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DISSOCIATED GAS FLOW IN THE BOUNDARY LAYER IN THE CASE OF A POROUS CONTOUR OF THE BODY WITHIN FLUID

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Abstract. *This paper studies the ideally dissociated gas flow in the boundary layer when the contour of the body within fluid is porous. Firstly, the momentum equation has been obtained from the corresponding starting boundary layer equations and the necessary set of porosity parameters has been introduced. Then, the boundary layer equations of the considered problem have been brought to a generalized form by means of transformations. The obtained equations have been numerically solved in a three-parametric approximation. A necessary program has been written to solve them. Based on the obtained solutions, conclusions concerning behaviour of certain boundary layer characteristics have been drawn.*

1. STARTING EQUATIONS

We have studied the ideally dissociated gas flow i.e. air in the case of the so-called frozen boundary layer, where the contour of the body within fluid is porous.

The primary aim of this investigation is to obtain generalised boundary layer equations of the considered problem by means of generalised similarity method and to solve them. Furthermore, based on the obtained conclusions, we should see the influences of certain parameters on physical values and characteristics of the boundary layer.

If we exclude the pressure from the equations of the laminar plane and steady boundary layer of the ideally dissociated gas (air) [2, 5], then the system of equations of the frozen boundary layer is:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0,$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\rho_e u_e\frac{du_e}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right),$$

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$$\rho u\frac{\partial\alpha}{\partial x}+\rho v\frac{\partial\alpha}{\partial y}=\frac{\partial}{\partial y}\left(\rho D\frac{\partial\alpha}{\partial y}\right),$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=-u\rho_e u_e\frac{du_e}{dx}+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2+\rho D(c_{pA}-c_{pM})\frac{\partial\alpha}{\partial y}\frac{\partial T}{\partial y},\qquad(1)$$

$$p=\rho T(1+\alpha)\frac{k}{2m_A}=\rho T(1+\alpha)R_M;$$

$$u=0,\quad \underline{v=v_w(x)},\quad T=T_w,\quad \alpha=\alpha_w\quad \text{for}\quad y=0\ ,$$
$$u\to u_e(x),\quad T\to T_e(x),\quad \alpha\to\alpha_e(x)\quad \text{for}\quad y\to\infty.$$

These equations represent respectively: the mixture continuity equation, the dynamic equation, the equation of diffusion of the atomic component, the energy equation and the state equation of the ideally dissociated gas. Notation common in the boundary layer theory has been used for certain physical values in this equations [2, 5], as follows:

$u(x,y)$ - longitudinal projection of the velocity in the boundary layer, $v(x,y)$ - transversal projection, ρ - density of the ideally dissociated gas (of the mixture), p - pressure, μ - dynamic viscosity, λ - coefficient of thermal conductivity, T - absolute temperature, α - mass concentration of the atomic component, D - coefficient of the diffusion atomic component, c_p - specific heat of the ideally dissociated gas, k - Boltzmann constant, m - mass and R - gas constant. The subscript "e" represents physical values at the outer edge of the boundary layer, "w" - conditions at the wall of the body within fluid, A and M - atomic and molecular component of the ideally dissociated gas.

Here, $v_w(x)$ stands for the given transversal velocity, which is also the velocity of the gas flowing transversally to and through the porous body contour and it may be positive or negative.

2. MOMENTUM EQUATION. POROSITY PARAMETERS

As with other solved problems of compressible fluid flow [3, 5], in order to apply the general similarity method, new variables $s(x)$ and $z(x,y)$ are introduced and they are in the form of the following transformations:

$$s(x)=\frac{1}{\rho_0\mu_0}\int_0^x\rho_w\mu_w dx,\qquad z(x,y)=\frac{1}{\rho_0}\int_0^y\rho dy\ ,\qquad(2)$$

where ρ_0, $\mu_0=\rho_0\nu_0$ represent the known constant values of the density and the dynamic viscosity of the dissociated gas, while ρ_w and μ_w are the known distributions of these values at the wall of the body within fluid, respectively.

By the usual procedure - an integration transversal to the boundary layer and a change of the variables, we obtain the momentum equation from the first two equations of the system (1). The general similarity method is based on the application of this equation. The momentum equation in this case of flow can be written in its three forms:

$$\frac{dZ^{**}}{ds}=\frac{F_{dp}}{u_e},\qquad \frac{df}{ds}=\frac{u_e'}{u_e}F_{dp}+\frac{u_e''}{u_e'}f,\qquad \frac{\Delta^{**'}}{\Delta^{**}}=\frac{u_e'F_{dp}}{2fu_e};\tag{3}$$

where the prime represents the derivation with respect to the longitudinal variable s.

