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EFFICIENT ENGINEERING APPROACHES FOR THE PREDICTION OF FATIGUE PROPAGATION OF CORNER CRACKS IN THE CASE OF MULTIPLE SITE DAMAGE

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Abstract. *The prediction of fatigue propagation of corner cracks in the case of Multiple Site Damage is a complicated task since the calculation of the Stress Intensity Factors requires three-dimensional stress analysis of the structure. In the present work, two methodologies aiming in treating the three-dimensional nature of a corner crack growth problem by performing equivalent two-dimensional analysis are presented. The first approach provides a simple procedure for the substitution of a corner crack by an equivalent through crack. In the second approach the corner crack propagation is treated by two-dimensional analysis, by exploiting the benefits of the sub-modelling technique of the Finite Element Method (FEM), thus the necessity of the three-dimensional analysis is limited only in the corner cracks adjacent areas. Both approaches are verified for the case of single corner crack propagation, as well as, Multiple Site Damage corner crack propagation.*

1. INTRODUCTION

In Multiple Site Damage (MSD) situations, simple and accurate techniques for the rapid assessment of the complex structural damage states are of major importance. MSD is by definition a difficult situation, as it implies interactions between multiple cracks and flaws. The part of the structure, which needs to be modelled and analysed is usually large, including many structural details and different types of stress concentrators. The presence of one or more corner cracks provides an additional complexity as the problem becomes three-dimensional.

Three-dimensional (3-D) crack analysis has attracted considerable attention in the literature. The most common methodologies, which are used to calculate the Stress Intensity Factors (SIFs) of corner cracks, include approximate analytical method [1-2] the alternating method [3-4], the Finite Element Method (FEM) [5-6] with hybrid displacement

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elements [7], or singularity elements [8-9], the FEM combined to the alternating method [10-17] and the boundary integral equation method [18]. From the above methodologies the most economic and efficient for application in practical complex 3-D crack configurations seems to be the FEM alternating method. However, the FEM alternating methodology is limited only to specific types of in-plane loading conditions. Therefore, the accurate prediction of crack propagation of panels with interacting corner and through cracks, needs full 3-D FEM analysis, meaning considerably high numerical effort.

In the present paper, in order to overcome the difficulty of analysing a complete 3-D model of a panel with corner cracks, which is necessary in realistic crack propagation situations, two alternative methodologies of corner cracks treatment are presented. In the first methodology the corner crack is emulated by an equivalent through crack, by taking into account that in thin sheets the corner crack becomes through, after a limited number of propagation cycles, which usually corresponds to a small percentage of the total fatigue life of the structure. The equivalence implies that both cracks will behave identically when fatigue loaded, thus the emulation concerns only the fatigue behaviour of the corner crack. However, although the emulation methodology may be applied in all types of loading, in the present paper it is limited to in-plane loading conditions, due to lack of suitable crack propagation experimental data of corner cracks under complex loading.

To extend the capability of efficient prediction of the behaviour of corner cracks to arbitrary loading conditions, a hybrid 2D-3D Finite Element approach for the calculation of SIFs in the case of MSD is developed. The method takes advantage of the sub-modelling techniques of the FE method and applies 2-D analysis for the entire MSD structure. When additional detailed information of the stress field around the corner crack is necessary, 3-D analysis may be performed afterwards, only locally, at the vicinity of the corner cracks. The commercial Finite Element code ANSYS [9] is used for the present methodology development, however, any other FE code providing sub-modelling capabilities may be applied. The hybrid 2D-3D methodology reduces considerably the numerical effort of the analysis, provides an engineering tool for the calculation of SIFs and makes the prediction of MSD evolution in the presence of corner cracks efficient and practically manageable for arbitrary loading type and arbitrary geometrical configurations.

2. EMULATION OF AN IN-PLANE FATIGUE LOADED CORNER CRACK TO A THROUGH CRACK

Consider a corner crack of length a and depth c in a panel of thickness t, as shown in Fig 1. Under fatigue loading the crack depth c will become equal to the thickness t and the corner flaw will become a through crack after a number of fatigue propagation cycles. Assuming that detailed information on the stress field around the corner flaw is not necessary during the propagation period of the corner crack to through crack, a methodology is proposed, for the emulation of the corner crack to an equivalent through crack. The equivalence implies that both cracks under the same fatigue loading, behave in an identical way, at least, as far as their propagation is concerned.

The emulation is based on the principle that the corner crack and a hypothetical through crack of suitable initial crack length a_{eq} will reach the same length a_{th}, at the same number of propagation cycles. The propagation of the corner crack along the surface and through the thickness is first calculated, based on the crack growth rules of the material. For each load cycle the crack increment of the surface crack is calculated equal to the crack propagation rate that corresponds to the stress intensity factor at the surface of the panel:

$$\Delta a = \frac{da}{dN}(\Delta K_{0^\circ}) \tag{1}$$

while the respective for the through the thickness crack is:

$$\Delta c = \frac{dc}{dN}(\Delta K_{90^\circ}) \tag{2}$$

with ΔK_{90° being the stress intensity factor at the edge of the crack inside the hole. From these calculations, the number of cycles N_{th}, which is required for the flaw depth c to reach the panel thickness t can be evaluated. At this stage of the fatigue life, the corner crack becomes almost immediately a through crack of length a_{th}. The unknown crack length a_{eq} can be evaluated by 'back propagating' a through crack of length a_{th} for N_{th} cycles. 'Back propagation' means the theoretical reverse process of the through crack propagation phenomenon from length a_{eq} to length a_{th}. For the calculation of the increment of the through crack for each load cycle a relationship of the form:

$$\Delta a = \frac{da}{dN}(\Delta K) \tag{3}$$

is used. The whole procedure is illustrated in the flow chart of Fig 2.

