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Invited Paper

EQUIVALENT ELECTRODES METHOD APPLICATION IN FLUID AND HEAT FLOW

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Abstract. *By using standard analogies between electrostatic and other potential fields, Equivalent Electrodes Method can be adopted to the corresponding potential fields of theoretical physics. In this paper the method is extended to the steady magnetic field, to the fluid flow, heat flow and grounding problems solving. The theoretical investigations are supported by several typical examples and the obtained numerical results are compared with known analytical or numerical values. The very good agreement is realized, because Equivalent Electrodes Method gives exact solutions in the limit process with respect to the EE number.*

1. INTRODUCTION

Some time ago the present author suggested a new numerical method, so-called the **Equivalent Electrodes Method** (EEM) [1], for non-dynamic electromagnetic fields and other potential fields of the theoretical physics solving. The first very good results were obtained in [2], when the method was used for calculating the equivalent radius of uniform antennas. Afterwards, the good results were obtained in the computations of electrostatic fields [3-6], in the theory of low-frequency grounding systems [7], in the static magnetic field solving [8, 9] and for transmission lines analysis [10-15, 21-23]. Also, the method was extended to other potential fields: to plan-parallel fluid flow [16] and for heat flow problems solving [17].

The basic idea of the proposed theory is that an arbitrary shaped electrode can be replaced by a finite system of **equivalent electrodes** (EE). Thus it is possible to reduce a large number of complicated problems to equivalent simple systems. Depending on the problem geometry, the flat or oval strips (for plan-parallel field) and spherical bodies (for

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three-dimensional fields), or toroidal electrodes (for systems with axial symmetry) can be commonly used. In contrast to the Charge Simulation Method [18], when the fictitious sources are placed inside the electrodes volume, the EE are located on the body surface. The radius of the EE is equal to the equivalent radius of electrode part, which is substituted. Also the potential and charge of the EE and of the real electrode part are equal. So it is possible, using boundary condition that the electrode is equipotential, to form a system of linear equations with charges of the EE as unknowns. By solving this system, the unknown charges of the EE can be determined and, then, the necessary calculations can be based on the standard procedures. It is convenient to use Green's functions for some of electrodes, or for stratified medium, in case when the system has several electrodes, or when multilayer medium exists, and after that the remaining electrodes substitute by EE. The general presentation of the direct EEM to the determining electric field of the electrodes systems with multilayer media is presented in [19]. In the formal mathematical presentations, the proposed EEM is similar to the Moment Method form [20], but very important difference is in the physical fundaments and in the process of matrix establishments. So it is very significant to notice that in the application of the EEM an integration of any kind is not necessary. In the Moment Method solutions the numerical integration is always present, which produces some problems in the numerical solving of nonelementer integrals having singular subintegral functions. If the Charge Simulation Method is used, the potential can be also put in similar form, but the difference between EEM and Charge Simulation Method is in the choice of the positions of the equivalent charges and in the choice of matching points.

2. Outline of the Method

In order to the simplicity, but with sufficient generality, the EEM application is presented on an electrostatic system of a single infinite conductor having uniform cross section and known electric scalar potential $\varphi = U$ (Fig. 1).

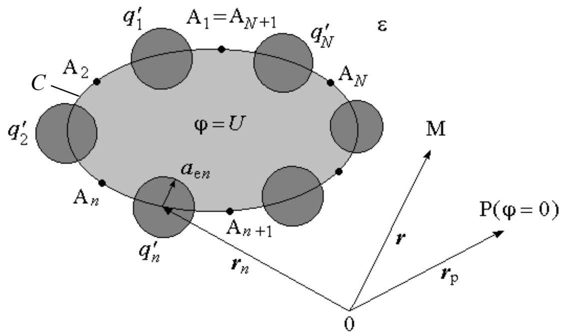

Fig. 1. Cross-section of single infinite conductor.

As it is known, the electric scalar potential in the conductor exterior space satisfies very well known Laplace's equation,

$$\Delta\varphi = 0\,, \tag{1}$$

and boundary condition on the surface body S, $\varphi = U$.

Let A_n, $n = 1,2,...,N$ ($A_{N+1} = A_1$) are N points, quite arbitrary selected on the curve C, which defines conductor cross section. So N oval strip electrodes, $A_n A_{n+1}$, $n = 1,2,...,N$, are formed on the electrode surface. The potential of strips is equal to the electrode potential and the line charge per unite strip length is q'_n. So total charge per unite electrode length is

$$q' = \sum_{n=1}^{N} q'_{\mathrm{n}} \,. \tag{2}$$

The presented strips can be replaced by cylindrical conductors having equivalent radius, a_{en}, in respect to the strips and equal potential, U, and charge per unit length, q'_n. The equivalent radius of flat strip, having length d, is $a_e = d/4$ (**Appendix I**). For oval strip, having angular width 2α and radius a, the equivalent radius is $a_e = a\sin(\alpha/2)$ (**Appendix II**).

So the real electrode can be replaced by equivalent cage structure, and, approximately, the potential can be expressed as

$$\varphi = \sum_{n=1}^{N} q'_{\mathrm{n}} G(\boldsymbol{r}, \boldsymbol{r}_n), \tag{3}$$

where:

$$G(\boldsymbol{r}, \boldsymbol{r}_n) = \frac{1}{2\pi\varepsilon} \ln \frac{|\boldsymbol{r}_{\mathrm{p}} - \boldsymbol{r}_n|}{|\boldsymbol{r} - \boldsymbol{r}_n|} \tag{4}$$

is potential Green's function of isolated uniform line charge;
$\boldsymbol{r}$ is the field point radius vector;
$\boldsymbol{r}_n$ is the radius vector of the electrical middle point (so called barycentre) of the strip, or of the EE;
$\boldsymbol{r}_p$ defines the zero potential point; and
ε is the electrical permittivity.

In the case when N is large, and the biggest width of the strip is small, the logarithmic potential theory (**Appendix III**) can be used and the electrode potential is

$$U = \sum_{n=1}^{N} q'_n G_{mn}, \quad m = 1,2,...,N, \tag{5}$$

where

$$G_{mn} = \frac{1}{2\pi\varepsilon} \ln \frac{|\boldsymbol{r}_{\mathrm{p}} - \boldsymbol{r}_n|}{\sqrt{|\boldsymbol{r}_m - \boldsymbol{r}_n|^2 + a_{\mathrm{e}m}^2 \delta_{mn}}} \tag{6}$$

and δ_{mn} is Kronecker's symbol.