While obtaining the momentum equation (3) we introduced: the parameter of the form f, the conditional displacement thickness $\Delta^*(s)$, the conditional momentum loss thickness $\Delta^{**}(s)$, non-dimensional function of the friction $\zeta(s)$, the porosity parameter $\Lambda(s)$ and the characteristic function of the boundary layer (the dissociated gas, porous contour) F_{dp}, in the form of the following relations:

$$Z^{**}=\frac{\Delta^{**2}}{\nu_0},\qquad f=f_1=\frac{u_e'\Delta^{**2}}{\nu_0}=u_e'Z^{**}=f(s),$$

$$\Delta^*(s)=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{u}{u_e}\right)dz,\qquad \Delta^{**}(s)=\int_0^\infty\frac{u}{u_e}\left(1-\frac{u}{u_e}\right)dz;\qquad H=\frac{\Delta^*}{\Delta^{**}},$$

$$F_{dp}=2\left[\zeta-(2+H)f\right]-2\Lambda\ ,\qquad \zeta(s)=\left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})}\right]_{z=0},\tag{4}$$

$$\Lambda(s)=-\frac{v_w\Delta^{**}}{\nu_0}\frac{\mu_0}{\mu_w}=-\frac{V_w\Delta^{**}}{\nu_0},\qquad V_w=v_w\frac{\mu_0}{\mu_w}=V_w(s).$$

Using the value Z^{**}, the porosity parameter can be written as:

$$\Lambda=\Lambda_1=-\frac{V_w}{\sqrt{\nu_0}}Z^{**\,1/2}=\Lambda_1(s)\ ,$$

where V_w is the conditional transversal velocity at the inner edge of the boundary layer.

Based on the obtained expression for the porosity parameter Λ_1 it is easily obtained that:

$$\frac{d\Lambda_1}{ds}=\frac{u_e'}{u_ef_1}\left(\frac{1}{2}F_{dp}\Lambda_1+\Lambda_2\right),\qquad \Lambda_2=-u_e\frac{V_w'}{\sqrt{\nu_0}}Z^{**\,3/2}.$$

Differentiating of the parameter Λ_2 with respect to the coordinate s and continuing the procedure of the differentiation of the following parameters we discover that the ***general porosity parameter*** of the dissociated gas has this form:

$$\Lambda_k=-u_e^{k-1}\left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(k-1)}\cdot Z^{**k-1/2},\qquad (k=1,2,3,...)\tag{5}$$

The porosity parameters satisfy the following simple recurrent differential equation

$$\frac{u_e}{u_e'}f_1\frac{d\Lambda_k}{ds}=\{(k-1)f_1+[(2k-1)/2]F_{dp}\}\Lambda_k+\Lambda_{k+1}\equiv\chi_k\ .\tag{6}$$

Expressions (5) and (6) for the general porosity parameter and for its derivation have completely the same form as the corresponding expressions for the case of homogenous gas which flows with great velocities around the porous contour.

It is also pointed out that the expressions, obtained in this paper, for the characteristic function F_{dp} and for the set of the porosity parameters $\Lambda_k(s)$ are of the same form as the corresponding expressions in the case of the incompressible fluid flow [4]. Furthermore, in the conditions of the liquid flowing across the porous contour (ρ = const, μ = const) it will be: $s(x) \to x$, $z(x,y) \to y$, $V_w \to v_w$, $\Delta^{**} \to \delta^{**}$, $\Lambda_1 \to \lambda_1, \ldots$ so all the values of the dissociated gas come down to the corresponding values of the incompressible fluid, as expected.

3. Transformation of the Boundary Layer Equations

In order to apply the generalised similarity method, as with other problems of fluid flow [3], we introduce a stream function $\psi(s,z)$ in accordance with the relations:

$$u = \frac{\partial \psi}{\partial z}, \qquad \tilde{v} = \frac{\rho_0 \mu_0}{\rho_w \mu_w}\left(u \frac{\partial z}{\partial x} + v \frac{\rho}{\rho_0}\right) = -\frac{\partial \psi}{\partial s}, \tag{7}$$

which are obtained based on the continuity equation.

