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DISSIPATED ENERGY IN CASE OF NONLINEAR DYNAMIC SYSTEMS SUBJECTED TO HARMONIC DISTURBING FORCES

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Abstract. *The paper puts into evidence the influence of elastic and viscous nonlinearity upon the dissipated energy in dynamic systems subjected to harmonic excitation. The physic aspect of the nonlinear behavior results in a very high energy dissipation as compared with the linear systems under the same conditions of dynamic load.*

1. INTRODUCTION

The paper is focused on the dynamic nonlinear systems subjected to harmonic excitation under steady regime. The force $F(t) = m_0 r\,\omega^2 \sin\omega t$ has the fundamental excitation pulsation ω and $m_0 r$ is the static moment for the unbalanced mass. The nonlinearity is put into evidence by the viscous force as well as by the elastic force.

The nonlinear dynamic systems model has been applied at the evaluation of the vibratory technological processes such as land, asphalt and concrete compaction, transport and screening of the granular materials and dynamic testing of the polymeric materials (plastics and rubber).

2. DYNAMIC SYSTEMS HAVING NONLINEAR VISCOSITY

The nonlinearity degree of the viscous behavior can be assessed by a viscous damping factor expressed as a power series, as follows:

$$\eta = \eta_0 + a\,|\,\dot{x}\,| + b\,|\,\dot{x}\,|^2 + \ldots \tag{1}$$

where η_0 is the viscous damping factor under steady regime (quasi-steady);
a, b – viscous damping factors under dynamic regime;

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$\dot{x}$ - instantaneous deflecting speed of the material on the x direction.

Thus, the viscous force $F_v = \eta\dot{x}$ depends on $\eta = \eta(\dot{x})$ and $\dot{x} = A\omega\cos\omega t$, where the first approximation has been adopted under the form $x = A\sin\omega t$ in the given differential nonlinear equation:

$$m\ddot{x} + \eta(\dot{x})\dot{x} + kx = P_0 \sin\omega t \tag{2}$$

In this case starting from the original function $f(t) = \dot{x} = r_0 \cos\omega t$ with $T\omega = 2\pi$ for $t \in (0, T)$ one can pass to the modulus function $g(t) = |\dot{x}| = v_0 \, |\cos\omega t|$ for $t \in (-T_p/2,\ T_p/2)$ where $T = 2T_p$ or $\omega_p = 2\omega$ represents a pulsating cycle.

Developing in a Fourier series the function g(t) and returning to the original pulsation ω, we obtain:

$$g(t) = \frac{2v_0}{\pi} + \frac{4v_0}{\pi}\sum_{j=1}^{\infty}\frac{(-1)^j}{4j^2-1}\cos 2j\omega t;\ j = 1, 2 \tag{3}$$

or

$$g(t) = \frac{2v_0}{\pi} + \frac{4v_0}{\pi}\left(\frac{1}{1\cdot 3}\cos 2\omega t - \frac{1}{5\cdot 7}\cos 6\omega t - \frac{1}{7\cdot 9}\cos 8\omega t - \ldots\right)$$

and finally we can write

$$g(t) = \ |\dot{x}|\ = a' + b'\cos 2\omega t + c'\cos 4\omega t + \ldots$$

where factors *a'*, *b'*, *c'* are the following:

$$a' = \frac{2v_0}{\pi};\quad b' = \frac{4v_0}{\pi}\frac{1}{1\cdot 3};\quad c' = \frac{4v_0}{\pi}\frac{1}{5\cdot 7}$$

so that η results in:

$$\eta = \eta(\dot{x}) = \eta_0 + a(a' + b'\cos 2\omega t + c'\cos 4\omega t) + b(a' + b'\cos 2\omega t + c'\cos 4\omega t)^2 + \ldots \tag{4}$$

The viscous force can be written as:

$$F_v = B_1 \cos\omega t + B_3 \cos 3\omega t + B_5 \cos 5\omega t + \ldots \tag{5}$$

where the factors B_1, B_3, B_5, … are:

$$B_1 = a'' + 0,5b'';\quad B_3 = 0,5(b'' + c'');\quad B_5 = 0,5c''$$

and

$$a'' = v_0\eta_0 + v_0 aa' + 0,5v_0(bb'^2 + bc'^2)$$

$$b'' = v_0(ab' + 2ba'b' + bb'c')$$

$$c'' = v_0(ac' + 2a'c'b + 0,5bb')$$

While recommending to limit at the first three terms we have only three odd harmonic components for the viscous force.

The differential equation of the nonlinear system becomes:

$$m\ddot{x} + B_1\cos\omega t + B_3\cos 3\omega t + B_5\cos 5\omega t + kx = P_0\cos\omega t \tag{6}$$

meaning the system is self-excited on the odd harmonics.

The motion amplitude on the j-order component excited by the superior harmonic jω is of the form:

$$A_j = \lambda r j^2 \omega^2 [(p^2 - j\omega^2)^2 + 4n_j^2 \omega^2 j^2]^{-1/2} \text{; j=1, 3, 5, 7, …} \tag{7}$$

with $p^2 = k/m$; $\lambda = m_0/m$; r - the unbalanced mass eccentricity.