After solving linear equations system (5) the unknown line charges of the EE can be determined and the values of interest can be evaluated in standard way. So the electric field strength can be put as

$$\boldsymbol{E} = -\operatorname{grad}\varphi \,. \tag{7}$$

The application of formula (7) gives the biggest error on the electrode surface. Therefore, the evaluation of the electric field strength on the electrode surface is based on the boundary condition for normal electric field component,

$$E_n = \eta/\varepsilon \,, \tag{8}$$

where $\eta = q'_n / l_n$ is surface charge density and l_n denotes the strip width.

In the presented treatment the EE are located on the electrode surface. They can be placed into conductor interior region. Then good results can also be obtained, but it is necessary to notice that the volume distribution of the EE is not physically correct and produces several numerical problems in matrix inversion.

2.1. EEM, Matrix Method and Charge Simulation Method

In the Matrix Method solution [13] of the presented problem on Fig. 1, the potential can be expressed as

$$\varphi(\boldsymbol{r}) = \oint_C \eta(\boldsymbol{r}')G(\boldsymbol{r},\boldsymbol{r}')\mathrm{d}l' = \sum_{n=1}^{N} \int_{\mathrm{A}_n\mathrm{A}_{n+1}} \eta(\boldsymbol{r}')G(\boldsymbol{r},\boldsymbol{r}')\mathrm{d}l' \,, \tag{9}$$

where C is the conductor cross-section curve.

If the strip elements are not large, the surface charge density of each element can be substituted by mean value,

$$\eta_n = \frac{1}{l_n} \int_{\mathrm{A}_n\mathrm{A}_{n+1}} \eta(\boldsymbol{r}')\mathrm{d}l' \tag{10}$$

so, approximately, the potential is

$$\varphi(\boldsymbol{r}) \approx \sum_{n=1}^{N} \eta_n \int_{\mathrm{A}_n\mathrm{A}_{n+1}} G(\boldsymbol{r},\boldsymbol{r}')\mathrm{d}l' \,. \tag{11}$$

After matching potential function (11) to the value of the electrode potential, U, in N matching points selected on the presented strip elements, the following linear equations system can be put

$$U = \sum_{n=1}^{N} \eta_n \gamma_{mn} \,, \quad \gamma_{mn} = \int_{\mathrm{A}_n\mathrm{A}_{n+1}} G(\boldsymbol{r} = \boldsymbol{r}_m, \boldsymbol{r}')\mathrm{d}l' \,, \quad m = 1,2,\ldots,N \,. \tag{12}$$

By solving this linear equations system, the unknown mean surface charge densities can be determined.

The difference between EEM and Moment Method is evident, although the similarity between basic linear equation systems (5) and (12) exists. In the application of the EEM an integration of any kind is not necessary. In the Moment Method solutions the numerical integration is always present (13), which produce some problems in the numerical solving of non-elementar integrals having singular subintegral functions.

If the Charge Simulation Method is used [12], the potential can be also put in the form (3), where q'_n. are unknown line charge densities, which are placed in the body interior. By matching approximate potential value to the real electrode potential in N points on the electrode surface, the linear equations system, having form (5) can be put. So the difference between EEM and charge simulation method is in the positions of the equivalent charges and in the choice of matching points.

2.2 Several examples

In order to illustrate the application of EEM in electrostatics, several examples (for plan-parallel, axially symmetric and 3D electrode system) will be presented in the next text.

2.2.1. Example I – Plan-parallel electrostatic system

In order to illustrate the EEM application the infinite isolated cylindrical conductor having circular cross section with radius a will be presented. Because of symmetry properties, it is convenient to form N EE having the same angular width $\alpha = 2\pi/N$, equivalent radius $a_e = a\sin(\pi/2N)$ and charge per unite length, $q' = Q'/N$, where Q' denotes the total charge per conductor unite length. So the potential is (**Appendix IV**).

$$\varphi = \varphi_0 - \frac{Q'}{2\pi\varepsilon N}\ln\prod_{n=1}^{N}\sqrt{r^2 + a^2 - 2ar\cos(\theta - \theta_n)} =$$

$$= \varphi_0 - \frac{Q'}{2\pi\varepsilon N}\ln\sqrt{r^{2N} + a^{2N} - 2a^N r^N \cos(N\theta)}\,, \qquad (13)$$

where:
φ_0 is an additive constant;
r and θ are cylindrical coordinates ($r = 0$ defines the position of the electrode axis.); and
EE axes are on the directions $r = a$, $\theta = (n-1)a$, $n = 1,2,...,N$.

By using the results from **Appendix III** the cylindrical electrode potential is

$$U = \varphi_0 - \frac{Q'}{2\pi\varepsilon}\ln A_e\,, \qquad (14)$$

where

$$A_e = a\sqrt[N]{N\sin\frac{\pi}{2N}} \qquad (15)$$

is the equivalent radius of cylindrical conductor.

As it is known, the equivalent radius exact value of cylindrical conductor is a. The Table 1. shows the ratio A_e / a for different number of EE, N.

Table 1. The ratio A_e / a for different number of EE, N

EE number, n	A_e / a
1	1.000 000 000 000 000
2	1.189 207 115 002 721
3	1.144 714 242 553 332
4	1.112 307 621 429 733
5	1.090 913 895 690 523
10	1.045 762 932 781 543
50	1.009 069 242 738 553
100	1.004 525 625 667 449
1 000	1.000 451 684 272 690
10 000	1.000 045 159 289768
100 000	1.000 004 515 837 249
1 000 000	1.000 000 451 582 807
10 000 000	1.000 000 045 158 272
100 000 000	1.000 000 004 515 827
1 000 000 000	1.000 000 000 451 583
∞	1.000 000 000 000 000

