FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol.3, N° 14, 2003, pp. 785 - 790

Invited Paper

ON SOME ANALOGIES *

UDC 530.7+531.3+517.546(075.1)+620.10

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Abstract. *The paper consideres the idea of two physical phenomena, which are essentially different, but can have the same linearized mathematical model. If one of two phenomena is well known, the analogy allwes for better understanding of the secon one. Analogy can be ascertained, but it may lead under some circumstances to new discoveries, at least from a theoretical point of view and if experimentally confirmed, they can be considered as new verities in science. The idea of analogies is illustrated by numerous examples.*

1. INTRODUCTION

1. **Analogy represents** the idea of two physical phenomena, which are essentially different, but can have the same mathematical model. If one of the two phenomena is well known, the analogy allows for a better understanding of the second one. Such an example is the famous analogy noticed by Kirchhoff in the 19th century between the movement of a rigid body with a fixed point (Euler's equations):

$$\frac{\mathrm{d}\mathbf{K}}{\mathrm{d}t}=\frac{\partial\mathbf{K}}{\partial t}+\boldsymbol{\omega}\times\mathbf{K}=\boldsymbol{M}_{\mathbf{0}} \tag{1}$$

and the relations between the efforts in the cross section of a bar, having as axis a curve in the three-dimensional space and the exterior load on the bar unit length:

$$\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s}+\mathbf{p}=\frac{\partial\mathbf{R}}{\partial\mathbf{t}}+\boldsymbol{\Omega}\times\mathbf{R}+\mathbf{p}=0 \tag{2}$$

Received May 23, 2002
* Paper presented at international Conference Mechanical Vibrations Timisoara 2002, as Invited Plenary Lecture. Paper is from program collaboratons between Faculty of Mechanical Engineering University Politehnika Timisoara, Rpmania and Faculty of Mecahanical Engineering University of Niš.

In the relation (1), **K** is the kinetic moment with respect to the fixed point, $\boldsymbol{M_0}$ is the moment of the exterior forces with respect to the same point, and $\boldsymbol{\omega}$ is the angular velocity vector.

In the relation (2), **R** is the resultant of the N_τ (axial), T_υ and T_β (shearing) efforts on the transversal section, **p** is the exterior load on the bar unit length, and $\boldsymbol{\Omega} = \frac{1}{R_\tau}\boldsymbol{\tau} + \frac{1}{R_c}\boldsymbol{\beta}$ is the Darboux vector, where R_τ and R_c represent the torsion radius and the curvature radius of the bar axis, respectively, and $\boldsymbol{\tau}$, $\boldsymbol{\upsilon}$, β are the vectors of Frenet's trihedron axes.

2. Analogy can be **ascertained** as in the above example, but it may lead under some circumstances to new discoveries, at least from a theoretical point of view and if experimentally confirmed, they can be considered as new verities in science. This is the case of the analogy that we could name *De Broglie's Analogy*. He was wondering why only the light had corpuscular and undulator features, whereas the other entities in nature were either corpuscles or waves.

Admitting the existence of an analogy between the corpuscular and the undulator features of light on the one hand, and the features of the electron (considered at that time to be a corpuscle) on the other hand, he has discovered in a theoretical way the undulator features of the electron.

A few years later, the interference of the electrons was discovered in laboratory, namely a clearly undulator feature resulting in the validation of De Broglie's presumed analogy.

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3. We will further give three examples of analogies: the first two have been acknowledged; the last example leads to a theoretical result waiting to be confirmed (or invalidated) by practical knowledge.

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4. Analogy between the small strains of a prismatic console acted by a constant moment couple and the rectilinear translation movement of a rigid body acted by a constant module force

The differential equations in the two cases are respectively:

$$EI \cdot \frac{d^2w}{dx^2} = -M \tag{3}$$

$$m \cdot \frac{d^2u}{dt^2} = F \tag{4}$$

In case of the boundary conditions $x = 0$, $w = 0$ and $x = 0$, $w' = 0$ for the console and in case of the initial conditions $t = 0$, $u = 0$ and $t = 0$, $u' = 0$ for the translation movement, the solutions are respectively:

$$w = -\frac{M}{2EI}x^2 \tag{5}$$

$$u = \frac{F}{2m}t^2 \tag{6}$$

In case of the console, we note with *EI* the rigidity of the bar, with *w* the arrow in any section and with *M* the couple moment In case of the translation movement, we note with m the body mass, with *u* the space displacement and with *F* the module of the force **F**. The analogy is obvious.