The damping factor $n = \eta/2m$ as a variable function of the harmonic order j has the form:

$$n_j^2 = \frac{B_j^2 (p^2 - j^2\omega^2)^2}{4j^2\omega^2[j^4\omega^4 m^2 \lambda^2 r^2 - B_j^2]} \tag{8}$$

The mean power necessary to maintain the forced motion on the fundamental excitation harmonic is:

$$N_{med} = \frac{(m_0 r)^2}{m} \frac{\eta\omega^6}{(p^2 - \omega^2)^2 + 4n^2\omega^6}$$

having maximum values for $\eta = \dfrac{\omega^2 - p^2}{2\omega}$ in post-resonance when $\omega > p$, so that we obtain:

$$N_{med}^{\max} = \frac{1}{4} \frac{(m_0 r)^2}{m} \frac{\omega^5}{p^2 - \omega^2}$$

and for the j-order harmonic:

$$N_{med,j}^{\max} = \frac{1}{4} \frac{(m_0 r)^2}{m} \frac{j^5\omega^5}{p^2 - j^2\omega^2} \tag{9}$$

The maximum dissipated internal energy is:

$$W_j^M = N_{med,j}^{\max} \cdot T_j = N_{med,j}^{\max} \frac{2\pi}{j\omega}$$

or

$$W_j^M = \frac{\pi}{2} \frac{(m_0 r)^2}{m} \frac{j^4\omega^4}{p^2 - j^2\omega^2} \tag{10}$$

The maximum energy dissipated on the fundamental excitation harmonic is:

$$W_0^M = \frac{\pi}{2} \frac{(m_0 r)^2}{m} \frac{\omega^4}{p^2 - \omega^2}$$

The maximum internal energy weight on the superior harmonic $j > 2$ can be defined by the weight factor $\nu_{j,}$ as follows:

$$\nu_j = \frac{W_j^M}{W_0^M} = j^4 \frac{\omega^2 - p^2}{j^2\omega^2 - p^2} \text{ ; j = 3, 5, 7, …} \tag{11}$$

3. DYNAMIC SYSTEMS HAVING NONLINEAR ELASTICITY

Let consider the elastic nonlinearity can be introduced by the elasticity factor depending on the system instantaneous displacement *x*. Thus:

$$k = k_0 + ax + bx^2 + cx^3 + dx^4 + ... \tag{12}$$

where k_0 is the static elasticity factor and *a*, *b*, *c*, *d*, … are the elasticity factors under dynamic regime. In this case, using the solution approximation by $x = A\sin\omega t$, we obtain:

$$k = k_0 + a'_0 + a'\sin\omega t - a''\cos 2\omega t - a'''\sin 3\omega t + a^{iv}\cos 4\omega t \tag{13}$$

and taking into account $F_{el} = kx$, the elastic force results in:

$$F_{el} = k_{01}x + a_0 + a_1\sin\omega t + a_2\cos 2\omega t + a_3\cos 3\omega t + a_4\cos 4\omega t + a_5\cos 5\omega t \tag{14}$$

The differential equation of the system having physic elastic nonlinearity is:

$$m\ddot{x} + b\dot{x} + k(x)x = P_0\sin\omega t$$

or

$$\begin{aligned} & m\ddot{x} + b\dot{x} + k_{01}x + a_0 + a_1\sin\omega t + a_2\cos 2\omega t + a_3\cos 3\omega t + a_4\cos 4\omega t + a_5\cos 5\omega t \\ & = P_0\sin\omega t \end{aligned} \tag{15}$$

The mean power for the pulsation ω, as a function of k becomes:

$$N_{med} = \frac{(m_0 r)^2}{m}\frac{n\omega^6}{\left(\frac{k}{m} - \omega^2\right)^2 + 4n^2\omega^2}$$

The condition for maximum at $k = m\omega^2$ leads to obtain the maximum power under the form:

$$N_{med}^M = \frac{1}{4}\frac{(m_0 r)^2}{mn}\omega^4$$

and maximum dissipated internal energy is:

$$W_0^M = \frac{1}{2}\pi\frac{(m_0 r)^2}{mn}\omega^3 \tag{17}$$

The maximum energy for the superior harmonic j will be:

$$W_j^M = \frac{1}{2}\pi\frac{(m_0 r)^2}{mn} j^3\omega^3$$

and the weight index e_j is determined with the following relation:

$$e_j = \frac{W_j^M}{W_0^M} = j^3 \; ; j = 3, 5, 7, \dots \tag{18}$$

4. CONCLUSIONS

The nonlinear viscous systems excited under steady harmonic regime are characterized by the fact that they self-excite leading to superior harmonics responsible for the dissipation internal energy amplification. We can conclude that:

a) in case of nonlinear viscosity systems the weighting index ν_j depends on the superior harmonic order, according to relation (11);
b) in case of nonlinear elastic systems the weighting index e_j is proportional to j^3. Under the same excitation and inertial conditions the dissipated energy in the second case is higher than in case of first case.

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RASIPANJE ENERGIJE U SLUČAJU NELINEARNIH DINAMIČKIH SISTEMA PODVRGNUTIH HARMONOJSKIM POREMEĆAJNIM SILAMA

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U radu si prikazani rezultati istraživanja uticaja elastičnih i viskoznih nelinearnosti na rasipanje nergije dinamičkih sistema podvrgnutih harmonijskim pobudama. Fizički aspekt u rezultatu nelinearnog ponašanja je veoma veliko rasipanje energije u poredjenju sa linearnim sistemima pod istim uslovima dinamičkog opterećivanja.