FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol. 3, N° 14, 2003, pp. 937 - 942

SYNTHESIS OF THE NONLINEAR ELASTIC CHARACTERISTICS IN A SYSTEM WITH MANY DEGREES OF FREEDOM IN SINPHASE OF HARMONIOUS OSCILLATION

UDC 531+534+534.141

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Abstract. *In this project, a synthesis of nonlinear elastic characteristics is processed into a one-sided bonded chain system with a final number of degrees of independency, for given periodic disturbances with a whole-number values of circular frequencies of inducement and at given sinphase of harmonious oscillation of mass system Using Chebishevliev`s polynomials, in a project exact analytical expressions are extracted for nonlinear elastic characteristics in a system that is being observed.*
The exactness of extracted analytical expressions for nonlinear elastic characteristics is confirmed by results derived from solving a specific example.

INTRODUCTION

In a phase of projecting phase-oscillation machines, whose organs when performing its functions should accomplish certain oscillation movement, synthesis of a certain number of mechanic characteristics is being executed [3,4].

In this project, a synthesis of nonlinear elastic characteristics is processed into a one-sided bonded chain system with a finite number of degrees of freedom, for given periodic disturbances with a whole-number values of circular frequencies of inducement and at given sinphase of harmonic oscillation of mass system. Using Chebishevliev`s polynomials, in a project exact analytical expressions are extracted for nonlinear elastic characteristics in a system that is being observed.

The exactness of extracted analytical expressions for nonlinear elastic characteristics is confirmed by results derived from solving a specific example.

Received June 10, 2003

MARKS IN A TEXT

r,j,k	- index,
s	- number of degrees of freedom,
t [T]	- time,
$q_1=q_{1a}$ [L]	- absolute generalized coordinates(movement at first phase),
q_r, (r=2,3,…,s) (L)	- relative generalized coordinates,
$\ddot{q}_r$, (r=1,2,…,s) (LT^{-2})	- generalized acceleration,
m_r, (r=1,2,…,s) (M)	- mass,
$f_r(q_r)$, (r=1,2,…,s) [MLT^{-2}]	- nonlinear elastic characteristics,
$F_r(t)$, (r=1,2,…,s) [MLT^{-2}]	- disturbance forces,
F_{or}, (r=1,2,…,s) [MLT^{-2}]	- amplitude of disturbance forces,
k; k=1,2,… [T^{-2}]	- whole-number values of circular frequencies of inducement,
A_r, (r=1,2,…,s) [L]	- amplitude of oscillation,
x_r, (r=1,2,…,s) [L]	- phase coordinates,
$x_{s+r} = \dot{x}_r$ (r=1,2,…,s) [LT^{-1}]	- phase coordinates.

DIFFERENTIAL EQUAZIONS IN A SYSTEM OF MOVEMENT. INDUCEMENT AND LAWS OF SINPHASE OSCILLATION SYSTEM

Differential equations of movement of observed one-sided bonded chain system with a finite number of degrees of freedom (see Figure 1) are as follows:

$$m_1 \ddot{q}_1 = -f_1(q_1) + f_2(q_2) + F_1(t)\,,$$

$$m_1 \ddot{q}_1 + m_2 \ddot{q}_2 = -f_2(q_2) + f_3(q_3) + F_2(t)\,,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$m_1 \ddot{q}_1 + m_2 \ddot{q}_2 + \ldots + m_s \ddot{q}_s = -f_s(q_s) + F_s(t)\,,$$

or briefly

$$m_j \sum_{i=1}^{j} \ddot{q}_r = -f_j(q_j) + f_{j+1}(q_{j+1}) + F_j(t)\,, \qquad (j=1,2,\ldots,s) \tag{1}$$

where q_1, $q_{2,\ldots,}$ q_s appear as relative generalized coordinates.

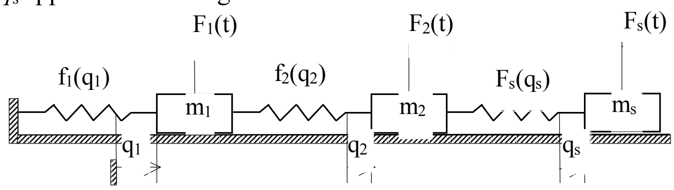

Fig. 1. The one-sided bonded chain system

Let's allow the disturbance forces of shape to effect a mass system:

$$F_r=F_{or}coskt\,,\ (r=1,2,\ldots,s;\ \ k=1,2,3,\ldots), \tag{2}$$

And let's consider it is needed for a system to produce sinphase oscillation process:

$$q_r = A_r \cos t\,, \qquad (r=1,2,\ldots,s). \tag{3}$$

Absolute mass movement of observed system will be:

$$q_{ja} = \sum_{r=1}^{j} q_r\,, \quad (j = 1,2,\ldots,s)\,. \tag{4}$$

CHART OF CHEBISHEVLIEV'S POLYNOMIALS

Chebishev's polynomials [3,5] is defined as

$$T_k\left(\frac{q}{A}\right) = \cos\left(k \arccos\frac{q}{A}\right), \quad (k = 0,1,2,\ldots)\,, \tag{5}$$

Taken that $\frac{q}{A} \in [-1,1]$.

