FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol.3, N^o 14, 2003, pp. 881 - 890

Invited Paper

TO MODELLING PROBLEM IN MECHANICS (THEORETICAL AND APPLIED ASPECTS)

UDC 531.011+621.8.031

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Abstract. *The research is devoted to the development of approximate methods in the complex systems dynamics on base of Lyapunov's stability theory methods for singular perturbations problems and their applications. It allows to obtain a solving urgent mathematical modelling problems in Mechanics; to work out the reduction principle in general qualitative analysis of complex systems.*
The research subject is complex large-scale systems, for which the original mathematical model, adequate real object, is extremely complex. The principal tasks are the elaborating universal methods of modelling; the constructing correct simplified models; the rigorous substantiating these reduced models in dynamics; the estimating errors and admissible parameters domains under using these reduced models (with keeping of qualitative equivalence).
Suggested method is combining the stability theory and perturbations theory methods based on two postulates (stability and singularity ones). It allows to work out the effective manners of rigorous analysis with general methodology of constructing simplified models and their analysing; with dividing of original problem on separate particular ones; with decomposing of original model and its dynamic characteristics; with building shortened models hierarchy; with revealing essential variables and freedom degrees. Besides the elaborated approach is extending the traditional statements of stability problems and perturbations theory, with the treating modelling problem as stability problem (in specific sense) under singular perturbations, with the extension of N.G.Chetayev's, K. P. Persidskiy ideas. Moreover the research results will allow to come nearer to fundamental modelling problem in complex systems dynamics. This problem is concerned with reduction principle in general qualitative analysis and with singularity problem in Mechanics. The results allow to get the developing theory of approximate methods for rigorous analysis; to give substantiation of approximate theories and models in Mechanics (both for traditional ones, well-known in engineering practice, and for new models, that are constructed by this developed method; including non-Newton models of Mechanics).

Received April 20, 2003

1. INTRODUCTION

Under the study and the designing of any real object in engineering practice very many various aspects arise from first stages. This research deals with the elaboration of general methods [1-5] in qualitative analysis of dynamical systems and the applications of these methods to the modelling problems. As rule, original technical object (IO) and initial mathematical model (IMM) is non-linear, multi-connected, high-dimensional one. The complexity is leading to the necessity of the simplification of original full model (FM), with the separation of the system parameters on the substantial and non-substantial ones, with the revealing of main degrees of system freedom, with the subsequent transition to the shortened model [6,7] (Sh.M)

$$\text{FM} \rightarrow \text{Sh.M.}$$

Besides the Sh.M is obtained "on intuition", without rigorous mathematical analysis of influence of rejected members on dynamic properties. The correctness problem and qualitative equivalence are non-discussed. The criterion of validity of this Sh.M of "intuitional level" is the experience.

As example:

In dynamics of the mechanical systems the original full model is Lagrange's equations in form [4]

$$\text{FM} \quad a q^{\bullet\bullet} + b q^{\bullet} + e q = \ldots\ldots \tag{1}$$

that describes the behaviour of the investigated object; k is the number of the freedom degrees of FM; $N=2k$ is the order of (1). In engineering practice the researchers neglect the inertial members [2,6] and transit to the shortened model of type

$$\text{Sh. M} \quad b q^{\bullet} + e q = \ldots\ldots \tag{2}$$

N_{Sh} is the order of (2); $N_{Sh}<N$ (here $N_{Sh}= N/2$).

From the view point of mechanics it means the transition from FM (with k – number of freedom degrees) to the simplified system with k_{Sh} – number of freedom degrees ($k_{Sh} = N_{Sh}/2$; $k_{Sh}<k$). But in general case the equations (2) are only formalized mathematical construction, that does not describe the behaviour of some real object [7]; and in general case k_{Sh} is non-integer number [6]. In this case the mathematical decomposition [8] of initial system is realized. As next example consider the case, when the researches transit from FM with non-absolutely rigid elements to the shortened model with absolutely rigid ones

$$\text{Sh.M}' \quad a_1 q_1^{\bullet\bullet} + b_1 q_1^{\bullet} + e_1 q_1 = \ldots\ldots \tag{2'}$$

in (2') q_1 is k'_{Sh} – vector; $k'_{Sh}<k$ is the number of the freedom degrees of Sh.M' (absolutely rigid model).