5. Analogy between the large deformations of a prismatic console load by a constant moment couple and the rectilinear translation movement large velocity of a rigid body driven by a constant force in the module

The differential equations in the two cases are respectively:

$$EI\frac{1}{\rho} = EI\frac{w''}{[1+(w')^2]} = -M \tag{7}$$

$$m_0\frac{u''}{\left[1-\frac{1}{c^2}(u')^2\right]} = F \tag{8}$$

(The differential equation (8) has been taken over from the Relativity Theory).

Apparently, there is no perfect analogy, the minus sign of the denominator of the relation (8) foreshadowing a "dramatic" situation in case of the translation movement. It is only an appearance. Both differential equations (7) and (8) result from the more general equation:

$$\frac{y''}{[1+a(y')^2]} = \frac{1}{b} \tag{9}$$

where *a* and *b* are two constants. The solution of the differential equation (9) for the initial conditions $x = 0$, $y = 0$ and $x = 0$, $y' = 0$, is:

$$x^2 + ay^2 - 2by = 0 \tag{10}$$

namely a conic, what can easily be verified. The differential equation (7) and its solution comply with $a = 1$, $b = -EI / M$, and the differential equation (8) and its solution comply with $a = -1 / c^2$, $b = m_0 / F$, the solutions in the two cases being respectively:

$$x^2 + w^2 + \frac{2Ei}{M} = 0 \tag{11}$$

$$t^2 - \frac{1}{c^2}u^2 - \frac{2m_0}{F}u = 0 \tag{12}$$

The diagrams are shown in the figures 1 and 2.

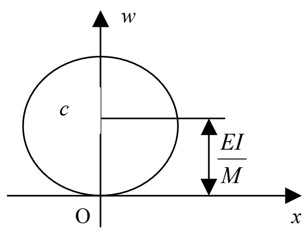

Fig. 1

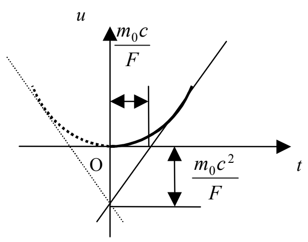

Fig. 2

In case of large deformations of the console, the diagram is a circle and in case of a high velocity translation movement, the diagram is a hyperbola. **In both cases, the diagrams represent conics**, that are the analogy is perfect, the circle and respectively the hyperbola conditions, resulting from the values assigned to different coefficients and not from different **natures** of the diagrams, which are conics.

6. The analogy between the small deformations of a prismatic console considering the influence of the shearing force and the rectilinear translation movement of a rigid body.

The differential equation of a medium deformed fibre of the console is in this case:

$$EI\frac{d^2w}{dx^2} = -M + 2(1+\upsilon)\cdot k \cdot i_s^2\,\frac{d^2M}{dx^2} \tag{13}$$

where υ is the coefficient of Poisson, k is a coefficient taking into consideration the nonuniform distribution of the shearing stress in a cross section and i_s is the inertia radius of the cross section.

The rectilinear translation movement of a rigid body (if such an analogy continued to exist) would accordingly imply the existence a differential equation, namely:

$$m\frac{d^2u}{dt^2} = F \pm \frac{1}{\omega_0^2}\cdot\frac{d^2F}{dt^2} \tag{14}$$

where ω_0 is a pulsation that has to be experimentally determined. The plus or minus sign in the second side of this relation has to be established experimentally, as well.

Remarks:

a) In practical designing, the influence of the shearing forces on the deformations of the prismatic console are neglected, being very small. Accordingly, through analogy, the ω_0 pulsation is expected to be very high, so that the term $\pm\frac{1}{\omega_0^2}\cdot\frac{d^2F}{dt^2}$ becomes negligible.

b) The deformations due to the shearing forces are nevertheless high in case of very short consoles acted by very high forces. Similarly, through analogy, the term $\pm\frac{1}{\omega_0^2}\cdot\frac{d^2F}{dt^2}$ can not be neglected in case of short term movements taking place under the action of rapidly changing forces, such as collisions, high frequency elastic waves etc.

7. Application in case of a longitudinal wave propagation in a prismatic bar.

By isolating an element of dx thickness from the respective bar and introducing the forces acting on it and using the differential equation $ma = F$, the equation with partial derivatives known as:

$$\frac{1}{c^2}\frac{\partial^2u}{\partial t^2} = \frac{\partial^2u}{\partial x^2} \tag{15}$$

is obtained. It is integrated using for instance the method of separation of the variables.

Considering the semiinfinite bar ($x \geq 0$), under the boundary condition $x = 0$, $u = u_0 \cos \omega t$, and retaining only the direct wave, the classic solution is:

$$u = u_0 \cos \frac{\omega}{c}(x - ct) \tag{16}$$

If the differential equation (14) is applied for the bar element with the thickness dx, after having introduced all the forces acting on it, the equation with partial derivatives of the fourth order is obtained further to elementary calculation:

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \pm \frac{1}{\omega_0^2}\frac{\partial^4 u}{\partial x^2 \partial t^2} \tag{17}$$

It can be integrated through the same method of the separation of variables.