2.2.2. Example II - Electrostatic systems with axial symmetry

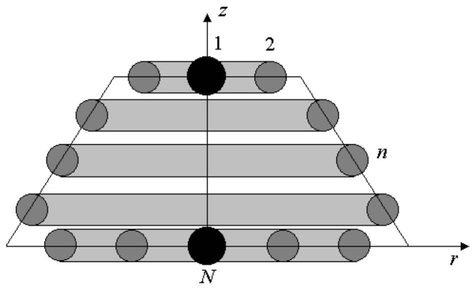

Fig. 2. Electrostatic system is with rotational symmetry

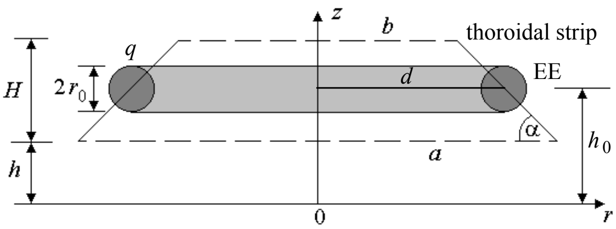

Fig. 3. Thoroidal EE

When electrostatic system is with rotational symmetry, the thoroidal strip elements on the electrode surface can be replaced by thin thoroidal EE (Fig. 2). Then the EE thorus radius, d, and radius of thorus cross section, r_0, are (Fig. 3):

$$d = \frac{a+b}{2} \tag{16}$$

and

$$r_0 = \sqrt{H^2 + (a-b)^2}\ ,\ H = (a-b)\mathrm{tg}\,\alpha\,. \tag{17}$$

The potential of the thoroidal EE is

$$\varphi = \frac{q}{2\pi^2\varepsilon}\frac{K(\pi/2,k)}{\sqrt{(d+r)^2+(z-h_0)^2}}\ , \tag{18}$$

where:
$K(\pi/2,k)$ is the complete elliptic integral of the first kind (**Appendix V**);
$k^2 = \dfrac{4rd}{(d+r)^2+(z-h_0)^2}$ are the modules of the elliptic integral;
q is the charge of the EE; and $h_0 = h + H/2$.

If $b = 0$ and $H = 0$ (EE number 1 and N in the Fig. 2) the observed strip element degenerates in thin disk with radius a. Then the EE is a small sphere having equivalent radius

$$a_{\mathrm{e}} = \frac{2}{\pi}a \tag{19}$$

and the EE potential is

$$\varphi = \frac{q}{4\pi\varepsilon\sqrt{r^2+(z-h)^2}}\,. \tag{20}$$

The total potential of all system can be expressed as

$$\varphi = \sum_{n=1}^{N} q_n G(r,z,r_n,z_n)\,, \tag{21}$$

where:
q_n is the charge of the EE;
$G(r, z, r_n, z_n)$ is Green's function having the following form:

$$G(r,z,r_n,z_n) = \frac{1}{4\pi\varepsilon\sqrt{r^2+(z-z_n)^2}}, r_n = 0 \tag{22}$$

for disk element, and

$$G(r,z,r_n,z_n) = \frac{1}{2\pi^2\varepsilon}\frac{K(\pi/2,k_n)}{\sqrt{(r+r_n)^2+(z-z_n)^2}}, k_n^2 = \frac{4rr_n}{(r+r_n)^2+(z-z_n)^2} \tag{23}$$

for thoroidal element;
r_n is the radius of thoroidal EE; and
$z = z_n$ defines the EE position.

By matching the potential expression (21) to the electrode potential value, U, in N points selected on the EE, $r'_m = r_m, z'_m = z_m + a_{\mathrm{e}m}\delta_{mn}$, where a_{em} denotes the equivalent radius of spherical EE, or of thoroidal EE cross section, the following linear equations system can be put

$$U = \sum_{n=1}^{N} q_n G(r = r'_m, z = z'_m, r_n, z_n)\ , m = 1,2,\ldots,N\ . \tag{24}$$

The EEM values of capacitance of thoroidal electrode (Fig. 4) are presented and compared with exact results,

$$C = 16\varepsilon\sqrt{d^2 - a^2}\sum_{n=0}^{\infty}\frac{1}{1+\delta_{n0}}\frac{Q_{n-1/2}(d/a)}{P_{n-1/2}(d/a)}, \tag{25}$$

in the Table 10, where $P_{n-1/2}$ and $Q_{n-1/2}$ are Legendre functions.

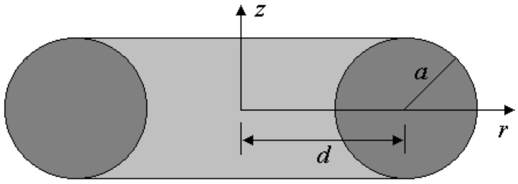

Fig. 4. Isolated thoroidal electrode.

Table 2. Capacitance of isolated toroidal electrode, $C/\varepsilon d$, for different ratio d/a and different number of EE, N

d/a	$N = 10$	$N = 50$	$N = 100$	$N = 150$	$N = 200$	exact
1.1	23.142	22.835	22.802	22.792	22.787	22.760
1.2	24.027	23.719	23.686	23.675	23.670	23.649
1.3	24.906	24.597	24.564	24.554	24.549	24.528
1.4	25.781	25.471	25.438	25.427	25.422	25.405
1.5	26.651	26.340	26.307	26.296	26.291	26.271
1.6	27.517	27.204	27.171	27.161	27.155	27.138
1.7	28.379	28.065	28.032	28.021	28.016	27.999
1.8	29.237	28.922	28.888	28.877	28.872	28.856
1.9	30.090	29.774	29.741	29.730	29.724	29.708
2.0	30.941	30.623	30.589	30.578	30.573	30.555
2.5	35.139	34.814	34.779	34.768	34.763	34.746
3.0	39.256	38.922	38.886	38.875	38.869	38.853
4.0	47.271	46.918	46.880	46.867	46.861	46.843
5.0	55.037	54.662	54.621	54.608	54.602	54.582
10.0	91.284	90.794	90.741	90.724	90.715	90.690

2.2.3. Example III – 3D electrostatic system

When the electrostatic system is three dimensional, the plate elements on the electrode surface can be replaced by small spherical EE (Fig. 5).

So the potential is

$$\varphi = \sum_{n=1}^{N} q_n G(\boldsymbol{r}, \boldsymbol{r}_n), \tag{26}$$

where:

$$G(\boldsymbol{r}, \boldsymbol{r}_n) = \frac{1}{4\pi\varepsilon|\boldsymbol{r} - \boldsymbol{r}_n|} \tag{27}$$

is potential Green's function of isolated point charge;

$\boldsymbol{r}$ is the field point radius vector; and

$\boldsymbol{r}_n$ is the radius vector of the electrical middle point of the strip surface element, or of the EE.

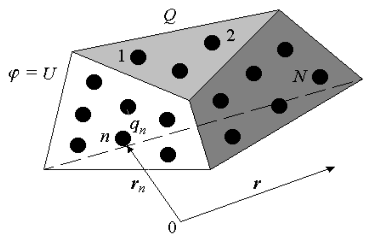

Fig. 5. 3D isolated electrode.