In this case the physical decomposition [8] is realized, and we have here also the reduction to the system of more low order.

But these situations are generating non-ordinary problems, that are very urgent both for theory and for engineering applications: the reduction procedure to the model of less order [1,2]; the special case of full mathematical decomposition [7,8].

These problems have no only mathematical aspects as singular problems [5], but also philosophical sense. Here interesting questions are generated:

– what is this "Sh.M"?
– what is "essential" and "non-essential" degree of freedom?
– what may be discarded?
– how understand "may be"?

2. Main statements

Our task is the principal problems:

– modelling problems (constructing the simplified model (Sh.M) by rigorous mathematical way);
– correctness problems (acceptability conditions in reducing the investigation of FM to the Sh.M);
– estimation problems (determining the domains of admissible parameters).

Note, rigorous mathematical statements, rigorous solutions for concrete dynamic properties, for concrete cases of Sh. M there are in works of many authors, beginning from early classical works of A. M. Lyapunov (in stability theory); A. A. Andronov (in theory of oscillations); M. V. Meerov, A. A. Feldbaum (in control theory); I. S. Gradstein, A. N. Tikhonov (in theory of differential equations); D. R. Merkin (in gyroscopes theory); ...It is different approaches, manners, methods for various statements of reduction problem. But it is necessary to get the general approach and the rigorous solution for fundamental modelling problem; to elaborate the engineering methods for our principal problems.

As initial assumptions we take Chetayev's stability postulate [9] and the singularity postulate (L. K. Kuzmina). Besides any object is treated as one of singularly perturbed class [10], for that IMM always may be represented in standard form as singularly perturbed system (SPS) [11], corresponding transformation of variables always may be constructed. Besides shortened model is treated as asymptotic model of s-level on small parameter μ. The constructing of this Sh.M is realized by elaborated method, that is developing the regular algorithm (simple scheme) of engineering level

$$\underset{(q,q^\bullet)}{\mathrm{IO}} \rightarrow \mathrm{IMM} \rightarrow \underset{(y)}{\mathrm{SPS}} \rightarrow \underset{(y)}{\mathrm{Sh.S}} \rightarrow \underset{(q,q^\bullet)}{\mathrm{Sh.M}} \tag{3}$$

$(q,q^\bullet)$ is vector of initial phase variables; y is vector of new variables, with what IMM is led to the standard form with the non-regular members

$$\mathrm{SPS} \qquad M(\mu)\frac{dy}{dt} = Y(t,\mu,y) \tag{4}$$

With reference to requirement of mechanics we assume here $\mu > 0$ is small dimensionless parameter; y is the vector of state variables, connected with the initial phase variables (generalized coordinates) by corresponding transformation; $M(\mu)$ is matrix, for that the elements may contain μ parameter in different powers (generally speaking, non-integer ones) $\mu^{\alpha i}$, $0 \le \alpha_i \le r$; $Y(t,\mu,y)$ is non-linear vector function, that is holomorphic one on the set of all variables, including μ.

As shortened system for (4) we shall introduce the simplified system of s-level (s-system) [12]

$$\text{Sh.S} \qquad M^s(\mu)\frac{dy}{dt} = Y^s(t,\mu,y) \tag{5}$$

System (5) is approximate one for (4), in what the members with $\mu^{\alpha i}$ are kept only for $\alpha_i \le s$; and $s < r$ is concrete, chosen in advance, number. Notice, we use, as generating system, this singularly perturbed system (5) for original SPS (4).