If $\psi(s,0) = \psi_w(s)$ is introduced for the stream function along the wall surface of the body ($z = 0$), the boundary conditions of the starting system of equations (1) change into:

$$\frac{\partial \psi}{\partial z} = 0, \quad \frac{\partial \psi}{\partial s} = \frac{d\psi_w(s)}{ds} = -\tilde{v}_w = -V_w, \quad T = T_w, \quad \alpha = \alpha_w \quad \text{for} \quad z = 0,$$

$$\frac{\partial \psi}{\partial z} \to u_e(s), \qquad T \to T_e(s), \qquad \alpha \to \alpha_e(s) \qquad \text{for} \qquad z \to \infty. \tag{8}$$

The boundary conditions can be preserved with the considered problem of the dissociated gas flow just like with the non-porous wall. That is why a new stream function $\psi^*(s,z)$ related to $\psi(s,z)$ is introduced:

$$\psi(s,z) = \psi_w(s) + \psi^*(s,z), \qquad \psi^*(s,0) = 0. \tag{9}$$

In this way, by application of the transformations given above (2), (7), (9), the starting system of equations (1) of the considered problem, is brought down to:

$$\frac{\partial \psi^*}{\partial z}\frac{\partial^2 \psi^*}{\partial s \partial z} - \frac{\partial \psi^*}{\partial s}\frac{\partial^2 \psi^*}{\partial z^2} - \frac{d\psi_w}{ds}\frac{\partial^2 \psi^*}{\partial z^2} = \frac{\rho_e}{\rho} u_e u_e' + \nu_0 \frac{\partial}{\partial z}\left(Q \frac{\partial^2 \psi^*}{\partial z^2}\right),$$

$$c_p\left(\frac{\partial \psi^*}{\partial z}\frac{\partial T}{\partial s} - \frac{\partial \psi^*}{\partial s}\frac{\partial T}{\partial z}\right) - c_p \frac{d\psi_w}{ds}\frac{\partial T}{\partial z} = -\frac{\rho_e}{\rho} u_e u_e' \frac{\partial \psi^*}{\partial z} + \nu_0 \frac{\partial}{\partial z}\left(\frac{Q}{\Pr} c_p \frac{\partial T}{\partial z}\right) +$$

$$+ \nu_0 Q \left(\frac{\partial^2 \psi^*}{\partial z^2}\right)^2 + \nu_0 \frac{Q}{\mathrm{Sm}} (c_{pA} - c_{pM}) \frac{\partial \alpha}{\partial z}\frac{\partial T}{\partial z}, \tag{10}$$

$$\frac{\partial \psi^*}{\partial z}\frac{\partial \alpha}{\partial s} - \frac{\partial \psi^*}{\partial s}\frac{\partial \alpha}{\partial z} - \frac{d\psi_w}{ds}\frac{\partial \alpha}{\partial z} = \nu_0 \frac{\partial}{\partial z}\left(\frac{Q}{\mathrm{Sm}}\frac{\partial \alpha}{\partial z}\right);$$

$$\psi^* = 0, \quad \frac{\partial \psi^*}{\partial z} = 0, \quad T = T_w, \quad \alpha = \alpha_w \quad \text{for} \quad z = 0,$$

$$\frac{\partial \psi^*}{\partial z} \to u_e(s), \quad T \to T_e(s), \quad \alpha \to \alpha_e(s) \quad \text{for} \quad z \to \infty.$$

In the obtained system of equations the non-dimensional function Q, Prandtl number and Schmidt number are determined with the expressions:

$$Q = \frac{\rho\mu}{\rho_w \mu_w}, \qquad \Pr = \frac{\mu c_p}{\lambda}, \qquad \mathrm{Sm} = \frac{\mu}{\rho D}. \tag{11}$$

Following the concepts of general similarity method [1, 6], another transformation of the variables is applied to the system of equations (10):

$$s = s, \quad \eta(s,z) = \frac{u_e^{b/2} z}{K(s)}, \quad \psi^*(s,z) = u_e^{1-b/2} K(s) \cdot \Phi(s,\eta),$$

$$\overline{T} = \frac{T}{T_1}, \quad K(s) = \left(a\nu_0 \int_0^s u_e^{b-1} ds \right)^{1/2}, \quad T_1 = \text{const.}, \quad a,b = \text{const.}; \tag{12}$$

where $\Phi(s,\eta)$ is the non-dimensional stream function, $\overline{T}$ - non-dimensional temperature, $\eta(s,z)$ - non-dimensional transversal coordinate, T_1 - temperature at the forward stagnation point of the body within fluid while a and b are real constants.

