system instantaneous displacement.

2. DYNAMIC RESPONSE OF THE NONLINEAR VISCOSITY SYSTEM

Let consider the viscous force of the form $F_v = \eta(\dot{x})\dot{x}$ where $\eta(\dot{x})$ changes according to the $|\dot{x}|$ powers, being expressed as:

$$F_v = B_1 \cos \omega t + B_3 \cos 3\omega t + B_5 \cos 5\omega t \tag{1}$$

and the differential motion equation is:

$$m\ddot{x} = F_v + kx = P_0 \cos \omega t \tag{2}$$

Received April 20, 2003

The solution $x = x(t)$ under steady forced regime for $\omega = const$ represents the dynamic response of the viscous nonlinear system, having the form:

$$x(t) = \sum_{j=1}^{n} A_j \sin(j\omega t - \varphi_j) \text{ ; j = 1, 3, 5, 7, …} \tag{3}$$

with

$$A_j = \lambda r j^2 \omega^2 /[(p^2 - j^2\omega^2)^2 + 4n_j^2\omega^2 j^2] \tag{4}$$

$$\varphi_j = arctg[2 j\omega n_j /(p^2 - j^2\omega^2)] \tag{5}$$

$$n_j^2 = B_j (p^2 - j^2\omega^2)^2 / 4 j^2\omega^2 (j^4\omega^4 m^2\lambda^2 r^2 - B_j^2) \tag{6}$$

It has been determined the response $x = x(t)$ expressed by relation (3) is a anharmonic periodical function having components according to the odd superior harmonics j = 1,3,5,7.

3. DYNAMIC RESPONSE OF NONLINEAR ELASTICITY SYSTEM

Let consider the elasticity expressed nonlinear by the elasticity factor $k(x)$ as a function of x powers, as follows:

$$k(x) = k_0 + ax + bx^2 + cx^3 + dx^4 + ... \tag{7}$$

and the elastic force $F_{el} = k(x)x$ can be expressed in spectral components if we adopt the first approximation $x = A\sin\omega t$. In this case it results in:

$$F_{el} = k_{01}x + a_0 + a_1 \sin\omega t + a_2 \cos 2\omega t + a_3 \sin 3\omega t + a_4 \cos 4\omega t + a_5 \sin 5\omega t \tag{8}$$

where the factors are defined by the following relations:

$k_{01} = k_0 + 0{,}5bA^2$

$a_0 = 0{,}5A^2(a + 0{,}75cA^2)$

$a_1 = 0{,}25A^3(b + dA^2)$

$a_2 = -0{,}5A^2(a + cA^2)$

$a_3 = -0{,}25A^3(b + 0{,}1875dA^2)$

$a_4 = 0{,}125cA^4$

$a_5 = 0{,}0625dA^5$

The differential equation with elastic and constant viscous ($\eta = const$) nonlinearity has the form:

$$m\ddot{x} + \eta\dot{x} + k_{01}x = -a_0 + (P_0 - a_1)\sin\omega t - a_2\cos 2\omega t - a_3 \sin 3\omega t - a_4 \cos 4\omega t - a_5 \sin 5\omega t$$

or

$$\ddot{x} + 2n\dot{x} + p^2 x = q_0 - q_1 \sin\omega t - q_2 \cos 2\omega t - q_3 \sin 3\omega t - q_4 \cos 4\omega t - q_5 \sin 5\omega t \tag{9}$$

where the following notations have been used:

$$q_0 = -\frac{a_0}{m}; \quad q_1 = \frac{1}{m}(P_0 - a_1); \; q_2 = \frac{1}{m}a_2; \quad q_3 = \frac{1}{m}a_3; \quad q_4 = \frac{1}{m}a_4; \quad q_5 = \frac{1}{m}a_5$$

The solution of the differential equation (9) having the form $x = x(t)$ represents the dynamic response under forced steady regime for $\omega = const$ can be expressed as:

$$x(t) = Q_1 \sin \omega t + Q_2 \cos \omega t + Q_3 \sin 2\omega t + Q_4 \cos 2\omega t + Q_5 \sin 3\omega t + Q_6 \cos 3\omega t + Q_0 \quad (10)$$

with

$$Q_1 = q_1(\omega^2 - p^2)/[(p^2 - \omega^2)^2 + 4n^2\omega^2]$$
$$Q_2 = 2n\omega q_1/[(p^2 - \omega^2)^2 + 4n^2\omega^2]$$
$$Q_3 = -4n\omega q_2/[(p^2 - \omega^2)^2 + 16n^2\omega^2]$$
$$Q_4 = q_2(4\omega^2 - p^2)/[(p^2 - \omega^2)^2 + 16n^2\omega^2]$$
$$Q_5 = q_3(9\omega^2 - p^2)/[(p^2 - \omega^2)^2 + 36n^2\omega^2]$$
$$Q_6 = 6n\omega q_3/[(p^2 - \omega^2)^2 + 36n^2\omega^2]$$
$$Q_0 = \frac{1}{p^2}q_0$$

The steady dynamic response expressed as instantaneous displacement in time consists of displacements of odd and even order superior harmonics.

4. CONCLUSIONS

The dynamic response of viscous systems subjected to external harmonic excitation $P(t) = P_0 \sin \omega t$ with $P_0 = m_0 r\omega^2$ is characterized by the presence of components superior to the excitation fundamental pulsation in the instantaneous displacement signal. Thus, two cases can be put into evidence, namely:

a) in case of nonlinear viscous systems with $\eta = \eta(|\dot{x}|)$ and linear elasticity $k = const$ we obtain the steady dynamic response as a instantaneous displacement signal having odd components superior to the excitation fundamental pulsation;
b) in case of nonlinear elastic systems with $k = k(x)$ and linear viscosity $\eta = const$ we obtain dynamic response as a instantaneous displacement signal having odd and even components superior to the excitation fundamental pulsation.

The time and frequency signal analyze can represent an important experimental evaluation for the nonlinear behavior of the dynamic systems.

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DINAMIČKI ODGOVOPR NELINEARNOG SISTEMA POD DEJSTVOM STACIONARNE HARMONIJSKE POBUDE

Polidor Bratu

Rad je usmeren na nelinearne dinamičke sisteme sa elastičnim i disipativnim elementima, a koji su porvrgnuti spoljašnje harmonijske pobudne sile. Istraživana je korelacija izmedju stepena nelinearnosti i vremena harmonijskog odziva u funkciji frekvencije sa harmonicima višeg reda u odnosu na harmonike spoljašnje pobude.