It is non traditional extended approach, more fruitful for engineering applications, very perspective for formulated above problems in mechanics. Many familiar Sh.M, widely used in applications, are not limit models, and the traditional technique of limiting transition does not give the possibility to obtain these models, also does not allow to investigate specific critical cases, characteristic for such systems. Therefore it is necessary new approaches that are developed here. In addition we receive the possibility to introduce the hierarchical sequence of different simplified systems, in accordance with the hierarchy of variables on small parameter μ

$$SS_0,\ SS_1,\ SS_2,\ldots,\ SS_{r\text{-}1} \tag{6}$$

and, after, returning to old variables (q, $q^\bullet$), we obtain the simplified models sequence

$$SM_0,\ SM_1,\ SM_2,\ldots,\ SM_{r\text{-}1} \tag{7}$$

It is effectual method, solving the important problem of the constructing comparison systems and comparison models.

Elaborated manner permits to detour many features in the singular systems theory and to solve the main problems: the building of correct simplified systems by strict mathematical way; the substantiation of acceptability for these shortened systems; the estimation of the acceptable domains. For the models validation, for determination of correctness conditions [13,14] we use the ideology of stability theory. Following to ideas of N.G.Chetayev, in accordance with the methods of Lyapunov, we solve here the dynamic problems (of stability, of proximity, of optimality, of quick-operating,..., both in non-critical and critical cases; in simple and multiple roots,...) as singular problems [15].

3. Acceptability problems

Let original differential equations, describing the state of initial object, are led to the standard form (4).

Let the s-system is accepted as the approximate (comparison) system in form (5).

It is the simplified system, that is different from traditional degenerate one in the singularly perturbed system theory. The system (5) has the less order than (4). The next problem: under which conditions the qualitative investigation of system (4) can be reduced to the investigation of system (5); when there is the proximity between the corresponding solutions of systems (4) and (5) on the infinite interval of time; when the dynamical properties possess by the decomposition and when ones can be determined on the shortened system (5) (in analysis of stability; quick-operating; optimal parameters; …); how may be estimated the μ-values, that will be permitting such transition.

For the solving of these tasks we shall use the methodology of stability methods. General approach, based of the methods of Lyapunov's theory, combining the methods of the theory of singular perturbations and the ideas of Chetayev, permits to consider the particular cases, which are interesting both for the theory and for the applications. Following Chetayev [2], we shall introduce in the considering the differential equations for the new variables $b = y - y^S$:

$$\tilde{M}(\mu)\frac{db}{dt} = B(t,\mu,b) \tag{8}$$

Here $y = y(t,\mu)$ is the solution of full system (4), $y^S = y^S(t,\mu)$ is the corresponding solution of shortened system (5).

Using Lyapunov's methods [2,11] (first and second), analyzing (8), we shall be able to determine the required conditions in the solving of singular problems, in the estimates of the parameter values and corresponding domains, and correspondingly – in the solving of acceptability problem in general.

It is necessary to note, here unlike perturbation theory we do not consider limit transition under $\mu \to 0$; here we introduce another statement: for given in advance value of $\varepsilon > 0$ it is necessary to determine the corresponding conditions and corresponding μ^* - value a such that with $\mu \leq \mu^*$ $\|y - y^S\| < \varepsilon$ will be on infinite time interval; and here Sh.S is SPS also.

4. Dynamic problems

Here we shall investigate the system of considered type for those specific cases, that are inherent in applied problems. Let the original system

$$M(\mu)\frac{dy}{dt} = Y(t,\mu,y) \tag{9}$$

where $y = \|y_1, y_2, y_3\|^T$ is vector; $M(\mu) = \|M_{ij}(\mu)\|$ is block matrix, $M_{ii}(\mu) = \mu^{\alpha i} I$; $\alpha_i \geq 0$ (i=1,2,3) are constant numbers; I are identity matrices; $Y(t,\mu,y)$ is holomorphic (on the all variables set) vector function with the continuous, limited (on t,μ) coefficients. Assuming the considering the critical cases [1,16], we take in (9) $y_1 = x_1$, $y_2 = x_2$, $y_3 = \| x_3, z \|^T$, x_i are n_i -dimensional vectors of basic variables; z is m -dimensional ($m \geq 0$) vector of critical variables [1]; $Y_i = \|P_i(\mu)x + X_i(t,\mu,z,x)\|$, (i = 1,2); $Y_3 = \| P_3(\mu)x + X_3(t,\mu,z,x); Z(t,\mu,z,x) \|^T$; $P_i(\mu)$ are matrices of the corresponding sizes; X_i, Z are non-linear vector functions; $Y(t,\mu,z,0) = 0$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\alpha_3 = 0$ (it is characteristic peculiarity of many dynamic systems in Newton's Mechanics). Acceptability problem is containing separate subproblems.