The introduced relations and characteristics of the boundary layer (4) can also, by means of the newly introduced transformations (12), be expressed as:

$$K(s) = \frac{u_e^{b/2} \Delta^{**}}{B(s)}, \quad \eta(s,z) = \frac{B(s)}{\Delta^{**}} z, \quad H = \frac{\Delta^*}{\Delta^{**}} = \frac{A}{B}, \quad \zeta = B \left(\frac{\partial^2 \Phi}{\partial \eta^2} \right)_{\eta=0},$$

$$A(s) = \int_0^\infty \left(\frac{\rho_e}{\rho} - \frac{\partial \Phi}{\partial \eta} \right) d\eta, \quad B(s) = \int_0^\infty \frac{\partial \Phi}{\partial \eta} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) d\eta, \quad \frac{f}{B^2} = \frac{a u_e'}{u_e^b} \int_0^s u_e^{b-1} ds; \tag{13}$$

presuming that the values A and B are continuous functions of the longitudinal coordinate s.

After a complex derivation, the system of equations (10), by means of the transformations (12), becomes:

$$\frac{\partial}{\partial \eta} \left(Q \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \frac{aB^2 + (2-b) f}{2B^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \frac{f}{B^2} \left[\frac{\rho_e}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^2 \right] + \underline{\frac{\Lambda}{B} \frac{\partial^2 \Phi}{\partial \eta^2}} =$$

$$= \frac{u_e}{u_e'} \frac{f}{B^2} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial s \, \partial \eta} - \frac{\partial \Phi}{\partial s} \frac{\partial^2 \Phi}{\partial \eta^2} \right),$$

$$\frac{\partial}{\partial \eta} \left(\frac{Q}{\Pr} c_p \frac{\partial \overline{T}}{\partial \eta} \right) + c_p \frac{aB^2 + (2-b) f}{2B^2} \Phi \frac{\partial \overline{T}}{\partial \eta} - \frac{\rho_e}{\rho} \frac{u_e^2}{T_1} \frac{f}{B^2} \frac{\partial \Phi}{\partial \eta} + \frac{u_e^2}{T_1} Q \left(\frac{\partial^2 \Phi}{\partial \eta^2} \right)^2 +$$

$$+\frac{Q}{\mathrm{Sm}}(c_{pA}-c_{pM})\frac{\partial\alpha}{\partial\eta}\frac{\partial\overline{T}}{\partial\eta}+\underline{c_p\frac{\Lambda}{B}\frac{\partial\overline{T}}{\partial\eta}}=\frac{u_e}{u_e'}\frac{f}{B^2}c_p\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{T}}{\partial s}-\frac{\partial\Phi}{\partial s}\frac{\partial\overline{T}}{\partial\eta}\right), \tag{14}$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\alpha}{\partial\eta}+\underline{\frac{\Lambda}{B}\frac{\partial\alpha}{\partial\eta}}=\frac{u_e}{u_e'}\frac{f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial s}-\frac{\partial\Phi}{\partial s}\frac{\partial\alpha}{\partial\eta}\right);$$

$$\Phi=\frac{\partial\Phi}{\partial\eta}=0,\quad \overline{T}=\overline{T}_w,\quad \alpha=\alpha_w\quad \text{for}\quad \eta=0,$$

$$\frac{\partial\Phi}{\partial\eta}\to 1,\quad \overline{T}\to\overline{T}_e,\quad \alpha\to\alpha_e\quad \text{for}\quad \eta\to\infty.$$

It is pointed out that each of the equations of the system (14) contains one term more (the underlined terms) than the corresponding equations of the dissociated gas boundary layer [5] in the case of the non-porous wall of the body within fluid. If the porosity parameter is $\Lambda = \Lambda_1 = 0$, the corresponding equations are the same.

4. The generalized boundary layer equations of the considered problem and their solutions

In the obtained system of equations (14) the outer velocity $u_e(x)$, its derivative and u_e^2/T_1 figure explicitly. That is why the solution of the system of equations would depend on each concrete form of the distribution of this velocity.

A detailed analysis of the equations (14) has shown that in order to obtain the generalized system of equations for the considered case of the dissociated gas flow, it is necessary to introduce the similarity transformations from the very beginning:

$$\psi^*(s,z)=\frac{u_e\Delta^{**}}{B}\Phi[\eta,\kappa,(f_k),(\Lambda_k)],$$

$$T(s,z)=T_1\cdot\overline{T}[\eta,\kappa,(f_k),(\Lambda_k)], \tag{15}$$

$$\alpha=\alpha[\eta,\kappa,(f_k),(\Lambda_k)].$$

In these similarity transformations the local compressibility parameter $\kappa=f_0$ and the set of parameters of Loitsianskii's type are determined by the following expressions

$$\kappa=f_0=\frac{u_e^2}{2c_{p1}T_1},\quad f_k=u_e^{k-1}u_e^{(k)}Z^{**k}\quad (k=1,2,3\,...) \tag{16}$$

and they satisfy the simple recurrent differential equations:

$$\frac{u_e}{u_e'}f_1\frac{d\kappa}{ds}=2\kappa f_1\equiv\theta_0,\qquad \frac{u_e}{u_e'}f_1\frac{df_k}{ds}=[(k-1)f_1+kF_{dp}]\,f_k+f_{k+1}\equiv\theta_k. \tag{17}$$

The set of the porosity parameters $\Lambda_k(s)$ is determined with the expressions (5) and (6).