From equation (5) for k=0 and k=1, in this order we get $T_o\left(\frac{q}{A}\right) = 1,\ T_1\left(\frac{q}{A}\right) = \frac{q}{A}$. Chebishev`s polynomials can be determined by recursive procedure based on first two polynomials (T_0 and T_1). By introducing $\theta = \arccos\frac{q}{A}$ in trigonometrically identity

$$\cos(k+1)\theta + \cos(k-1)\theta \equiv 2\cos k\theta \cos\theta,$$

We get a recursive formulae

$$T_{k+1}\left(\frac{q}{A}\right) = 2\frac{q}{A} T_k\left(\frac{q}{A}\right) - T_{k-1}\left(\frac{q}{A}\right),$$

based on which, chart of Chebishev's polynomials is formed:

$$T_o\left(\frac{q}{A}\right) = 1\,,$$

$$T_1\left(\frac{q}{A}\right) = \frac{q}{A}\,,$$

$$T_2\left(\frac{q}{A}\right) = 2\left(\frac{q}{A}\right)^2 - 1\,,$$

$$T_3\left(\frac{q}{A}\right) = 4\left(\frac{q}{A}\right)^3 - 3\left(\frac{q}{A}\right),$$

$$T_4\left(\frac{q}{A}\right) = 8\left(\frac{q}{A}\right)^4 - 8\left(\frac{q}{A}\right)^2 + 1\,,$$

$$T_5\left(\frac{q}{A}\right) = 16\left(\frac{q}{A}\right)^5 - 20\left(\frac{q}{A}\right)^3 + 5\left(\frac{q}{A}\right),$$

$$T_6\left(\frac{q}{A}\right) = 32\left(\frac{q}{A}\right)^6 - 48\left(\frac{q}{A}\right)^4 + 16\left(\frac{q}{A}\right)^2 - 1\ \ldots$$

NONLINEAR ELASTIC CHARACTERISTICS

By introducing (2) and (3) into (1), for a system with one degree of freedom ($s = 1$), follows

$$-m_1 A_1 \cos t = -f_1(t) + F_{01} \cos kt,$$

or

$$f_1(t) = m_1 A_1 \cos t + F_{01} \cos kt,$$

from where (taking into consideration (3)), the exact analytical expression with nonlinear elastic characteristic is derived

$$f_1(q_1) = m_1 A_1 \frac{q_1}{A_1} + F_{01} T_k\left(\frac{q_1}{A_1}\right). \tag{a}$$

Using the some procedure for a system with two degrees of freedom the following expression is reached:

$$f_1(q_1) = [m_1 A_1 + m_2(A_1 + A_2)]\frac{q_1}{A_1} + (F_{01} + F_{02})T_k\left(\frac{q_1}{A_1}\right),$$
$$f_2(q_2) = m_2(A_1 + A_2)\frac{q_2}{A_2} + F_{02} T_k\left(\frac{q_2}{A_2}\right). \tag{b}$$

Further more, for a system with three degrees of freedom, follows:

$$f_1(q_1) = [m_1 A_1 + m_2(A_1 + A_2) + m_3(A_1 + A_2 + A_3)]\frac{q_1}{A_1} + (F_{01} + F_{02} + F_{03})T_k\left(\frac{q_1}{A_1}\right),$$
$$f_2(q_2) = [m_2(A_1 + A_2) + m_3(A_1 + A_2 + A_3)]\frac{q_2}{A_2} + (F_{02} + F_{03})T_k\left(\frac{q_2}{A_2}\right), \tag{c}$$
$$f_3(q_3) = m_3(A_1 + A_2 + A_3)\frac{q_3}{A_3} + F_{03} T_k\left(\frac{q_3}{A_3}\right).$$

Based on derived expressions (a), (b) and (c), from which are observed the laws, general exact analytical expressions for nonlinear elastic characteristics for a system with a finite number of degrees of freedom, expressions (6), can be briefly written, which is exactly what was meant to be indicated by this paper.

$$f_j(q_j) = \left(m_j \sum_{r=1}^{j} A_r + m_{j+1} \sum_{r=1}^{j+1} A_r + \ldots + m_{s-1} \sum_{r=1}^{s-1} A_r + m_s \sum_{r=1}^{s} A_r\right)\frac{q_j}{A_j} + \sum_{r=j}^{s} F_{or} T_k\left(\frac{q_j}{A_j}\right), \tag{6}$$
$$T_k\left(\frac{q_j}{A_j}\right) = \cos\left(k \arccos \frac{q_j}{A_j}\right), \quad (j=1,2,...,3 \;\; ; \;\; k=0,1,2,...).$$

From expression (6) it can be observed that at the synthesis of nonlinear elastic characteristics can, if needed in specific case, combine the following parameters: F_{or} , A_r, m_r, k (r=1,2,...,s). Taking into consideration the analogy between the chain and torsion systems (1), expression (6) can be applied on one-sided bonded torsion system.