In order to determine the unknown charges of the EE, q_n, $n = 1,2,...,N$, the following linear equations system, governing boundary condition on the electrode surface, can be put

$$U = \sum_{n=1}^{N} \frac{q_n}{4\pi\varepsilon\sqrt{\left|\boldsymbol{r}_n - \boldsymbol{r}_m\right|^2 + a_{\mathrm{e}m}^2 \delta_{mn}}}\,, \quad m = 1,2,...,N\,, \tag{28}$$

where $a_{\mathrm{e}n}$ denotes the EE equivalent radius.

The capacitance of the isolated electrode is

$$C = \frac{Q}{U}\,, \quad Q = \sum_{n=1}^{N} q_n\,. \tag{29}$$

In the EEM application it is often necessary to determine equivalent radius of thin plate elements, frequently with rectangular form (Fig. 6).

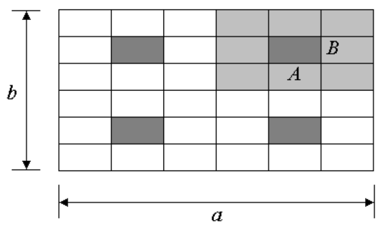

Fig. 6. Thin rectangular plate electrode.

Then it is possible to form the following dependence $A_{\mathrm{e}}/a = f(a_{\mathrm{e}}/A)$, where A_{e} is the equivalent radius of the rectangular plate with side a and b, and a_{e} is the equivalent radius of EE, having sides A and B. Evidently, because of the existing similarity, $A_{\mathrm{e}}/a = a_{\mathrm{e}}/A$. So it is possible to determine the equivalent radius of the thin rectangular plate. In practice the following approximation is very useful

$$\frac{A_{\mathrm{e}}}{a} = \begin{cases} 0.373(b/a)^{0.3882}, 0.5a < b \le a \\ 0.404(b/a)^{0.5}, 0.3a < b \le 0.5a \\ 0.696(b/a)^{0.957}, 0 < b \le 0.3a. \end{cases} \tag{30}$$

In the Table 3. the approximate values of A_{e}/a are compared with exact results. By using moment method, the equivalent radius of thin square plate with side a is $A_{\mathrm{e}} = 0.37a$ [20]. This result agrees very well with the presented value, $A_{\mathrm{e}} = 0.373a$.

Table 3. Exact and approximate values of equivalent radius of thin rectangular plate

b/a	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A_e/a exact	0.000	0.077	0.153	0.220	0.257	0.285	0.308	0.328	0.346	0.361	0.373
A_e/a formula (30)	0.000	0.077	0.149	0.220	0.256	0.285	0.306	0.325	0.342	0.358	0.373

For isolated electrode (Fig. 7), having parallelepiped shape with sides a, b and c the small spherical EE replace the rectangular electrode surface elements. Table 4. shows the values of parallelepiped electrode capacitance for different EE numbers, where M, N and L are the rectangular element numbers on the half parallelepiped sides.

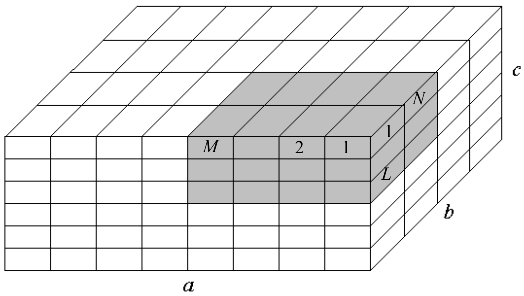

Fig. 7. Parallelepiped electrode.

Table 4. The values of the parallelepiped electrode capacitance, C for different EE numbers

	M	N	L	$C/4\pi\varepsilon a$
$a = b = c$	3	3	3	0.677
	4	4	4	0.674
	5	5	5	0.672
	6	6	6	0.670
$a = b = 10c$	7	7	2	0.432
	7	7	3	0.434
$a = 10b = 10c$	3	2	2	0.198
	4	2	2	0.205
	5	2	2	0.208
	6	2	2	0.210
	6	3	3	0.204

3. Analogies and EEM application to the other fields

Using adequate analogies between electrostatic and other potential fields the EEM can be successfully extended to the theory of low-frequency grounding systems [5], in the static magnetic field solving [6, 7], in electroheat [17] and for plan-parallel fluid flow solving [16]. The following exposition is destined to present the analogies of interest in the EEM application.

3.1. EEM application to low-frequency grounding system

By using analogies between electrostatic and electric grounding field in electrostatic formula the ratio q/ε, where q denotes electrode charge and ε is the electric permittivity, should be replaced with $\underline{I}/\underline{\sigma}$. $\underline{I}$ is the grounding electrode current and $\underline{\sigma} = \sigma + \mathrm{j}\omega\varepsilon$ denotes complex conductivity (j is imaginary unite, σ and ε are the soil conductivity and permittivity and ω is angular frequency). Then the potential of single grounding electrode can be put in the following form

$$\underline{\varphi} = \sum_{n=1}^{N} \underline{I}_n \underline{G}(\boldsymbol{r}, \boldsymbol{r}_n) \,, \tag{31}$$

where $\underline{I}_n$ is the current of the EE on the grounding electrode surface, N is the number of the EE and $\underline{G}(\boldsymbol{r}, \boldsymbol{r}_n)$ is the corresponding Green's functions.

By example, for depth point grounding source placed in the point, which radius vector is $\boldsymbol{r}'$, the Green's function is

$$\underline{G}(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{4\pi\underline{\sigma}|\boldsymbol{r} - \boldsymbol{r}'|} \,. \tag{32}$$

If the influence of the soil surface to the grounding system is not neglectable, the Green's functions is (Fig. 8):

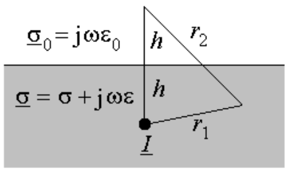

Fig. 8. Point electrode in two-layer media.

$$\underline{G} = \begin{cases} \dfrac{1}{4\pi\underline{\sigma}}\left(\dfrac{1}{r_1} + \dfrac{\underline{\sigma} - \underline{\sigma}_0}{\underline{\sigma} + \underline{\sigma}_0}\dfrac{1}{r_2}\right), \text{in the soil} \\ \dfrac{1}{2\pi(\underline{\sigma} + \underline{\sigma}_0)r_1}, \text{in the air}. \end{cases} \tag{33}$$

By solving linear equations system,

$$\underline{U} = \sum_{n=1}^{N} \underline{I}_n \underline{G}(\boldsymbol{r} = \boldsymbol{r}_n + \boldsymbol{a}_{\mathrm{e}n}\delta_{nm}, \boldsymbol{r}_m) \,, m = 1,2,\ldots,N \,, \tag{34}$$

where $a_{\mathrm{e}n}$ is the equivalent radius of the EE and $\underline{U}$ denotes the electrode potential, the unknown currents of the EE can be determined. The ratio

$$\underline{Y} = G + \mathrm{j}B = \underline{I}/\underline{U} \,, \; \underline{I} = \sum_{n=1}^{N} \underline{I}_n \,, \tag{35}$$

defines the grounding electrode admittance.