According to Fay-Riddell [5], the specific heats of the ideally dissociated air, as well as the specific heats of the atomic and molecular components, can be determined using these expressions:

$$\frac{c_p}{c_{p1}}=\frac{C^*}{C_1^*},\qquad \frac{c_{pA}-c_{pM}}{c_{p1}}=\frac{D^*}{C_1^*}, \tag{18}$$

where the non-dimensional functions C^*, D^* and C_1^* are determined with the relations:

$$C^*(\alpha,\overline{T})=\frac{10}{7}\alpha+(1-\alpha)\left[1+\frac{2}{7}e^{-(\overline{T}_v/\overline{T})^2}\right],\qquad C_1^*(\alpha_1)=\frac{10}{7}\alpha_1+(1-\alpha_1)\left[1+\frac{2}{7}e^{-(\overline{T}_v)^2}\right],$$

$$D^*(\overline{T})=\frac{3}{7}-\frac{2}{7}e^{-(\overline{T}_v/\overline{T})^2},\qquad \overline{T}_v=\frac{T_v}{T_1}=\frac{800}{T_1},\qquad c_{p1}=c_{p\infty}C_1^*,\qquad c_{p\infty}=\frac{7}{2}R_M, \tag{19}$$

respectively. Here α_1 represents the concentration of the atoms at the front stagnation point, and c_{p1} the specific heat at that point.

So, if the similarity transformations (15) are applied to the system (10) then, by means of (5), (6), (16), (17) and (18), this system of equations finally transforms into:

$$\begin{gathered}
\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f_1}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+\underline{\frac{\Lambda_1}{B}\frac{\partial^2\Phi}{\partial\eta^2}}=\\
=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)\right],\\
\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{C^*}{C_1^*}\frac{\partial\overline{T}}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\frac{C^*}{C_1^*}\Phi\frac{\partial\overline{T}}{\partial\eta}-\frac{\rho_e}{\rho}\frac{2\kappa f_1}{B^2}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+\frac{Q}{\mathrm{Sm}}\frac{D^*}{C_1^*}\frac{\partial\alpha}{\partial\eta}\frac{\partial\overline{T}}{\partial\eta}+\\
+\underline{\frac{C^*}{C_1^*}\frac{\Lambda_1}{B}\frac{\partial\overline{T}}{\partial\eta}}=\frac{1}{B^2}\frac{C^*}{C_1^*}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{T}}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial\overline{T}}{\partial\eta}\right)+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{T}}{\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial\overline{T}}{\partial\eta}\right)\right],\\
\frac{\partial}{\partial\eta}\left(\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial\alpha}{\partial\eta}+\underline{\frac{\Lambda_1}{B}\frac{\partial\alpha}{\partial\eta}}=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial\alpha}{\partial\eta}\right)+\right.\\
\left.+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial\alpha}{\partial\eta}\right)\right];\\
\Phi=\frac{\partial\Phi}{\partial\eta}=0,\quad \overline{T}=\overline{T}_w,\quad \alpha=\alpha_w\quad \text{for}\quad \eta=0,\\
\frac{\partial\Phi}{\partial\eta}\to 1,\quad \overline{T}\to\overline{T}_e=1-\kappa,\quad \alpha\to\alpha_e\quad \text{for}\quad \eta\to\infty.
\end{gathered} \tag{20}$$

Since there is no distribution of the outer velocity $u_e(s)$ in the system (20) obtained in this paper, this system of equations is generalized.

As for the ratio of the densities ρ_e/ρ and ρ/ρ_w which exist in the obtained system of equations and in the function Q, it is easily found from the state equation (1) of the ideally dissociated gas that in the boundary layer:

$$\frac{\rho_e}{\rho}=\frac{1+\alpha}{1+\alpha_e}\frac{\overline{T}}{\overline{T}_e}=\frac{1+\alpha}{1+\alpha_1}\frac{\overline{T}}{1-\kappa}, \qquad \frac{\rho}{\rho_w}=\frac{1+\alpha_w}{1+\alpha}\frac{\overline{T}_w}{\overline{T}}; \tag{21}$$

under the condition [5] that $\alpha_e=\alpha_1$ and $c_{pe}=c_{p1}$.