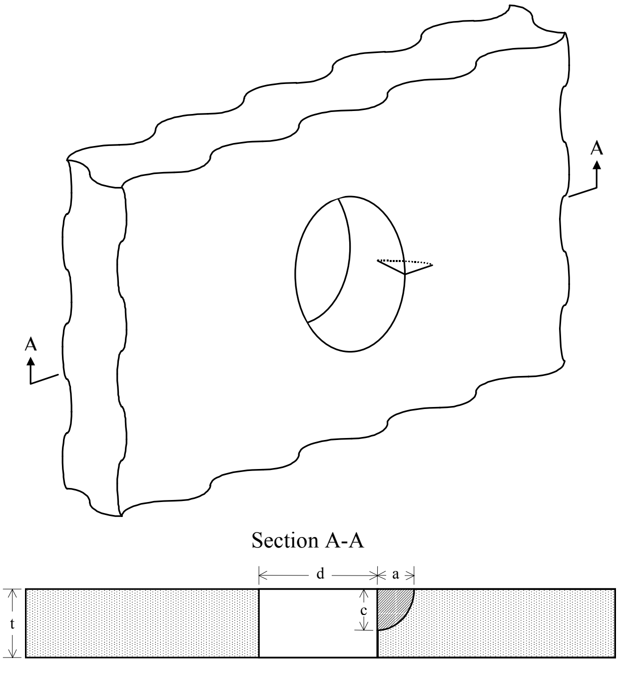


Fig 1. Configuration of a corner crack of length a and depth c.

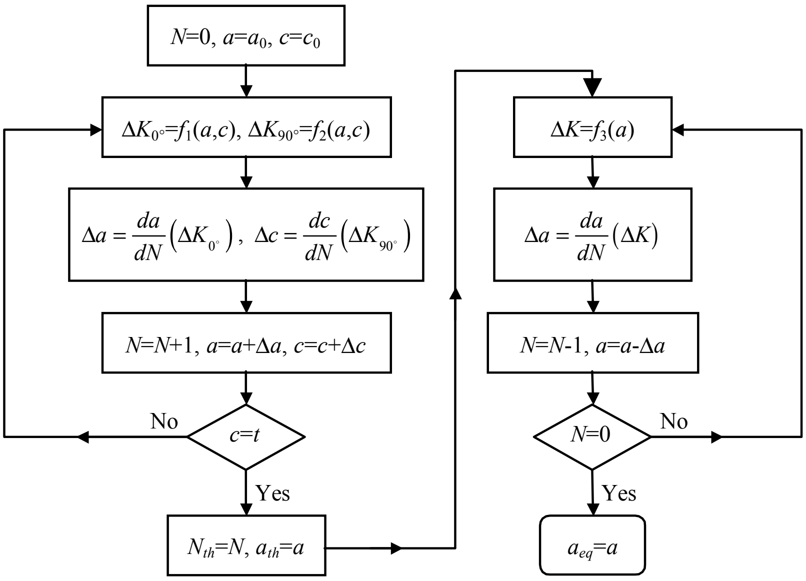


Fig 2. Flow-chart of the emulation methodology of a corner to a through crack

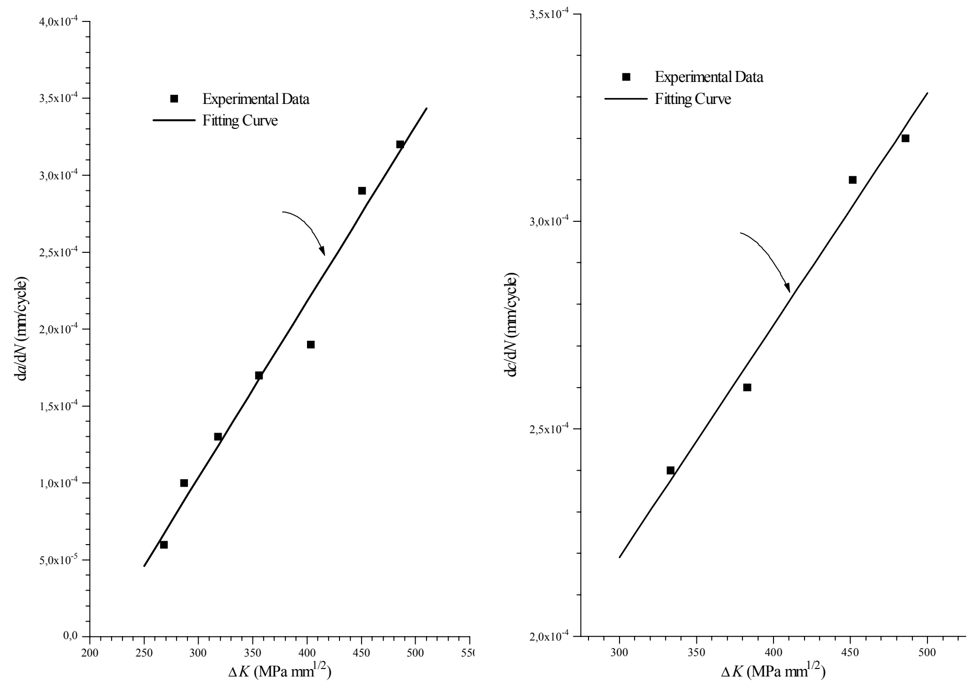
2.1. Demonstration of the emulation methodology in the case of a single corner crack

The emulation methodology described above, is demonstrated for the case of quarter elliptical corner cracks of various initial lengths *a* and various initial depths *c*. The cracks are emanating from a circular hole of diameter d = 6.63mm, in an Al 7075-T73 plate of thickness t = 6.35mm. The crack propagation rules along the surface and through the thickness of the corner cracks, for tension-tension fatigue loading (σ_{max}= 103.5Mpa and $R = 0.1$), used in the emulation, are presented in Fig 3. These rules are based on the experimental data of reference [10], performed at panels with corner cracks of initial length $a = 1.7$mm and initial depth c =1.96mm. The crack propagation rule for the equivalent through crack is presented in Fig 4, based also on the experimental results of reference [10]. The SIFs of the corner crack and the equivalent through crack are calculated by analytical relationships of Linear Fracture Mechanics.

Using the procedure described in the flow chart of Fig 2, equivalent through crack lengths are determined for corner cracks of various initial lengths, varying from 5% of the panel thickness to 90% of the panel thickness. The initial ratio of the corner crack length *a* to the crack depth *c* remains constant, at the value of 1. The results of the emulation are shown in Fig 5, where dimensionless crack lengths by dividing to the panel thickness. In that way, Fig 5 can be applied to panels with different thickness.