A. Problem of stability

We introduce as a comparison system for such system (9) the s-system (let s = 1), denoting here the members set, containing μ in power no more than "s", by "s"-index:

$$M^s(\mu)\frac{dy}{dt} = Y^s(t,\mu,y) \tag{10}$$

We shall say, that (9) describes the slow motion of "s-level" (with the corresponding characteristic equation $\lambda_m D_S(\lambda,\mu) = 0$). Then the fast motions characteristics are determined by the auxiliary equation

$$d(\beta) = |\,\beta I - P_{11}(0)\,| = 0 \tag{11}$$

The problem: in which conditions the stability property for full system (9) is ensuing from the stability property for this s-system.

Theorem A. When $|P(0)| \neq 0$ and the equations $D_S(\lambda,\mu) = 0$, $d(\beta) = 0$ satisfy the Hurwitz's conditions, then with sufficiently small μ-values the zero-solution stability of system (9) follows from the zero-solution stability of system (10).

Here we have the asymptotic stability (for $m = 0$) or non-asymptotic one (for $m > 0$: the critical case [1]). We shall call this property for full system (9) as s-stability property.

Remark 1A. Not dwelling on the proof, we note only, that the system (9) with sufficiently small μ-values is the Lyapunov's system [1]. Using the corresponding Lyapunov's manners, we receive this statement **A**.

Remark 2A. Let the auxiliary equation (11) does not satisfy the theorem conditions, having the imaginary roots (special case) β_0. Then for s-stability property we shall have to take the second requirement in theorem **A** in form:
the equations $D_S(\lambda,\mu) = 0$ and

$$d_S(\beta,\mu) = \begin{vmatrix} \beta\ I - P_{11}^S; & -P_{12}^S \\ -\mu P_{21}(0); & \beta\ I - \mu P_{22}(0) \end{vmatrix} = 0 \tag{12}$$

satisfy the Hurwitz's conditions, and also

$$\frac{\partial}{\partial\mu} d_S(\beta = \beta_0,\ \mu = 0) \neq 0 \tag{13}$$

Here we have the critical spectrums for the matrices of slow and fast variables. This result gives important supplement for critical cases of singular system.

B. Problem of proximity (set stability)

The question: in which cases the corresponding solutions of systems (9) and (10) will remain near to each other for all $t > 0$? This problem is directly connected with **A** [1,11].

Let $y = y(t,\mu)$ is the solution of system (9) with the initial conditions $y_0 = y(t_0,\mu)$; $y^S = y^S(t,\mu)$ is the solution of system (10) with the initial conditions $y^S{}_0 = y^S(t_0,\mu)$. Here $y^S{}_1 = x^S{}_1 = f_1(t,\mu,z^S,x^S{}_2, x^S{}_3)$ is determined from the equation $0 = P^S{}_1\, x + X^S{}_1$.

Following to N. G. Chetayev [2] we shall consider the equations of type (8) for the deviations. Employing (as in previous cases) stability methods, we shall be able to obtain the results for the particular cases. Not producing the details (and proof), we adduce the general results for the corresponding case.