Exacts of derived expression (6) confirms results received from solving a specific example, for: s=2, m_1=m_2=1, A_1=A_2=1, F_{01}=0, F_{02}=0,5 and k=3. Notice: all parameters with dimensions are taken in base units of International system of measures.

In this example analytical expressions for elastic characteristics are:

$$f_1(q_1) = 3q_1 + F_{02}(4q_1^2 - 3q_1)\,,$$

$$f_2(q_2) = 2q_2 + F_{02}(4q_2^2 - 3q_2)\,.$$

The geometric visualization of nonlinear elastic characteristics is given in Figure 2.

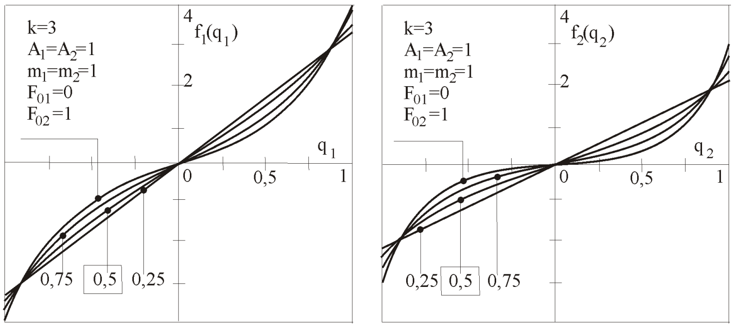

Fig. 2. The geometric visualization of nonlinear elastic characteristics.

In Figure 3 – inducement is shown.

Lows of expected sinphase oscillation of a system, received by numeric integrating corresponding differential equations of movement (6):

$$\dot{x}_1 = x_3\,,$$

$$\dot{x}_2 = x_4\,,$$

$$\dot{x}_3 = -1{,}5x_1 - 2x_1^3 + 0{,}5x_2 + 2x_2^3\,,$$

$$\dot{x}_4 = 1{,}5x_1 + 2x_1^3 - x_2 - 4x_2^3 + 0{,}5\cos 3t\ ;$$

For initial conditions of movement $x_1(0)$=$x_2(0)$=1 and $\dot{x}_1(0) = \dot{x}_2(0) = 0$, are given on pictures 4 and 5 or 6.

CONCLUSION

In our paper, the exact analytical expressions for nonlinear elastic characteristics in one-sided bonded mechanical chain system with a finite number of degrees of freedom are derived, for given periodic disturbances forces with whole-number values of circular frequencies of inducement and at given sinphase harmonious oscillation of mass system. These expressions can be used in a case of one-sided bonded torsion system.

The exactness of extracted expressions confirms the results derived from solving a specific example.

In our future work on synthesis of nonlinear elastic characteristics it would be interesting to determine synthesis of these characteristics in a case of one-sided bonded chain system with many degrees of freedom for given complex periodic disturbances and at given multiple harmonic (biharmonic and polyharmonic) oscillation of mass system.

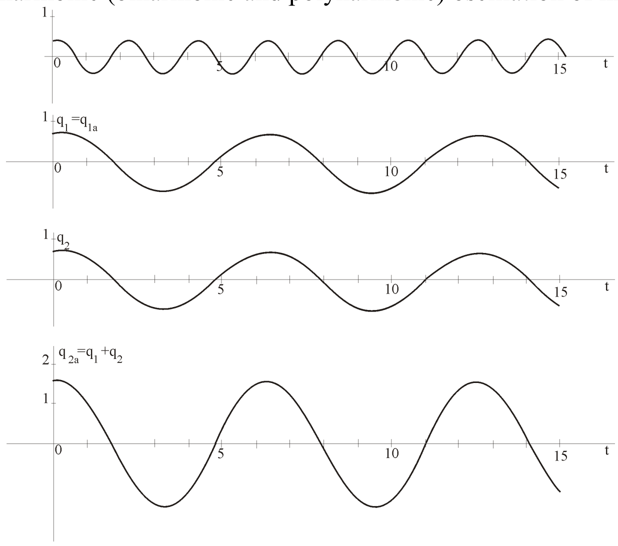

Fig. 3, 4, 5. and 6. Elongation-time diagrams for generalized coordinates

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SINTEZA NELINEARNIH ELASTIČNIH KARAKTERISTIKA U SISTEMU SA VIŠE STEPENI SLOBODE PRI SINFAZNOM HARMONIJSKOM OSCILOVANJU

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U ovom radu se obradjuje sinteza nelinearnih elastičnih karakteristika u jednostrano-vezanom lančanom sistemu sa konačnim brojem stepeni slobode za zadate periodične poremećaje sa celobrojnim vrednostima kružnih frekvencija pobude i pri zadatom sinfaznom harmonijskom oscilovanju masa sistema. Korišćenjem Čebiševljevih polinoma, u radu se izvede tačni analitički izrazi za nelinearne elastične karakteristike u posmatranom sistemu. Tačnost izvedenih analitičkih izraza za nelinearne karakteristike potvrdjuje se rezultatima dobijenim rešavanjem konkretnog primera.