2.2. Demonstration of the emulation methodology in the case of MSD

To identify whether the emulation methodology described above, is also valid in the case of MSD, the problem of a corner crack propagation, interacting with a through crack is considered. The Al 7075-T73 square panel of 100mm edge and 1.6mm thickness, shown in Fig 6, is subjected to uniform remote fatigue tension (σ_{max}=100Mpa, R=0) at its edges. A corner crack ($c = a = 1.2$mm) and a through crack (a=3.2mm) are emanating from the two circular holes of the panel, interacting between themselves during their propagation. If the effect of the interaction of the big through crack and the corner crack is neglected for the propagation period of the transformation of the corner crack to through crack, the emulation methodology described in the previous paragraph can be used to emulate the corner crack with a through crack and treat the entire problem as two-dimensional afterwards. According to Fig 5, a corner crack of $c = a = 1.2$mm, at a panel of thickness of 1.6mm may be substituted by an equivalent through crack of 0.96mm length. To evaluate the accuracy of this engineering approach, the equivalent length will also be calculated numerically by means of the FE method. The phase of the corner crack propagation to a through crack will be simulated first and the virtual back propagation of the through crack will follow, taking completely into consideration the effect of the interaction between the two cracks, by treating the entire panel model.



(a) (b)

Fig 3. Experimental corner crack propagation data (σ_{max}=103.5Mpa and $R = 0.1$) as taken from [10]; (a) along the crack length, and (b) through the thickness

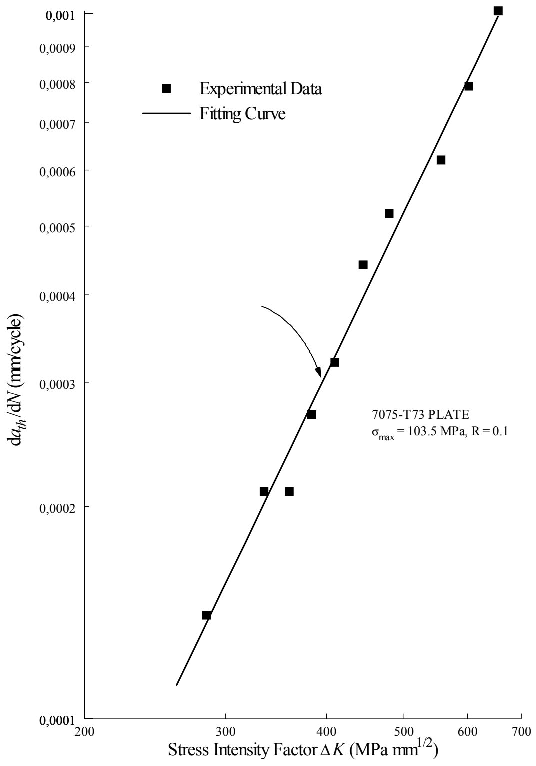


Fig 4. Experimental through crack propagation data, as taken from [10].

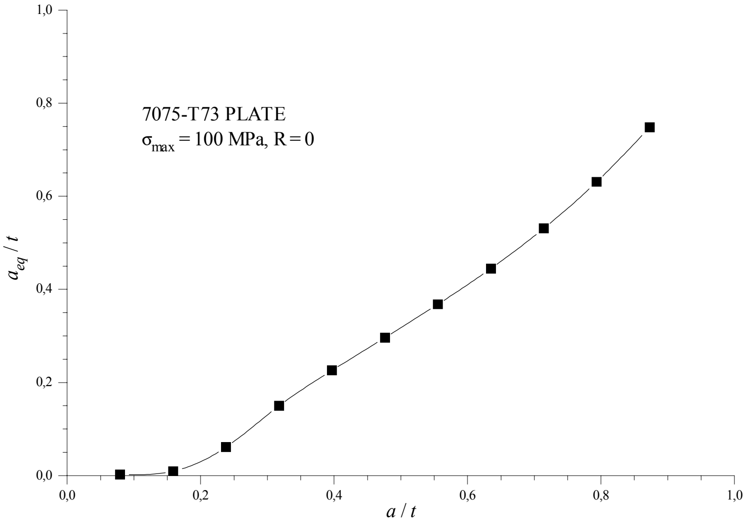
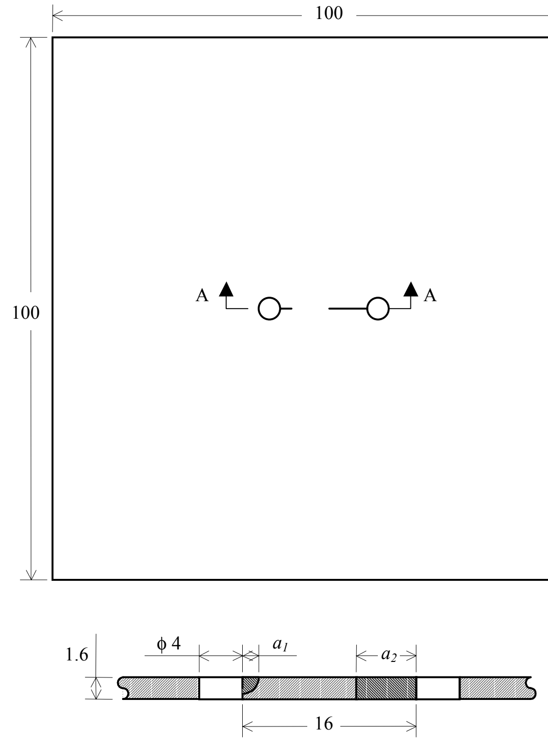


Fig 5. Dimensionless length of equivalent through crack (a_{eq}/t), versus dimensionless initial length of corner crack (a/t)

The SIFs of the corner and the through crack of the problem of Fig 6 can not be calculated analytically, as in the case of a single corner and a single through crack of the previous paragraph. Therefore, a numerical solution of the problem using the ANSYS FE code is performed. The developed FE model is three-dimensional, describing the geometrical configuration of the corner crack and the through crack, as shown in Fig 7. As the problem is symmetrical along the horizontal axis, a half symmetrical FE model is developed.