Theorem B. If $|P(0)| \neq 0$, equations $D_S(\lambda,\mu) = 0$ and $\begin{vmatrix} P_{11}(0); & P_{12}(0) \\ -P_{21}(0); & \alpha\ I - P_{22}(0) \end{vmatrix} = 0$ satisfy Hurwitz'sconditions and matrix $P_{11}(\mu)$ is stable matrix (when $\mu = 0$), then for sufficiently small μ - values, for the given in advance numbers $\varepsilon > 0$, $\delta > 0$, $\gamma > 0$, there is such μ_* - value, that in perturbed motion for $0 < \mu \le \mu_*$, for all $t \ge t_0 + \gamma$ we have $\| y - y^S \| < \varepsilon$, if $\| y_{10} - y^S_{10} \| < \delta$ $y_{20} = y^S_{20}$, $y_{30} = y^S_{30}$.

We shall call this property for original system the *s*-proximity property (*s*-set stability property).

Remark 1B. If μ-value is sufficiently small, then $\| y_i - y^S_i \| < \varepsilon$, (i=2,3) for all $t \ge t_0$.

Remark 2B. We can receive more general statement analogous to statement **B** but when all initial conditions for the solutions of systems (9), (10) do not coincide: $\| y_{i0} - y^S_{i0} \| < \eta$, (i=2,3), where η is the small value.

Remark 3B. Also (as in **A**) there is the corresponding result in critical particular case of imaginary eigenvalues of matrix of fast variables.

Remark 4B. Analogous results were obtained also when we as shortened system for (9) will take the *s*-system with $s = 0$:

$$M\,(0)\,\frac{dy}{dt} = Y\,(t, 0, y) \tag{14}$$

These results, obtained by Lyapunov's methods, supplement and generalize the known results for the singular systems in considered here critical cases.

From the point of stability theory it is the analogue of set stability property.

C. Problem of quick-operating

Now we shall consider the problem: under which conditions we shall be able to determine the quality of full system (the quick-operating, the optimality,...) on the shortened *s*-system (let for systems (9) and (14)).

Here the system quick-operating is characterized by the degree of stability [17]

$$\delta = \min_j |\operatorname{Re} \lambda_j|$$

λ_j is the root of the system characteristic equation. The parameters, with which δ has the maximum value, are called optimal ones (on quick-operating).

Let δ is the stability degree for full system (9) (for $m = 0$);

$$D(\lambda,\mu) = D_0(\lambda) + \mu D_1(\lambda) + \mu^2 D_2(\lambda) + ... = 0 \tag{15}$$

(15) is the characteristic equation for (9).

Let δ^s is the stability degree *s*-system (14) and the characteristic equation for (14) is

$$D_0(\lambda) = D(\lambda,0) = 0 \tag{16}$$

Let $\lambda = \lambda(\mu)$ is the root of equation (15); λ_0 is the root of equation (16). We have for corresponding roots:

$$\lambda(\mu) = \lambda_0 + \Delta(\mu).$$

Let δ_* is a given stability degree (required one in advance).

Using analogous methods, we can show

Theorem C. If $|P(0)| \neq 0$, and equations $D(\lambda,0) = 0$, $\begin{vmatrix} P_{11}(0); & P_{12}(0) \\ -P_{21}(0); & \alpha \quad I - P_{22}(0) \end{vmatrix} = 0$, $|\beta E - P_{11}(0)| = 0$ satisfy Hurwitz's-conditions, and roots λ_0 are simple ones, then the property of stability for the full system (9) with the given stability degree δ_* follows from the stability property for the shortened system (14) with the stability degree $\delta^s = 2\delta_*$, when $0 < \mu \leq \mu^*$.

Here μ^*-value is determined from the solution of equation

$$\max_j |\operatorname{Re}\Delta_i(\mu)| = \delta_* \tag{17}$$

Here $\Delta = \Delta(\mu)$ is holomorphic function of μ : $\Delta(\mu) = l_1 \mu + l_2 \mu^2 + \ldots$.

We shall call this property for original system the *s*-quick-operating property.