In this study Fay-Riddell's formula [5] has been used for the ratio of the viscosities of the ideally dissociated air

$$\frac{\mu}{\mu_w}=(1+\alpha)^{-1/2}\cdot\Pi(\overline{T}).$$

So, with a non-catalytic wall ($\alpha_w>0$) we obtain this expression for the non-dimensional function Q (11):

$$Q=(1+\alpha)^{-3/2}(1+\alpha_w)\frac{\overline{T}_w}{\overline{T}}\Pi(\overline{T}), \tag{22}$$

where:

$$\Pi(\overline{T})=\left(\frac{\overline{T}}{1-\kappa}\frac{T_1}{300}\right)^{3/2}\frac{413}{\dfrac{\overline{T}}{1-\kappa}T_1+113}+3{,}7\left(\frac{\overline{T}}{1-\kappa}\frac{T_1}{10000}\right)^2-2{,}35\left(\frac{\overline{T}}{1-\kappa}\frac{T_1}{10000}\right)^4. \tag{23}$$

In the three-parametric ($f_0=\kappa\neq 0,\ f_1\neq 0,\ \Lambda_1\neq 0,\ f_2=f_3=...=\Lambda_2=\Lambda_3=...=0$) twice-localized ($\partial/\partial\kappa=0,\ \partial/\partial\Lambda_1=0$) approximation the obtained system of equations (20) simplifies remarkably and reduces to:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f_1}{B^2}\left[\frac{1+\alpha}{1+\alpha_1}\frac{\overline{T}}{1-\kappa}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+\frac{\Lambda_1}{B}\frac{\partial^2\Phi}{\partial\eta^2}=$$
$$=\frac{F_{dp}f_1}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f_1}-\frac{\partial\Phi}{\partial f_1}\frac{\partial^2\Phi}{\partial\eta^2}\right),$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{C^*}{C_1^*}\frac{\partial\overline{T}}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\frac{C^*}{C_1^*}\Phi\frac{\partial\overline{T}}{\partial\eta}-\frac{1+\alpha}{1+\alpha_1}\frac{2\kappa\overline{T}}{1-\kappa}\frac{f_1}{B^2}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+$$
$$+\frac{Q}{\mathrm{Sm}}\frac{D^*}{C_1^*}\frac{\partial\alpha}{\partial\eta}\frac{\partial\overline{T}}{\partial\eta}+\frac{C^*}{C_1^*}\frac{\Lambda_1}{B}\frac{\partial\overline{T}}{\partial\eta}=\frac{F_{dp}f_1}{B^2}\frac{C^*}{C_1^*}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{T}}{\partial f_1}-\frac{\partial\Phi}{\partial f_1}\frac{\partial\overline{T}}{\partial\eta}\right), \tag{24}$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial\alpha}{\partial\eta}+\frac{\Lambda_1}{B}\frac{\partial\alpha}{\partial\eta}=\frac{F_{dp}f_1}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial f_1}-\frac{\partial\Phi}{\partial f_1}\frac{\partial\alpha}{\partial\eta}\right);$$

$$\Phi=\frac{\partial\Phi}{\partial\eta}=0,\quad \overline{T}=\overline{T}_w=\text{const.},\quad \alpha=\alpha_w\quad \text{for}\quad \eta=0,$$

$$\frac{\partial\Phi}{\partial\eta}\to 1,\quad \overline{T}\to\overline{T}_e=1-\kappa,\quad \alpha=\alpha_e=\alpha_1\quad \text{for}\quad \eta\to\infty.$$

Therefore, based on this study, it has been determined that the solution of the flow problems in the so-called frozen boundary layer in the case of the porous contour of the body within fluid comes down to the solution of the obtained generalized approximate system of equations (24).

Numerical computation of the system of equations (24) has been made using the finite differences method, i.e. using the "passage method". For the concrete computation of the obtained equation system a corresponding program, similar to one applied in paper [6], has been written.

Since the physical nature of Prandtl number is such that it negligibly depends on the temperature [3], in this study it is held constant. Based on [3], its value has been accepted to be Pr = 0,712, while Schmidt number is Sm = 0,509. The usual values have been accepted [4] for the constants a and b: a = 0,4408, b = 5,7140.