In the case of isolated grounding electrode in the homogeneous infinite soil it is

$$\underline{Y} = \underline{\sigma} C / \varepsilon \ , \tag{36}$$

where C is the electrode capacitance.

3.2 EEM Application to the static magnetic field solving

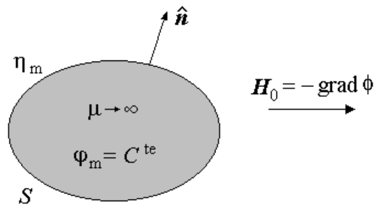

Fig. 9. Ferromagnetic body in external magnetic field.

Let an ideal ferromagnetic body ($\mu \to \infty$) is placed in the exiting static magnetic field, $\boldsymbol{H}_0 = -\text{grad}\phi$, where $\phi = \phi(\boldsymbol{r})$ is the exiting magnetic scalar potential (Fig. 9). Then the total magnetic scalar potential,

$$\varphi_m = \varphi_p + \phi \, , \tag{37}$$

satisfies Laplace's equation and on the body surface S the boundary condition $\varphi_m = C^{te}$ or $\hat{\boldsymbol{n}} \times \boldsymbol{H} = 0$. φ_p denotes the perturbed component of the magnetic scalar potential, which can be expressed as

$$\varphi_p = \oint_S \eta_m(\boldsymbol{r}') G(\boldsymbol{r}, \boldsymbol{r}') \mathrm{d}S' \, , \tag{38}$$

where η_m defines magnetic charge of the body unite surface and $G(\boldsymbol{r},\boldsymbol{r}')$ is the correspon-ding Green's function. By example, for a single point magnetic charge

$$G(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{4\pi|\boldsymbol{r} - \boldsymbol{r}'|} \, . \tag{39}$$

$\boldsymbol{H} = -\text{grad}\,\varphi_m$ is total magnetic field strength. Total magnetic charge on the body surface is always equal to zero,

$$\oint_S \eta_m(\boldsymbol{r}') \mathrm{d}S' = 0 \, , \tag{40}$$

The present analysis brings to the analogies as it is in the Table 5. By using these analo-gies, it is very easy to extend the EEM application to magnetostatic problems solving.

Table 5. Analogies between magnetostatic and electrostatic field

Magnetostatic field	Electrostatic field
Magnetic scalar potential, φ_m	Electric scalar potential, φ
Magnetic charge, q_m	Electric charge, q
Magnetic surface charge, η_m	Electric surface charge, η
Magnetic field strength, $\boldsymbol{H} = -\text{grad}\ \varphi_m$	Electric field strength, $\boldsymbol{E} = -\text{grad}\ \varphi$

3.3 EEM application in the heat conduction

The analogies between temperature field and electrostatic field (Table 6) are useful in the EEM application to the heat conduction problems solving.

Table 6. Analogies between temperature field and electrostatic field

Temperature field	Electrostatic field
Temperature, θ	Electrostatic potential, φ
Temperature field strength, $\boldsymbol{E}_\theta = -\mathrm{grad}\,\theta$	Electrostatic filed strength, $\boldsymbol{E} = -\mathrm{grad}\,\varphi$
Heat flux across the surface, $\boldsymbol{D}_\theta = \lambda \boldsymbol{E}_\theta$	Electric displacement, $\boldsymbol{D} = \varepsilon \boldsymbol{E}$
Thermal conductivity, λ	Permittivity, ε
Thermal power, $P = C_\theta \Delta\theta = \oint_S \boldsymbol{D}_\theta \,\mathrm{d}\boldsymbol{S}$	Electric charge, $q = C\Delta\varphi = \oint_S \boldsymbol{D}\,\mathrm{d}\boldsymbol{S}$
Thermal capacitance, C_θ	Electrical capacitance, C

Example I: The thermal capacitance of the thin, perfectly thermal conducting tube with length h and circular cross section having radius a in the homogeneous medium with thermal conductivity λ is

$$C_\theta/\lambda h = \begin{cases} 1.939 \ , \ \text{for } a/h = 0.01 \\ 3.068 \ , \ \text{for } a/h = 0.05 \\ 4.009 \ , \ \text{for } a/h = 0.10 \ . \end{cases}$$

Example II: The perfectly thermal conducting tube having zero temperature is in an uniform axial temperature electric field, $E_{\theta 0}$. h denotes the tube length, a and b are the interior and exterior radius of the tube cross-section, respectively. The temperature values on the tube axis are presented in the Table 7, when $a/h = 0.6$ and $b = h$. $\theta_0 = -E_{\theta 0} z$ is the non-perturbed temperature defying uniform thermal field. The cylindrical coordinate origin is in the tube centre and z – axis coincides with the tube axis.

In the Table 8. are the values of the ratio $D_\theta / D_{\theta\max}$ on the tube surface, where $D_{\theta\max}$ denotes the maximal value of the absorbed heat flux across the tube surface.