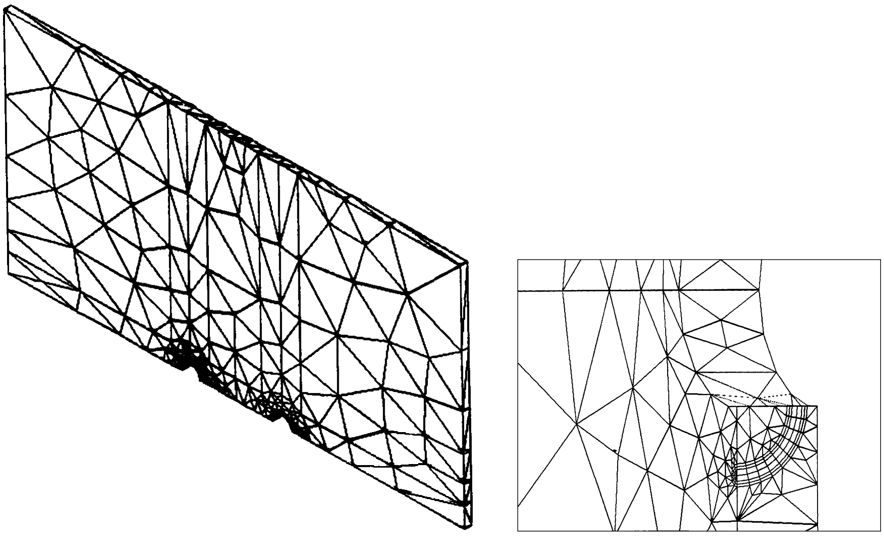
Section A-A (magnified)

Fig 6. Geometrical configuration of the fatigue loaded panel containing two interacting cracks (all dimensions in mm)

Solution of the FE model of Fig 7, leads to the calculation of the stress intensity factors K_a and K_c, along the plate surface and through the plate thickness of the corner crack, respectively. The SIF of the through crack is also calculated. For an incremental increase of the cycles (N_i) the new crack lengths (a_i) and (c_i) are calculated using the propagation rules of Fig 3 and the new crack configuration arises. It should be noted that because of the difference in the propagation rates along the plate surface and through the plate thickness, the initial quarter-circular configuration becomes a quarter-elliptical one, introducing additional modelling effort. In table 1, the number of cycles (N_i), crack lengths c_i and a_i and the corresponding Stress Intensity Factors are shown. It arises from the calculations that after (N_{th}) = 1403 cycles the depth c reaches the plate thickness value (c_{th} = 1.6mm), while the crack surface length (a_{th}) becomes 1.465mm.

Table 1. Numerically calculated corner crack length and SIFs versus crack propagation cycles

N (cycles)	c (mm)	K_c (Mpa $\sqrt{\text{mm}}$)	a (mm)	K_a (MPa $\sqrt{\text{mm}}$)
0	1.200	400	1.200	348
72	1.220	403	1.212	350
144	1.240	405	1.224	352
216	1.260	410	1.237	355
288	1.280	417	1.250	358
360	1.301	419	1.264	360
432	1.322	423	1.278	363
504	1.343	429	1.294	366
576	1.364	433	1.309	369
648	1.385	437	1.326	371
⋮	⋮	⋮	⋮	⋮
1403	1.600	481	1.465	399



(a) (b)

Fig 7. (a) 3-D FE model for the calculation of SIFs at the corner crack and through crack, and (b) detail at corner crack region

At this propagation stage it is considered that the quarter elliptical corner crack is immediately transformed into a through crack of length (a_{th}). To 'back propagate' this through crack and calculate the equivalence crack length (a_{eq}), a 2-D finite element model of the half symmetrical part of the plate is developed, as shown in Fig 8. The 2-D model consists of singular plane elements for the area around the crack tip and regular plane elements for the rest of the plate.

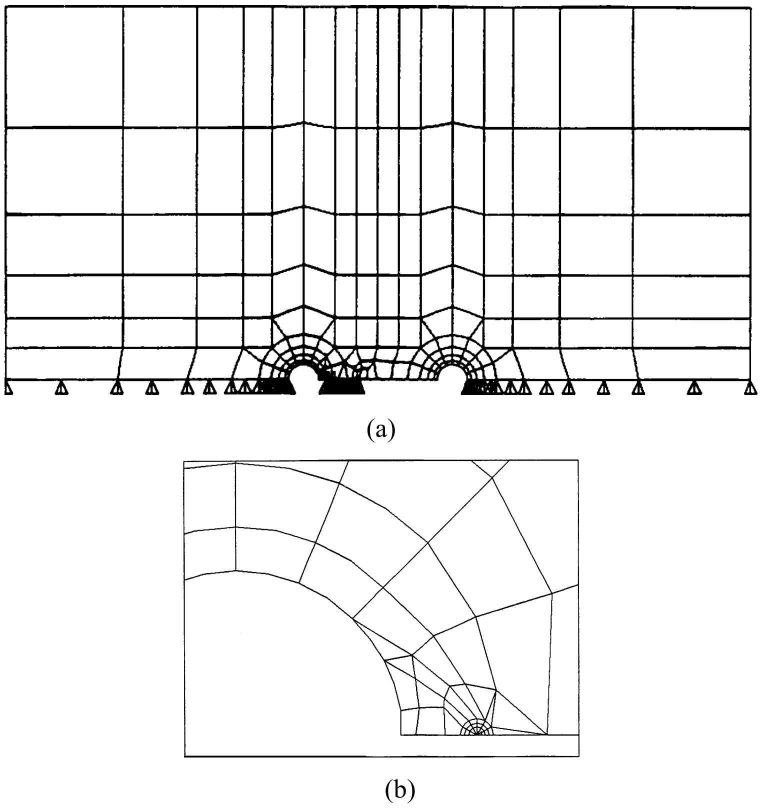


Fig 8. (a) 2-D FE model used for the 'back propagation' of the through crack and (b) detail of the model

In table 2, the 'back propagation' process of the through crack is presented. More specifically, the number of cycles N_i, crack lengths a_i and stress concentration factors K_a, as calculated from the 2-D FE model of Fig 8 are shown. As it can be seen from table 2, at $N_i = 0$, the crack length, i.e. the equivalent crack length (a_{eq}) is 1.06mm. If the effect of the interaction of the through crack and the corner crack is neglected, the equivalent length has been calculated to 0.96mm. The difference between the full-model crack length and the emulation length is only 9.4%, which corresponds to about 400 propagation cycles. It can be concluded that the interaction between the two cracks may be neglected during the propagation period that the corner crack becomes through. Therefore, it can be stated that the emulation methodology gives the required engineering accuracy, also in the case of MSD situations, taking into account the great reduction of computing effort achieved. It is also apparent that the emulation benefit will become higher, if the number of corner cracks in the MSD structure increases, or when the structure geometrical complexity is higher.