Only some of the results, which are here obtained by numerical computations of the system of equations (24), have been presented in this paper. We have shown the diagrams of the non-dimensional velocity $u/u_e = \partial\Phi/\partial\eta$ (Fig. 1), the non-dimensional temperature $\overline{T}$ (Fig. 3, Fig. 7), the profile of the atomic component of concentration of the ideally dissociated air α (Fig. 2, Fig. 6) for different values of the input parameters (T_1, T_w, α_w, α_1).

The diagrams of the boundary layer characteristics are also given: the non-dimensional function ζ (Fig. 4), the characteristic function F_{dp} (Fig. 5) and the non-dimensional functions A (Fig. 8) and B (Fig. 9).

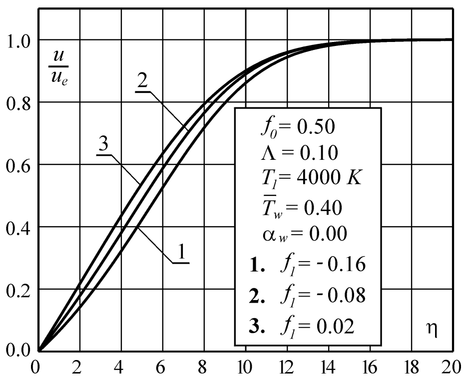

Fig. 1. The distribution of the non-dimensional velocity u/u_e

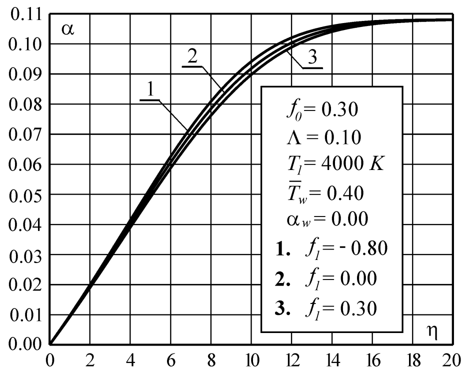

Fig. 2. The graphics of the atom concentration α

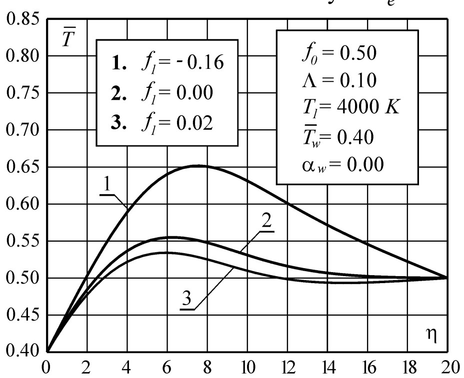

Fig. 3. The distribution of the non-dimensional temperature $\overline{T}$

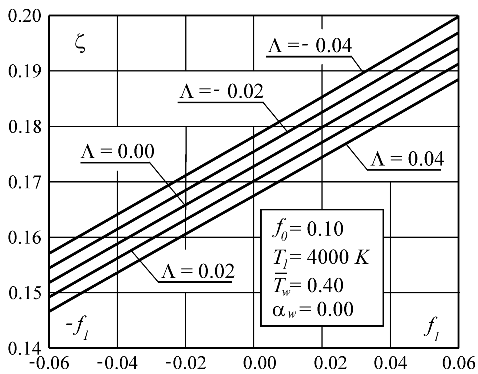

Fig. 4. The characteristics of the boundary layer ζ

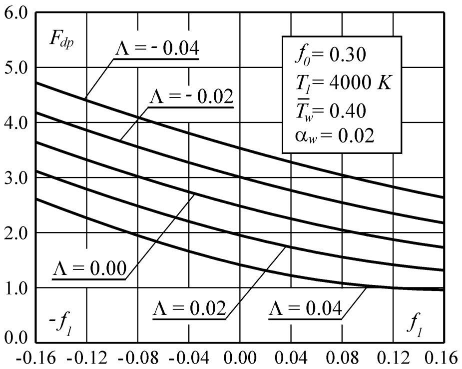

Fig. 5. The characteristics of the boundary layer F_{dp}

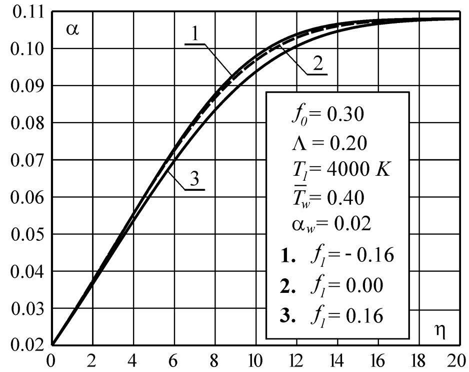

Fig. 6. The graphics of the atom concentration α

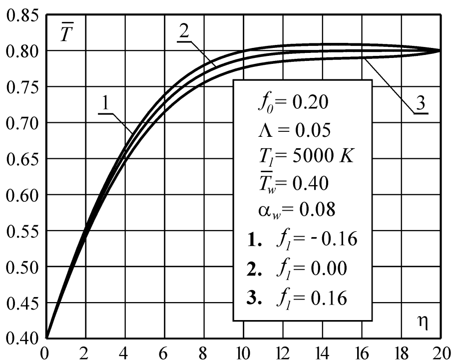

Fig. 7. The distribution of the non-dimensional of the temperature $\overline{T}$