Table 7. The temperature distribution on the tube axis

$2z/h$	$\theta/E_{\theta 0}h$	θ/θ_0	$2z/h$	$\theta/E_{\theta 0}h$	θ/θ_0
0.00	0.000	1.000	0.65	0.397	0.610
0.05	0.014	0.278	0.70	0.453	0.648
0.10	0.029	0.285	0.75	0.512	0.683
0.15	0.044	0.296	0.80	0.572	0.715
0.20	0.062	0.312	0.85	0.633	0.745
0.25	0.083	0.332	0.90	0.694	0.771
0.30	0.107	0.375	0.95	0.756	0.795
0.35	0.135	0.386	1.00	0.817	0.817
0.40	0.167	0.419	1.25	1.115	0.892
0.45	0.204	0.454	1.50	1.399	0.933
0.50	0.246	0.493	1.75	1.673	0.956
0.55	0.293	0.532	2.00	1.939	0.970
0.60	0.343	0.572	∞	$-\infty$	1.000

Table 8. The ratio $D_\theta/D_{\theta\max}$ on the tube surface

Interior side $r = a$, $0 \le z \le h/2$		End side $z = h/2$, $a \le r \le b$		Exterior side $r = b$, $0 \le z \le h/2$	
z/h	$D_\theta/D_{\theta\max}$	r/b	$D_\theta/D_{\theta\max}$	z/h	$D_\theta/D_{\theta\max}$
0.0125	0.002	0.64	0.710	0.4875	0.986
0.0625	0.011	0.72	0.578	0.4625	0.639
0.1125	0.021	0.80	0.595	0.4125	0.424
0.1625	0.033	0.88	0.679	0.3625	0.321
0.2125	0.047	0.96	1.000	0.3125	0.252
0.2625	0.065			0.2625	0.198
0.3125	0.091			0.2125	0.153
0.3625	0.128			0.1625	0.114
0.4125	0.193			0.1125	0.077
0.4625	0.342			0.0625	0.042
0.4875	0.592			0.0125	0.008

3.4 EEM application in the hydrodynamics

The EEM application to the plan-parallel ideal fluid flow problems solving is based on the analogies between hydrodynamic field and electrostatic field. Let

$$\Phi(\underline{z} = x + \mathrm{j}\,y) = \varphi(x, y) + \mathrm{j}\,\psi(x, y) \tag{41}$$

be an analytical function. So the Cauchy-Riemann equations are adopted and the real and imaginary part present analytic functions, which satisfy Laplace's equation. The surfaces families $\varphi(x,y) = C^{\mathrm{te}}$ and $\psi\ (x,y) = C^{\mathrm{te}}$ are mutual perpendicular and may be used to define equipotentials and stress lines in a field of force.

For example, if in electrostatics $\varphi(x,y)$ present electric scalar potential function, then $\varphi(x,y) = C^{\mathrm{te}}$ are equipotentials, $\psi\ (x,y) = C^{\mathrm{te}}$ are lines of force and $\boldsymbol{E} = -\mathrm{grad}\ \varphi$ is the electric field strength.

In the hydrodynamics of ideal two-dimensional fluids, $\psi\ (x,y)$ is the velocity potential, $\varphi(x,y)$ defines stream function and $\boldsymbol{v} = -\mathrm{grad}\ \psi$ is the fluid velocity.

So, the essence of EEM application to the fluid flow problem solving is the generation of complementary electrostatic system. After solving electrostatic system the complex potential (63) can be formed and the velocity potential can be determined as imaginary part of the complex potential. For example, if it solves the problem of infinite cylindrical body in uniform transversal fluid flow with velocity $\boldsymbol{v}_0 = v_0\hat{\boldsymbol{y}}$ (Fig. 10) the complementary electrostatic system has the form as on the Fig. 11.

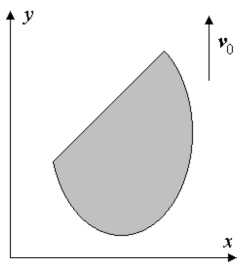

Fig. 10. Fluid flow problem

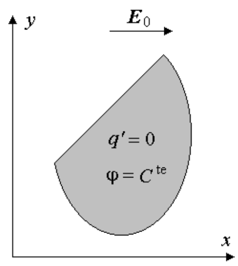

Fig. 11. Complementary electrostatic problem

By using the EEM the potential of the electrostatic system can be put in the following form

$$\varphi = -E_0 x + \sum_{n=1}^{N} q'_n G(x, y, x_n, y_n), \tag{42}$$

including charge condition

$$\sum_{n=1}^{N} q'_n = 0, \tag{43}$$

where

$$G(x, y, x_n, y_n) = -\frac{1}{2\pi\varepsilon} \ln \sqrt{(x - x_n)^2 + (y - y_n)^2} \tag{44}$$

is the Green's function of the EE infinite line charge, q'_n, placed on the direction $\boldsymbol{r}_n = x_n \hat{\boldsymbol{x}} + y_n \hat{\boldsymbol{y}}$.

So the complex potential is

$$\underline{\Phi} = -E_0 \underline{z} - \frac{1}{2\pi\varepsilon} \sum_{n=1}^{N} q'_n \ln|\underline{z} - \underline{z}_n|, \underline{z} = x + \mathrm{j}\, y, \underline{z}_n = x_n + \mathrm{j}\, y_n. \tag{45}$$

By replacing E_0 with v_0, the velocity potential can be determined as

$$\psi = \mathrm{Im}\{\underline{\Phi}\} = -E_0 y - \frac{1}{4\pi\varepsilon} \sum_{n=1}^{N} q'_n \operatorname{arctg} \frac{y - y_n}{x - x_n}, \text{for}\, E_0 \to v_0. \tag{46}$$

4. Conclusion

A new numerical method, so called the equivalent electrodes method, for the potential fields problems solving, is presented. The basic idea of the proposed theory is: an arbitrary shaped electrode can be replaced by a finite system of equivalent electrodes. So it is possible to reduce a large number of complicated problems to the equivalent simple systems. Depending on the problem geometry, the flat or oval strips (for plan-parallel field) and spherical bodies (for three-dimensional fields), or toroidal electrodes (for systems with axial symmetry) can be commonly used. In contrast to the charge simulation method, when the fictitious sources are placed inside the electrodes volume, the equivalent electrodes are located on the body surface. The radius of the equivalent electrodes is equal to the equivalent radius of electrode part, which is substituted. Also the potential and charge of the equivalent electrodes and of the real electrode part are equal. So it is possible, using boundary condition that the electrode is equipotential, to form a system of linear equations, with charge densities of the equivalent electrodes as unknown. Solving this system, the unknown charge densities of the equivalent electrodes can be determined and, then, the necessary calculations are based on the standard procedures. In the case when the system has several electrodes, or when the multilayer medium exists, it is convenient to use Green's functions for some electrodes, or for stratified medium, and after the remaining electrodes substitute with equivalent electrodes. In the formal mathematical presentations, the proposed equivalent electrodes method is similar to the moment method form, but very important difference is in the physical fundaments and in the process of matrix establishment. So it is very significant to notice that in the application of the equivalent electrodes method an integration of any kind is not necessary. In the moment method solutions, the numerical integration is always present, which produce some problems in the numerical solving of non-elementar integrals having singular subintegral functions. The method is very simple and exact and in limit process with the number of the equivalent electrodes gives exact results. The present method is very useful in electrostatics, magnetostatics, in the theory of low-frequency grounding problem, in the analysis of transmission lines and in the solving the problems of heat flow and fluid flow.