Remark 1C. It is known, that the problem of optimality on quick-operating is connected with the case of multiple roots λ_0 [17]. This case corresponds to optimal parameters (here δ has the maximum value). Theorem **C** does not include this case. But, using similar approach, we can obtain the corresponding conditions, under which the transition from full system to *s*-system in the problem of quality is possible also in the case of multiple roots. But here we have: $\Delta(\mu) = b_1 \mu^{1/N} + b_2 \mu^{2/N} + \ldots$ (N is the root multiplicity). In this case the usage of *s*-system in the determining system quick-operating gives more error than in case of simple roots λ_0. Moreover the domain of admissible μ-values is diminished here. Therefore the every case of optimal parameters is requiring the special analysis.

Remark 2C. This new result is very important one at present, because in last years the criteria of maximal stability degree is broadly used [18] for the synthesis of control systems.

5. EVALUATION PROBLEM

Correctness problem is connected with the of μ-value estimation problem, with permissible parameters domains, with obtaining errors.

The stability methods allow to obtain the μ-values estimates, that are permitting the transition to *s*-system in dynamic problem. For this we can use both first and second method of Lyapunov [1,2]. The example of some estimating by method of the Lyapunov's functions was given in early work of Chetayev (1957).

Here for considered systems we show results for the estimation of permissible μ-values in the stability problem. We shall introduce in the consideration all subsystems, on which the initial system is divided: the system, corresponding to slow variables of *s*-level ("*s*-system" in this paper) and the auxiliary subsystems, describing the fast variables. Following Chetayev, determining the conditions, that ensure the stability property for the full system by the conditions on the stability property for the every subsystem, we shall obtain the correlation's for the estimating μ.

Let the full system is (9); let the comparison system is (14), corresponding to slow variables y_3. Then, considering in first the subsystem of slow variables, we receive the estimation in form:

$\mu \le \mu_{3*}$, where μ_{3*} is the solution of the equation

$$\underset{j}{Sup} \,|\, \mathrm{Re}\,\Delta_j(\mu) \,| \;\; = \underset{j}{Inf} \,|\, \mathrm{Re}\,\lambda_{j0} \,| \tag{18}$$

In (18) $\lambda_j(\mu)$ is the root of equation (15); λ_{j0} is the corresponding root of (16) and $\lambda_j(\mu)=\lambda_{j0} + \Delta_j(\mu)$. Analogous estimations we obtain from the equations, corresponding to components y_2 and y_1 (middle and fast variables): $\mu \le \mu_{2*}$ and $\mu \le \mu_{3*}$. The sought estimate, determining the upper boundary of admissible μ-values domain, is

$$\mu_* = \underset{i}{Inf}\, \mu_{i*} \tag{19}$$

With all $\mu \le \mu_*$ the transition from the full system to the *s*-system is possible (in taken here sense: the stability property is conserved and there is the proximity between corresponding solutions on the infinite interval of time).

Remark. The estimation of μ-values, allowing the transition for all kinds problems, is the inexpedient aim (and, generally speaking, practically unsolvable).

6. Conclusion

These results are perspective both for the perturbations theory (the singular problems in specific cases are solved) and for applications in mechanics (the problems of mathematical modelling in oscillation theory; in systems theory with fast rotors, with non-rigid elements, with quick-responsive drives, with big friction,..., are discussed). This proposed approach corroborated the universality: for all investigated systems we constructed the corresponding transformation of variables, brought IMM to the standard form (4); obtained the simplified models using the scheme (3) with comparison models in form (5), that are acceptable in dynamic problems; determined the conditions of their correctness.

We considered the different engineering objects, using unified approach, as objects of singularly perturbed class: mechanical systems (MS) with non-rigid elements; gyroscopic systems (GS) with fast gyroscopes; with small gyroscopes; electromechanically systems (EMS) with small-inertia electrical circuits; systems of gyrostabilization (modeled as EMS) with quick-response follow subsystems; with fast gyroscopes in control subsystems; with big stabilized mass; robotic systems with fast-response drives; with non-absolute rigidity of elements; MS with big friction; point mass dynamics;...

Here we obtain the possibility to investigate a compound systems by analytical methods, to introduce the simplification of initial model with the division of variables on different-frequent groups; to construct new approximate models as comparison models. The investigations are revealing the interesting physical interpretation of these formalized mathematical "constructions".