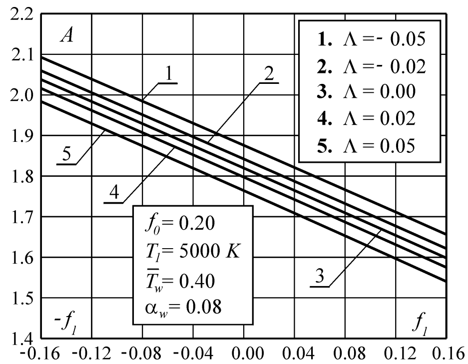

Fig. 8. The characteristics boundary layer A

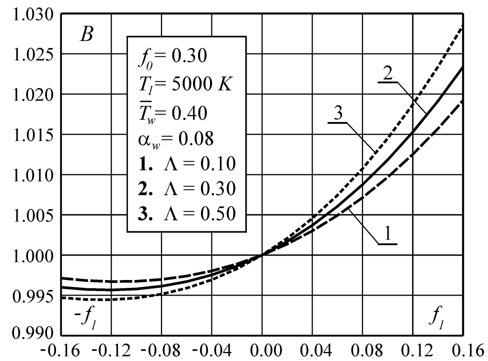

Fig. 9. The characteristics of the boundary layer B

5. Conclusions

Based on the given (and other) diagrams, it has been concluded that the obtained solutions and graphics of the boundary layer are of the same behaviour as with the ideally dissociated gas flow in the case of a balanced dissociation [3].

- The non-dimensional velocity at different cross sections of the boundary layer converges towards unity (Fig. 1).
- The profile of atom concentration α also rapidly converges towards the value at the outer edge of the boundary layer $\alpha_e = \alpha_1$ (Fig. 2, Fig. 6).

It is pointed out that the characteristic behaviour of the profile of the non-dimensional temperature has been observed in the boundary layer. As with other fluid flow problems for different values of the compressibility parameter f_0, the non-dimensional temperature $\overline{T}$ reaches the maximum value possible either in the boundary layer itself (Fig. 3) or at the outer edge of the boundary layer (Fig. 7).

- So, the local compressibility parameter f_0 has a great influence on the non-dimensional temperature $\overline{T}$ thus changing even the general character of the behaviour of this temperature.

It has been observed that the behaviour of the most important boundary layer characteristics - the non-dimensional functions ζ (Fig. 4) and B (Fig. 9) is as expected.

The diagrams Fig. 4 clearly shows the influence of the porosity parameter Λ_1 on the boundary layer separation point.

- With a decrease of the parameter Λ_1, i.e. with an increase of the transversal velocity of injection vw(x), the boundary layer separation point moves down the flow.

Finally, it is stressed that also in this case of the compressible fluid flow, a certain instability of numerical solution of the system of equations (24) is noticed. As a matter of fact, for some values of the input parameters the characteristic function F_{dp} inexplicably rapidly converges towards low values and the program stops at relatively small values of the positive parameter of the form f_1. For negative values of the parameter of the form f_1, this phenomenon has not been noticed.

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STRUJANJE DISOCIRANOG GASA U GRANIČNOM SLOJU ZA SLUČAJ POROZNE KONTURE OPSTRUJAVANOG TELA

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U radu se istražuje strujanje idealno disociranog gasa u graničnom sloju, pri čemu je kontura opstrujavanog tela porozna. Najpre je iz odgovarajućih polaznih jednačina graničnog sloja izvedena impulsna jednačina i uveden neophodni skup parametara poroznosti. Zatim su jednačine graničnog sloja razmatranog problema, pomoću svrsishodnih transformacija, dovedene na uopšteni oblik. Dobijene jednačine su numerički rešene u troparametarskom približenju. Za njihovo rešavanje je sastavljen neophodan program. Na bazi dobijenih rešenja izvedeni su zaključci o ponašanju pojedinih karakteristika graničnog sloja.