Table 2. Numerically calculated through crack length and SIFs versus crack 'back propagation' cycles

N (cycles)	a_{th} (mm)	K_a (MPa $\sqrt{mm}$)
1403	1.465	399
1263	1.422	397
1123	1.380	396
983	1.378	394
843	1.329	392
703	1.255	389
563	1.214	387
423	1.174	385
282	1.135	383
142	1.098	381
0	1.060	378

3. HYBRID CORNER CRACKS PROPAGATION ANALYSIS UNDER ARBITRARY LOADING CONDITIONS

The emulation methodology developed in the previous paragraphs is valid for the cases of in-plane fatigue loading and provides engineering accuracy in cases of MSD. However, in cases of arbitrary loading conditions and if detailed information on the stress field around the corner flaw is required, the emulation methodology of the corner crack by an equivalent through crack is not applicable. In such cases, full 3-D FE analysis is required to capture for bending effects and provide detailed solutions of the stresses and strains around the corner crack. However, especially in presence of MSD, a full 3-D analysis requires high modelling and computing effort, which becomes even higher during the phase that the corner crack propagates and becomes a through crack, mainly due to the change of the crack shape from semi-circular to semi-elliptical. In such cases a full 3-D analysis may become impractical as its efficiency is considerably reduced.

Sub-modelling techniques are usually used in Finite Element (FE) analysis in order to obtain more accurate results in high stress concentration area. This is achieved by generating a separate, more finely meshed model at the area of interest (e.g. Fig 9). The boundary line (for a 2-D model) or the boundary area (for a 3-D model) of the structure of interest, which has to be refined, is called 'cut boundary'. This independent model is then solved by applying to its 'cut boundary' the generalised displacements calculated by the initial rough model.

In the present paper the sub-modelling technique is utilised in order to treat three-dimensional (3-D) parts of a rather two-dimensional (2-D) structure. In cases that in a panel both corner and through cracks are propagating, the required model for the crack propagation is basically three-dimensional. However, to avoid a three-dimensional model of the whole panel, which would be very demanding, a hybrid methodology is proposed based on sub-modelling FE techniques. Under this approach, the entire structure is initially treated by 2-D modelling, while the areas around the corner cracks are refined afterwards using local 3-D models. Basic requirement for the application of the sub-modeling technique is that the 'cut boundary' is defined at such a sufficient distance away of the corner crack tip, so that any three-dimensional effects on the stress-field vanish at the 'cut-boundary'.

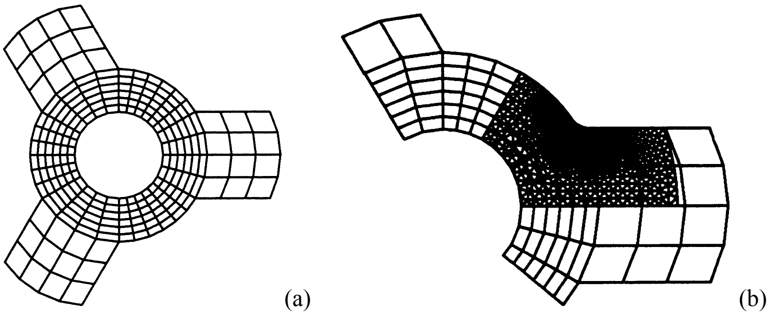


Fig 9. Example of sub-modelling technique; (a) Coarse model, (b) Sub-model area

In the proposed hybrid methodology, all corner cracks existing in the panel are transformed to through cracks of suitable length. It is possible then to develop a 2-D model of the panel, taken into account the areas around the corner cracks and the required 'cut-boundary' lines, which will enable the development of 3-D models afterwards. Following solution of the 2-D model of the entire structure, the SIFs of the through cracks can be calculated. Furthermore, the 'cut-boundary' solution may be applied as boundary condition in the detailed 3-D models of the areas around the corner cracks. The FE code ANSYS, which is used in the present work, provides the capability of the transition of a 2-D model to a 3-D one, by calculating the through thickness displacements of the 'cut boundary', which will be imposed to the local 3-D model, utilising not only the 2-D displacements of the 'cut boundary', but also its rotations. Therefore, an accurate shell-to-solid numerical transition can be performed, provided that the calculated displacements and rotations in the 2-D model are sufficiently accurate, which may be controlled by using the common error norm calculations. Solution of the 3-D models leads to an accurate calculation of the SIFs at the crack phases of the corner cracks. The propagation of both types of cracks may be calculated with sufficient accuracy, using propagation rules for both corner and through cracks. The hybrid procedure is repeated until all corner cracks of the structure will become through cracks, thus no 3-D modelling is further required. A flowchart of the hybrid methodology is shown in Fig 10.

The most important questions on the above procedure are:

a) Which is the equivalent through crack length of the corner crack to be applied in the global 2-D model, and
b) Which is the sufficient distance away from the corner crack tip, at which the effects of the three-dimensionality nature of the corner crack vanishes, at least as far as the stress field is concerned, i.e. which is the suitable distance of the cut-boundary from the corner crack face.

Concerning the first of the above two issues, the corner crack equivalence in the cases of in-plane loading may be treated by the emulation curve of Fig 5. However, as the present sub-modelling methodology is developed for arbitrary loading conditions and different material types, it is assumed that in a reasonable distance away of the crack front, the corner crack behaves as a through crack of length $a_{eq} = (a_c + c_c)/2$. Although, this assumption seems to be quite arbitrary, it is proven in the following analysis that it is practically valid.