Some general aspects of modelling, concerned with Newton's model in mechanics, are ones at issue in work (with accepted points, within the scope of Chetayev's stability postulate). Here the interesting gnosiological results are gained in regard to Newton's Mechanics model.

The author thanks the Russian Foundation of Fundamental Investigations for support of this research.

REFERENCES

1. A. M. Lyapunov, General problem on motion stability, Papers Coll., Vol.2, USSR Academy of Science, Moscow, 7-263, 1956.
2. N. G. Chetayev, "On evaluations of approximate integration", Appl.Math.and Mech., Vol.21, No.6, 1957, pp.419-421.
3. N. N. Bogolyubov, Yu.A.Mitropolsky, Asymp-totic methods in theory of non-linear oscillations, Moscow: St.publish., 1963.
4. L. K. Kuzmina, "General Modelling Problem in Mechanics", SAMS, vol.29, 1997, pp.105-118.
5. S. L. Campbell, Singular Systems of differential equations, London: Pitman Advanced Publishing Program, 1980.
6. A. A. Andronov, A.A.Vitt, S.E.Khaikin, Oscillations theory, Moscow: St. publish., 1959.
7. D. R. Merkin, Gyroscopic Systems, Moscow: St. publish., 1956.
8. D. D. Shiljak, Decentralized control of complex systems, N.Y.: Ac. Press, Inc., 1991.
9. N. G. Chetayev, "On stable trajectories of dynamics", Proceedings of Kazan Aviation Institute, No.5, 1936, pp.3-8.
10. L. K. Kuzmina, "Stability Theory Methods and Modelling Problems in Mechanics", Proc. of VIII-th National Congress of Theor. And Appl. Mechanics, Perm, 2001, p.379.
11. I. S. Gradstein, "Application of Lyapunov's stability theory to the differential equations theory with small multipliers before derivatives", Math.collect, Vol.32, No.2, 1953, pp.263-286.
12. L. K. Kuzmina, "Stability Methods in Theory of the Singularly Perturbed Systems", Advances in Model. and Analysis, ASME Press, 44(4), 1994, pp.56-64.
13. T. Bohlin, "Model validation", in Encycl.of Systems and Control (M.Singh, ed.), Oxford, Pergamon Press, 1987.
14. Lennart Ljung, System Identification: Theory for the User, Sweden: Prentice-Hall, 1987.
15. L. K. Kuzmina, "To Solving Singularly Perturbed Stability Problem", Appl. Math. and Mechanics, 55(4), 1991, pp.594-601.
16. L. K. Kuzmina, "To question about the approximate methods in analysis of singularly perturbed systems", Appl.Math.and Mech., Vol.49, No.6, 1985, pp.909-915.
17. I. S. Tsipkin, P.V.Bromberg, "To the degree stability of linear systems", News of Sciences Academy USSR, Tech.sc., No.2, 1945, pp.1163-1167.
18. G. I. Zagari, A.M.Shubladze, Control system synthesis on criteria of maximal stability degree, Moscow: Energoatomizd., 1988.

MODELIRANJE PROBLEMA U MEHANICI (TEORIJSKI I PRIMENJENI ASPEKTI)

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Istraživanje je posvećeno razvoju aproksimativnih metoda u kompleksnim sistemima dinamike na bazi metoda Ljapunovljeve teorije stabilnosti za singularne perturbacione probleme i njihove primene. Taj razvoj uvek zahteva neophodna rešenja modeliranja problema mehanike. Rad se bavi i redukcijom principa u generalizaciji kvalitativne analize kompleksnih sistema. Predložena metoda je kombinacija metoda teorije stabilnosti i teorije perturbacija (poremećaja) bazirana na dva postulata (stabilnosti i singularnosti). Pristup se sastoji u dekompoziciji originalnog modela i njegovih karakteristika, kao i gradjenju skraćenih modela hijerarhije sa odgovarajućim promenljivim i stepenima slobode.