Considering also this time the semiinfinite bar ($x \geq 0$) under the boundary conditions $x = 0$, $u = u_0 \cos \omega t$, and retaining only the direct wave (which is propagating in the positive sense of the axis Ox), we obtain the solution:

$$x = u_0 \cos \frac{\omega / c}{\sqrt{1 \pm (\omega/\omega_0)^2}}[x - c\sqrt{1 \pm (\omega/\omega_0)^2}\,] \cdot t \tag{18}$$

pointing out a wave propagating by the speed:

$$c_\omega = c\sqrt{1 \pm (\omega/\omega_0)^2} \tag{19}$$

Remarks:

a) The propagation speed of the elastic longitudinal waves is not constant $(c = \sqrt{E/\rho})$, but it depends on the pulsation ω.

b) The sign of the relation (ω/ω_0) has to be established through practical experiments. We assume this sign is "minus", that is the formula (19) would become:

$$c_\omega = c\sqrt{1 - (\omega/\omega_0)^2} \tag{20}$$

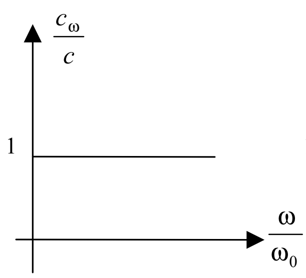

Fig. 3

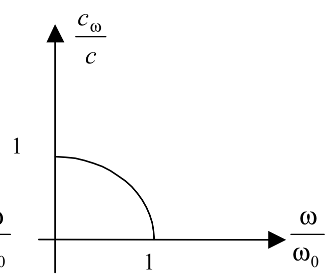

Fig. 4

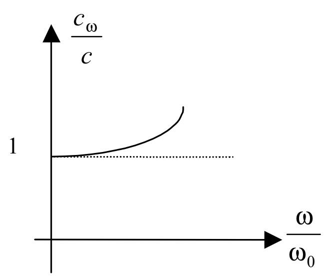

Fig. 5

In the figures 3, 4 and 5, the three variants are presented, namely:

- the classic: $c_\omega = c = ct.$ (fig.3),
- circle quarter: $c_\omega = c\sqrt{1 - (\omega/\omega_0)^2}$ (fig.4)
- hyperbola arc: $c_\omega = c\sqrt{1 + (\omega/\omega_0)^2}$ (fig.5)

c) As a consequence of the wave propagating speed depending on the pulsation ω is the fact that in case the boundary disturbance ($x = 0$) is the sum of different harmonics pulsations, each harmonic will be propagated by another speed generating a new phenomenon of *elastic wave dispersion*.

d) The fact that no variation of the wave propagation speed depending on the pulsation was stated is accounted for by the fact that the expansion in series of the relation $\sqrt{1 \pm (\omega/\omega_0)^2}$ begins with second order terms:

$$c_\omega = c\sqrt{1 \pm (\omega/\omega_0)^2} = 1 \pm \frac{1}{2}(\omega/\omega_0)^2 \mp \frac{3}{8}(\omega/\omega_0)^4 \pm .... \tag{21}$$

the order of magnitude of these terms and of the next ones being the same with that of the possible measurement errors due also to the very high pulsation value ω_0.

The equation (14) is obviously a pure theoretical result based on the assumption that the analogy between the small deformations of a prismatic console and the spaces covered by a rigid body in rectilinear translation, would continue to persist even if considering the influence of the shearing forces on the deformations of the console.

This fact has anyway to be confirmed by practical experience in order to become reality. If confirmation does not follow, a simple betraying dream will remain like so many other betraying dreams in my career. And this is all there is to it.

REFERENCES

1. Voinea R. P., *An analogy,* Proceedings of the Romanian Academy – series A, no.1, January-April, 2000, pp.51-53.

O NEKIM ANALOGIJAMA

Radu P. Voinea

U radu je rezrađena ideja dva fizička fenomena, esencijalnop različitih, ali koji imaju isti linearizovani matematički model. Ako je jedan od ova dva fenomena veoma dobro poznat, analogije omogućavaju bolje razulevanje drugog fenomena. Analogija može biti potvrđena, ali pod određenim uslovima može dovesti do određenih otkrića, barem sa teorijske tačke gledišta, kao i ako se eksperimentalno potvrdi, može se smatrati novim verifikacijama u nauci. Ideja analogije je ilustrovana brojnim primerima.