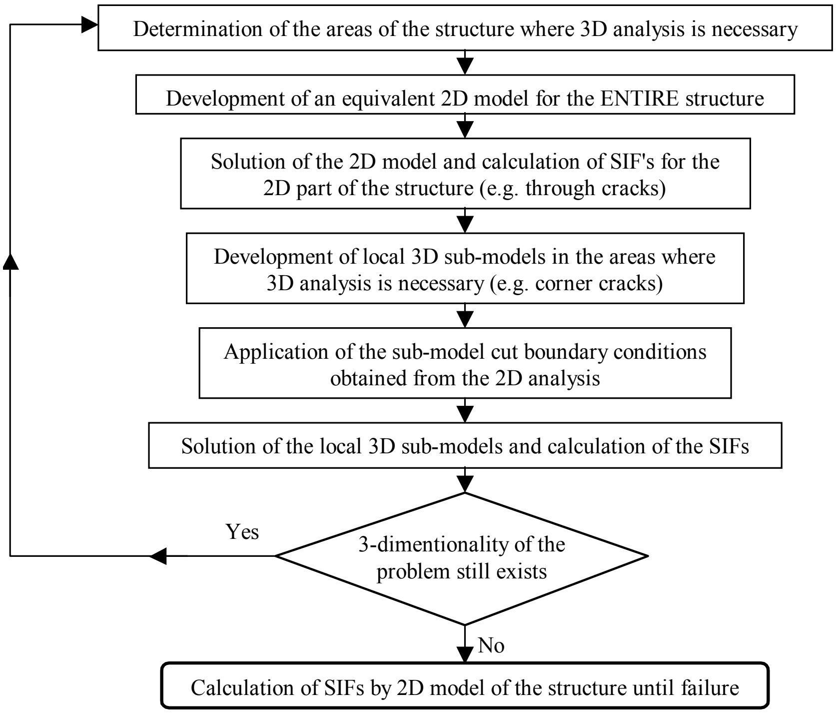


Fig 10. Corner crack propagation by sub-modelling methodology

Concerning the second issu, the suitable distance away from the corner crack that the three-dimensionality effect of the corner crack vanishes is treated in the present work numericaly, by comparing SIF results obtained by the proposed methodology to the corresponding SIFs obtained by a full 3D analysis, for different 3-D cut boundary distances, ranging from 1.5 to 5 times the equivalent crack length a_{eq}. By a parametric analysis in both single corner crack and MSD situations it is found that if the cut-boundary is located 2.5 to 3 times the equivalent crack length a_{eq} away from the crack front, then the difference between SIF results obtained by full 3-D analysis and the proposed sub-modelling methodology are different by less than 1%. Therefore, in the following demonstration cases of the sub-modelling methodology, the cut-boundary distance is defined to be 2.5 to 3 times the equivalent crack length a_{eq}.

3.1. Demonstration of the sub-modelling methodology in the case of a single corner crack

The proposed sub-modelling methodology is demonstrated in the case of a single corner crack emanating from a hole. A panel of 2024-T3 material, having the geometry shown in Fig 11, is subjected to uniform fatigue tension. To represent the panel with a two-dimensional model, it is assumed that in a reasonable distance away of the crack front, the corner crack behaves as a through crack of length $a_{eq} = (a_c + c_c)/2$.

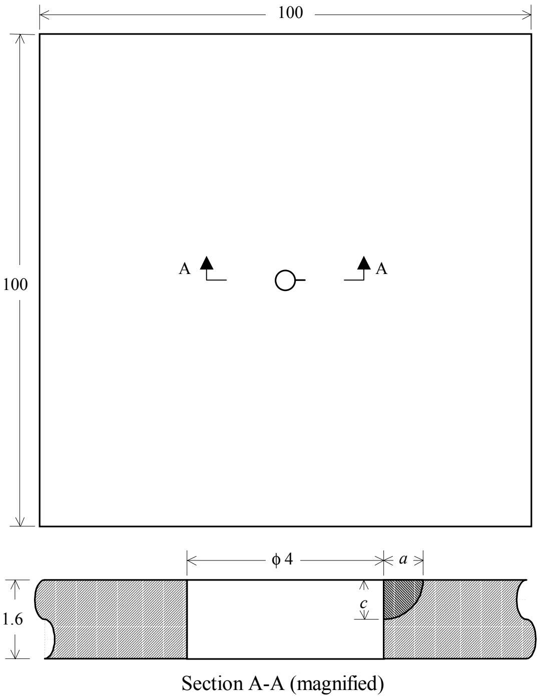


Fig 11. Single corner crack emanating from a circular hole (all dimensions in mm).

Upon this assumption, the 2-D FE model shown in Fig 12 is developed, consisting of 208 shell elements. A detail in Fig 12(b) shows the 'cut boundary' and the panel area,

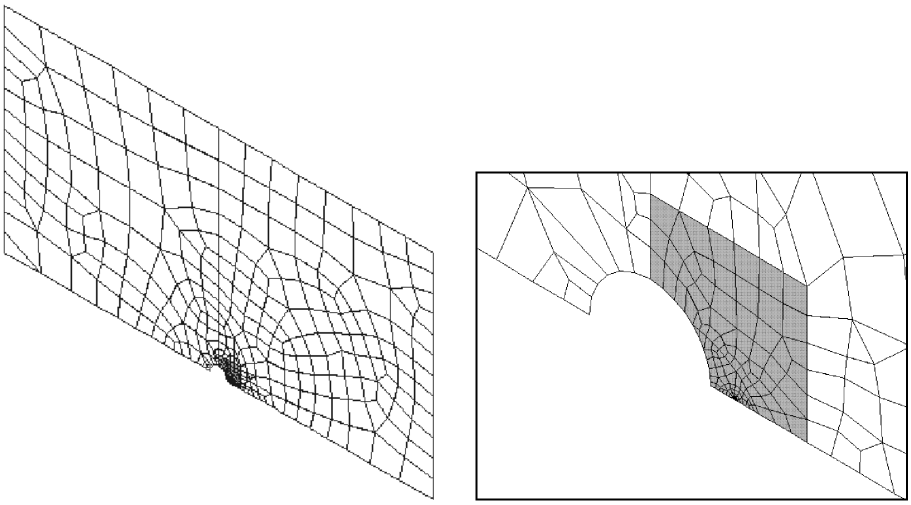

(a)
(b)

Fig 12.2-D model of a corner cracked panel;
(a) FE-mesh at the hole region, (b) detail of the 'cut-boundary' area

which will be reanalysed to calculate the SIF along the crack front. The 'cut boundary' is considered to be at a distance of 2.5-3 times of length a_{eq}. The solution of this model leads to the determination of the displacements and rotations of the 'cut boundary'. Using Mindlin assumptions for the deformation shape of the panel, the displacements through the thickness of the 'cut area' are calculated.

A detailed 3-D model of the area around the corner crack is developed afterwards, which models exactly the physical reality, as shown in Fig 13.