APPENDIX I

By using conformal mapping, Joukowsky transform,

$$\underline{z} = x + \mathrm{j}y = \frac{c}{2}\left(\underline{w} + \frac{1}{\underline{w}}\right), \quad \underline{w} = u + \mathrm{j}v = R\mathrm{e}^{\mathrm{j}\psi}, \tag{a1.1}$$

the exterior region of the flat strip having zero width can be mapped to exterior domain of the cylindrical conductor having circular cross section with unite radius (Fig. A1.1).

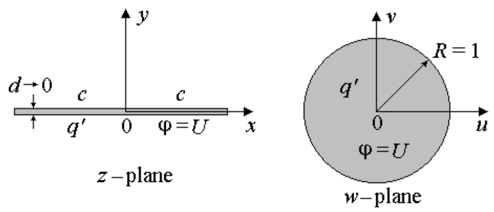

Fig. A1.1

Because the electrode potential and the total electrode line charge are unchanged in the mapping process, the complex potential can be expressed as

$$\underline{\Phi} = U - \frac{q'}{2\pi\varepsilon}\ln \underline{w} \cdot \tag{a1.2}$$

If $z >> c$, $\underline{z} \approx \frac{c}{2}\underline{w}$ and the complex potential is

$$\underline{\Phi} = U - \frac{q'}{2\pi\varepsilon}\ln\frac{2\underline{z}}{c} = U - \frac{q'}{2\pi\varepsilon}\ln\frac{\underline{z}}{a_{\mathrm{e}}}, \tag{a1.2}$$

where

$$a_{\mathrm{e}} = c/2 = 2c/4 \tag{a1.3}$$

is equivalent radius of thin flat strip conductor.

Appendix II

Evaluation of equivalent radius of thin oval circular strip is based on two following conformal mappings (Fig. A2.1):

$$\underline{w}_1 = R_1 \, \mathrm{e}^{\mathrm{j}\psi_1} = u_1 + \mathrm{j} v_1 = \sqrt{\underline{w}} \tag{a2.1}$$

and

$$\underline{w} = u + \mathrm{j} v = R \, \mathrm{e}^{\mathrm{j}\psi} = \mathrm{e}^{-\mathrm{j}\alpha} \frac{a \, \mathrm{e}^{\mathrm{j}\alpha} - \underline{z}}{\underline{z} - a \, \mathrm{e}^{-\mathrm{j}\alpha}} \, , \, \underline{z} = x + \mathrm{j} y = r \, \mathrm{e}^{\mathrm{j}\theta} \, . \tag{a2.2}$$

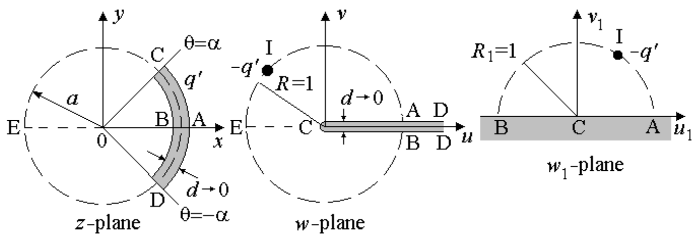

Fig. A2.1

Electrostatic system is uncharged. So in infinity, for $\underline{z} \to \infty$, point I, exists line charge having density $-q'$. By using image theorem, the complex potential is

$$\underline{\Phi} = U - \frac{q'}{2\pi\varepsilon} \ln \frac{\underline{w}_1 - \underline{w}_{10}}{\underline{w}_1 - \underline{w}_{10}^*} , \underline{w}_{10} = \mathrm{e}^{\mathrm{j}(\pi - \alpha)/2} \tag{a2.3}$$

For $z >> a$ it is

$$\underline{\Phi} = U - \frac{q'}{2\pi\varepsilon} \ln \frac{\underline{z}}{a_\mathrm{e}} \, , \tag{a2.4}$$

where

$$a_\mathrm{e} = a \sin(\alpha/2) \tag{a2.5}$$

is the equivalent radius of oval strip conductor.

If strip conductor is arbitrary oval, noncircular, then three points (two end points, 1 and 3, and one on the middle strip, 2) can be used to generate circular form, as it is shown in Fig. A2.2.

Table TA2.1 The positions of several points of interest in the present conformal mapping

points	z – plane	w – plane	w_1 – plane
A	$r = a + 0, \theta = 0$	$R = 1, \ \psi = +0$	$R_1 = 1, \ \psi_1 = +0$
B	$r = a - 0, \theta = 0$	$R = 1, \ \psi = 2\pi$	$R_1 = 1, \ \psi_1 = \pi$
C	$r = a, \theta = \alpha$	$\underline{w} = 0$	$\underline{w}_1 = 0$
D	$r = a, \theta = -\alpha$	$\underline{w} \to \infty$	$\underline{w}_1 \to \infty$
E	$r = a, \theta = \pi$	$R = 1, \ \psi = \pi$	$R_1 = 1, \ \psi = \pi/2$
I	$\underline{z} \to \infty$	$R = 1, \ \psi = \pi - \alpha$	$R_1 = 1, \ \psi_1 = (\pi - \alpha)/2$
0	$\underline{z} = 0$	$R = 1, \ \psi = \pi + \alpha$	$R_1 = 1, \ \psi_1 = (\pi + \alpha)/2$

The circle center and radius are defined as

$$x_0 = \frac{(x_1^2 + y_1^2)(y_3 - y_2) + (x_2^2 + y_2^2)(y_1 - y_3) + (x_3^2 + y_3^2)(y_2 - y_1)}{2[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]} \, , \tag{a2.6}$$

$$y_0 = \frac{(x_1^2+y_1^2)(x_2-x_3)+(x_2^2+y_2^2)(x_3-x_1)+(x_3^2+y_3^2)(x_1-x_2)}{2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]} \tag{a2.7}$$

and

$$r_0 = \sqrt{(x_1-x_0)^2+(y_1-y_0)^2}\ . \tag{a2.8}$$

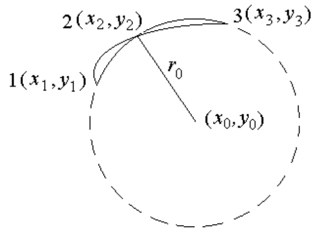

Fig. A2.2

Appendix III

It considers electrostatic system of two parallel, infinite line conductors, as on Fig. A3.1. If conductor charges per unit length are located on the conductor axis, the potential is

$$\varphi = \frac{q_1'}{2\pi\varepsilon}\ln\frac{r_p}{\sqrt{x^2+y^2}}+\frac{q_2'}{2\pi\varepsilon}\ln\frac{r_p}{\sqrt{(x-d)^2+y^2}}\ . \tag{a3.1}$$