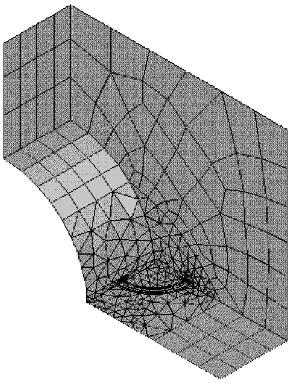
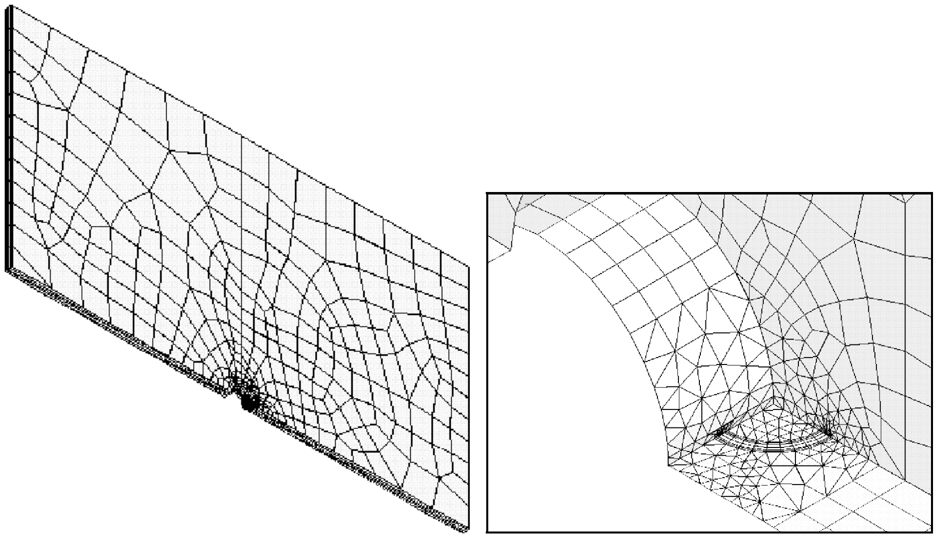


Fig 13. 3-D FE model of the region around the corner crack

For the verification of the results obtained by the use of the sub-modelling methodology a full 3-D finite element model of the corner cracked panel of Fig 11 is developed (Fig 14). Stress intensity factors are calculated for different lengths of corner cracks for the case of pure tension of the structure as well as for bending. The results for SIFs obtained from the two FE models, for different combination of crack lengths and different load cases are presented at Fig 15 in the normalised form of SIF, i.e. ($K / \sigma \sqrt{\pi a}$) versus corner crack angle φ. An excellent agreement between SIFs calculated from full 3-D model and from hybrid 2-D/3-D FE model, in all the cases of loading and crack lengths, can be observed from the curves of Fig 15.



(a) (b)

Fig 14. (a) Full 3D model of the panel, (b) detail of the corner crack region

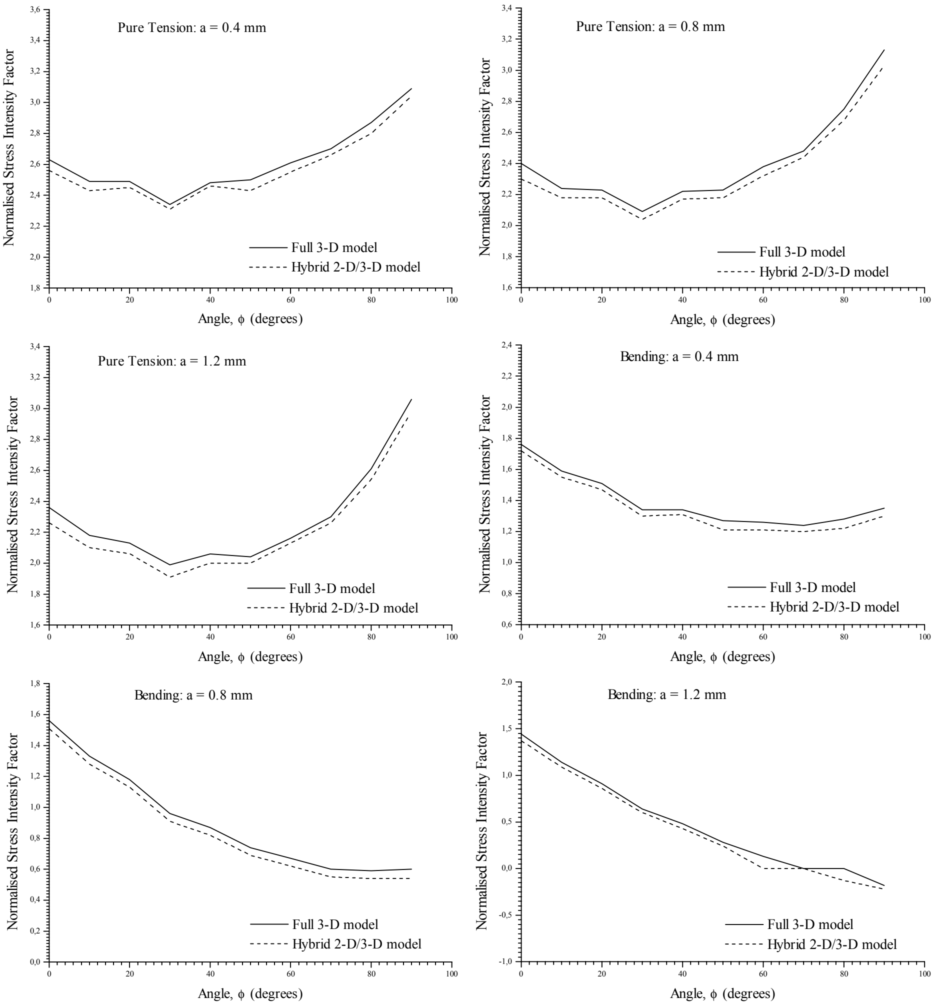


Fig 15. Calculated SIF results from full 3-D model and from hybrid 2-D/3-D FE model for the case of a single corner crack

3.2. Application of the sub-modelling methodology in the case of MSD crack interaction

For the validation of the methodology of the previous paragraph in crack interaction problems, the panel considered in section 2.2 is treated by the hybrid method. From the left circular hole of the panel a corner crack is emanating, while from the right circular hole a large through crack propagates, as shown in Fig 6. The developed FE models used in the analysis are presented in Fig 16. Stress Intensity Factors for different load combinations and crack lengths are calculated by both the hybrid methodology, as well as, a full 3-D analysis. The corresponding graphs of Fig 17 show a very good agreement between SIFs calculated by the proposed hybrid analysis and SIFs calculated by the full

3-D analysis, for all the cases of loading and crack lengths. Therefore, the hybrid method can be efficiently applied also in the case of the crack interaction problem and MSD situations.

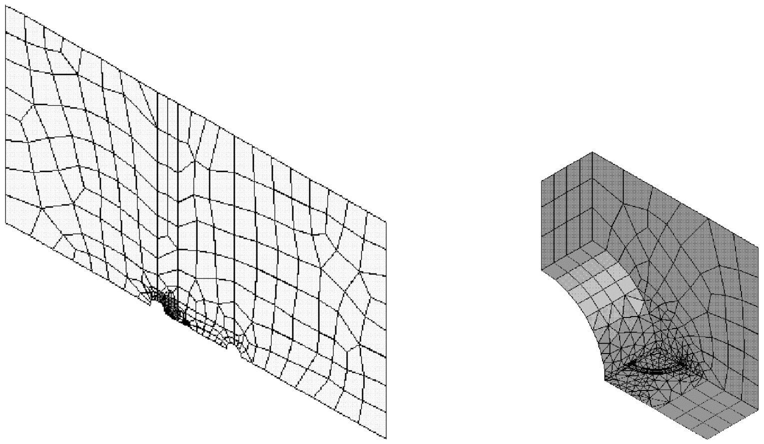

(a) (b)

Fig 16. (a) Equivalent 2-D model containing only through cracks, (b) 3-D sub-model representing the corner crack

4. CONCLUSIONS

The accurate prediction of fatigue crack growth in MSD situations requires high modelling and computational effort. Two efficient engineering methodologies are developed in the present paper, aiming to reduce the effort by providing the required accuracy. Both approaches take advantage of the idea of emulating one or more corner cracks of the structure to equivalent through cracks.

The emulation methodology based on the crack length equivalence with respect to the number of fatigue loading cycles, provides the basis for an efficient corner crack propagation analysis. The equivalence is developed for in-plane fatigue loading, but may be extended to arbitrary loading conditions, upon availability of the required experimental data. In the MSD case studied, the effect of crack interaction during propagation of the corner crack to through crack does not affect significantly the emulation accuracy.

The hybrid method is more sophisticated and may be applied in all types of fatigue load types, i.e. in-plane and out-of-plane. Numerical results of SIFs obtained from the submodeling hybrid methodology are in excellent agreement to those obtained from a full 3-D analysis, both in the case of a single and MSD corner crack problems. The reduction of the modelling and the computation time is radical, as only local areas of the model, i.e. areas around corner cracks, need to be treated as three-dimensional, while the other areas are treated by two-dimensional models. Both methodologies are easily automated and easily applicable to corner crack propagation analysis.

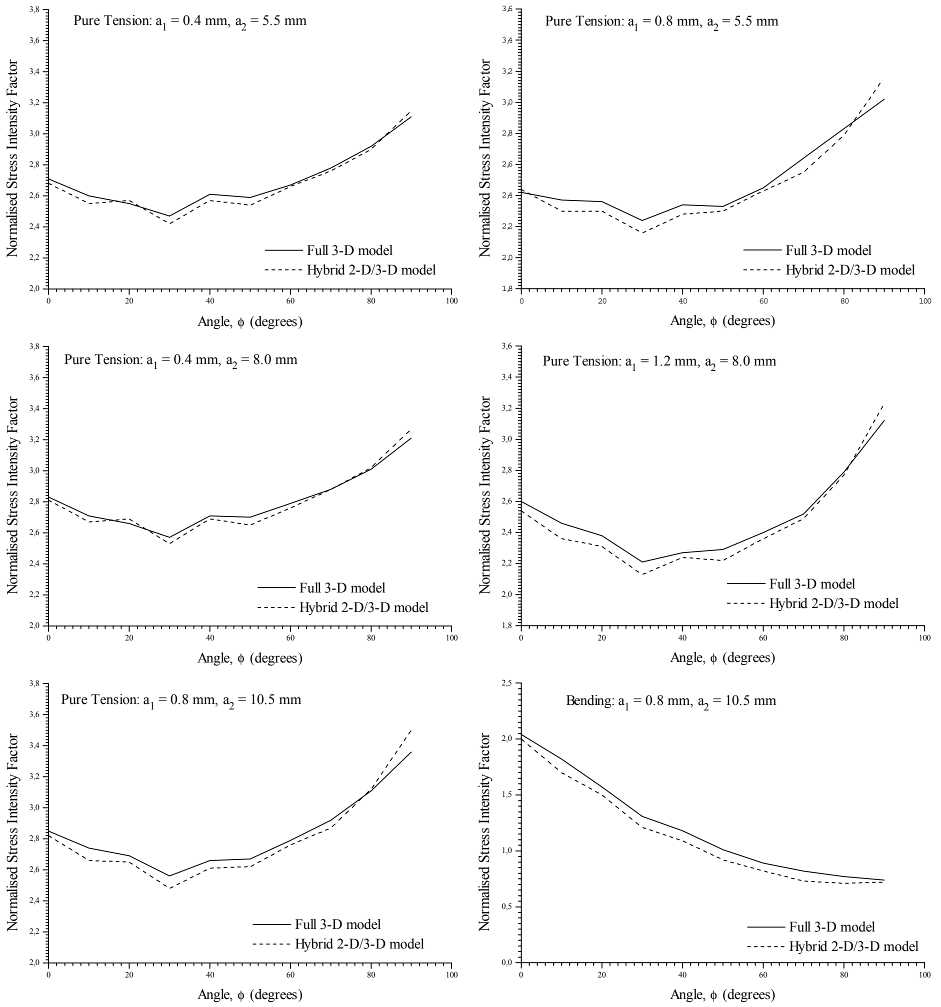


Fig 17. Normalized SIF results versus corner crack angle φ, as calculated from full 3-D FE model and from hybrid 2D/3D model, for the case of MSD crack interaction

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EFIKASNI TEHNIČKI PRISTUPI PREDVIDJANJU ŠIRENJA ZAMORA UGAONIH PRSLINA U SLUČAJU OŠTEĆENJA NA VIŠE POLOŽAJA

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Predvidjanje širenja zamora ugaonih prslina u slučaju oštećenja na više položaja složen je zadatak, pošto izračunavanje faktora intenziteta napona zahteva trodimenzionalnu analizu napona strukture. U ovom radu prikazane su dve metode usmerene ka tretiranju trodimenzionalne prirode problema napredovanja ugaone prsline putem vršenja ekvivalentnih dvodimenzionalnih analiza. Prvi pristup pruža jednostavnu proceduru zamene ugaone prsline ekvivalentnom prolaznom prslinom. U drugom pristupu, prostiranje ugaone prsline tretira se preko dvodimenzionalne analize, korišćenjem prednosti tehnike podmodeliranja metode konačnog elementa (FEM), čime je neophodnost trodimenzionalne analize ograničena samo u oblastima neposredno uz ugaone prsline. Oba pristupa proverena su za slučaj prostiranja jedne ugaone prsline, kao i za prostiranje ugaone prsline kod oštećenja na više položaja.