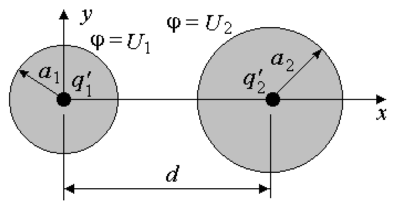

Fig. A3.1

Evidently, the presented potential expression does not satisfy boundary condition that the conductors are equipotential. In order to minimize the error in the boundary condition, the conductor potentials will be determined as mean values of approximate potential solution on the electrode surfaces. So it is

$$U_1 = \frac{1}{2\pi}\int_0^{2\pi}\varphi(x=a_1\cos p, y=a_1\sin p)\,\mathrm{d}p =$$

$$= \frac{q_1'}{2\pi\varepsilon}\ln\frac{r_p}{a_1}+\frac{q_2'}{4\pi^2\varepsilon}\int_0^{2\pi}\ln\frac{r_p}{\sqrt{d^2+a_1^2-2a_1d\cos p}}\,\mathrm{d}p\,. \tag{a3.2}$$

By expanding logarithmic function into Fourier series,

$$\ln\frac{r_p}{\sqrt{d^2+a_1^2-2a_1d\cos p}} = \ln\frac{r_p}{d}+\sum_{m=1}^{\infty}\left(\frac{a_1}{d}\right)^m\frac{\cos(mp)}{m},\ a_1\le d\ , \tag{a3.3}$$

the present integration gives

$$U_1 = \frac{q_1'}{2\pi\varepsilon}\ln\frac{r_\mathrm{p}}{a_1} + \frac{q_2'}{4\pi^2\varepsilon}\ln\frac{r_\mathrm{p}}{d} \cdot \tag{a3.4}$$

In similar way it can determine the mean value of other electrode mean potential,

$$U_2 = \frac{q_1'}{2\pi\varepsilon}\ln\frac{r_\mathrm{p}}{d} + \frac{q_2'}{4\pi^2\varepsilon}\ln\frac{r_\mathrm{p}}{a_2} \cdot \tag{a3.4}$$

Appendix IV

Let

$$\underline{z}_n = a\,\mathrm{e}^{\mathrm{j}\theta_n} \text{ and } \theta_n = (n-1)\frac{2\pi}{N},\ n = 1,2,...,N$$

denote rolls of algebraic equation

$$\underline{z}^n - a^n = 0\ , \tag{a4.1}$$

then

$$\underline{z}^n - a^n = \prod_{n=1}^{N}(\underline{z} - \underline{z}_n) \tag{a4.2}$$

and

$$\left|\underline{z}^n - a^n\right| = \sqrt{r^{2N} + a^{2N} - 2a^N r^N \cos(N\theta)} = $$
$$= \prod_{n=1}^{N}\left|\underline{z} - \underline{z}_n\right| = \prod_{n=1}^{N}\sqrt{r^2 + a^2 - 2ar\cos(\theta - \theta_n)}\,. \tag{a4.2}$$

Appendix V

By using the process of the arithmetic and geometric mean [18], the value of the complete elliptic integral of the first kind can be calculated very exact and quickly. It starts with a given number triple

$$a_0 = 1, b_0 = k' = \sqrt{1-k^2}, c_0 = a_0 - b_0, \tag{a5.1}$$

and it determines number triple a_n , b_n , c_n according to the following scheme

$$a_{n+1} = \frac{a_n + b_n}{2}, b_{n+1} = \sqrt{a_n b_n}, c_{n+1} = a_{n+1} - b_{n+1}. \tag{a5.2}$$

If $n = N$ is sufficient large, then $a_N \approx b_N$ and $c_N \to 0$ and the value of the elliptic integral of the first kind is

$$K(\pi/2, k) \approx \frac{\pi}{a_N + b_N}\ . \tag{a5.3}$$

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PRIMENA METODA EKVIVALENTNE ELEKTRODE U MEHANICI FLUIDA I PRI PRENOŠENJU TOPLOTE KONDUKCIJOM

Dragutin M. Veličković

Zadnjih godina na Elektronskom fakulktetu u Nišu razvijen je jedan nov metod, takozvani Metod Ekvivalente Elektrode, za približno numeričko rešavanje problema bezvrtložnih polja Teorijske fizike. Iako je prvenstveno ovaj metod razvijen za rešavanje problema elektrostatičkih polja, on je, zahvaljujući postojećim analogijama, sa uspehom primenjen i na probleme stacionarnih magnetnih polja, niskofrekventnog uzemljenja, prostiranja po vodovima, prenošenja toplote kondukcijom, kao i strujanja nestišljivih fluida. U suštini metoda je ideja da se stvarne elektrode posmatranog sistema zamene pomoću ekvivalentnih elektroda. Ove ekvivalentne elektrode se postavljaju po površini posmatranih tela i tako biraju i dimenzionišu da ga u potpunosti reprezentuju i zamenjuje. U zavisnosti od geometrije posmatranog sistema, kao ekvivalentne elektrode koriste se uniformni linijski izvori neograničene dužine (kod plan-paralelnih sistema), male loptaste ekvivalenente elektrode (kod trodimenzionih sistema) i tanki kružni lineični obruči (kod sistema sa aksijalnom simetrijom). U slučajevima kada sistem poseduje veći broj tela ili postoje višeslojne sredine, koriste se Grinove funkcije pobrojanih izvora u odnosu na prisutna tela, odnosno slojevite sredine. Metod je veoma jednostavan za primenu i tačan. U toku njegove primene nema potrebe za numeričkom integracijom, tako da eventualne loše uslovljenosti sistema obrazovanih linearnih jednač!ina nisu do sada uočene. U graničnom slučaju, kada je uzet neograničeno veliki broj ekvivalentnih elektroda, metod dovodi do tačnog rešenja. Pored teorijskog izlaganja, u radu je prikazan i veći broj podesno odabranih primera, koji ilustruju način primene i ostvarenu tačnost i konvergenciju rezultata.