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RESONANCE FREQUENCIES OF PZT PIEZOCERAMIC DISKS: A NUMERICAL APPROACH

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Abstract. *In this paper the frequency spectra of axisymmetric extensional vibrations in PZT piezoceramic disk plates having Poisson's ratio (σ) of 0.294 have been investigated theoretically and experimentally. The theoretical frequency spectra were calculated according to Hutchinson's theory of vibrations for isotropic elastic disk (i.e. cylinder) with thickness to diameter ratio (d/2a) from 0 to 3, without consideration the piezoelectric properties of disks. Experimental spectra were observed for the disk with ratio d/2a of 0.06 to 1.2. Experimental spectra are coincided with theoretical spectra and all observed parts of spectra might be completely predicted on the basis of the vibration theory of isotropic disk.*

Key words: *piezoceramic disk, model, resonance frequency*

1. INTRODUCTION

In industrial applications of ultrasonic vibrations, the transducers of disk shapes are often used for generating high-power waves at the thickness extensional mode. In such transducers, the thickness displacement component can be used to radiate sound into the medium, possibly via one or several encapsulation/matching layers. Fig. 1 shows a disk type transducer which is used generally at high-frequency with thickness extensional mode.

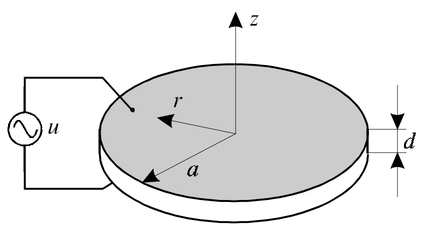

Fig. 1. Piezoceramic thickness-poled and electroded circular disk element

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Oscillator vibrations in the thickness extensional mode, in common case, are very complicated in the surrounding of fundamental thickness resonance frequency because of influences of the oscillator finite boundaries. For practical application of these oscillators it is necessary to determine clearly the frequency spectra of the resonant vibrations.

Numerous authors have studied vibrations of isotropic disks. Shaw's experimental studies of vibrational modes in the piezoelectric ceramic disks, have been reported in the literature [1], where axially symmetric extensional vibrations in the disk plates studied by using piezoelectric $BaTiO_3$ ceramics.

Gazis and Mindlin [2] attempted to interpret Shaw's results by modeling the vibrational modes of an isotropic, non-piezoelectric disk, with an effective isotropic Poisson's ratio σ adjusted to fit the experimental data.

In the literature [3], [4], authors observed in detail the frequency spectra in $PbTiO_3$ ceramic disks with Poisson's ratio $\sigma = 0.20$ and $Pb(ZrTi)O_3$ piezoceramic disk with $\sigma = 0.37$ for wide range diameter/thickness ratio. These workers obtained a reasonably good theoretical fitting to their data using the isotropic disk model of Gazis and Mindlin and the isotropic Poisson's ratio of 0.18 [3].

Holland and EerNisse have developed a variational formulation of piezoelectricity applicable to a large class of electromechanically resonant structures [5].

The natural vibrational modes of axially symmetric piezoelectric ceramic disk by the finite-element method have been calculated in [6]. An analysis of the complete spectrum of piezoelectrically active modes as a function of diameter/thickness ratio is presented for PZT-5H ceramic disks.

In the last papers the models of piezoelectric disks were made, which present the system with tree or four accesses.

For thickness extensional mode transducers, a number of analytical one-dimensional (1D) models have been presented which describe the thickness vibration in thin piezoceramic disks with additional front and back layers, such as the Mason thickness extensional model [7]. The traditional 1D model for radial modes in thin piezoceramic disks [8] does neither account for coupling to the thickness vibration component, nor other effects of finite disk thickness.

A Brissaud model has been proposed as an alternative to these traditional 1D thin disk radial and thickness extensional mode models [9]. The proposed three-dimensional (3D) model has been stated to yield a better description of radial and thickness extensional modes, including a description of their coupling. However, important inconsistencies in the results predicted by Brissaud's model have been pointed out [10].

In literature [11] an approximated 3D model of cylinder shaped piezoceramics disks of arbitrary diameter/thickness ratio is described. The model is able to describe the radial and thickness extensional modes and the coupling between them. Model predicts with sufficient accuracy only the first radial and thickness modes, which are mainly used in practical applications.

The present paper reports the frequency spectra of axially symmetric extensional vibration calculated for the elastically isotropic disk with σ of 0.294, as the example of $\sigma \approx 1/3$, and compares with experiments on PZT (lead-zirconium-titanate) ceramic disks corresponding σ. The earlier works [3], [4] have studied for the case where σ is fairly different from 1/3. It would be interesting to see how this σ influences on the pattern spectra. Theoretical spectrum for isotropic disk plate was calculated for same Poisson's ratio of disk material on the basis of Hutchinson's theory [12]. To our knowledge, this theory has never been used to study the full spectrum of modes in piezoelectric disks. The theoretical and experimental spectra show a good agreement within examined thickness/ diameter ratio. Observed spectra might be predicted completely by Hutchinson's theory, then it isn't needed to correct σ as in case [2], [3].

2. NUMERICAL CALCULATIONS OF THE FREQUENCY SPECTRA

The material constants necessary for the present calculation are Poisson's ratio σ, Lame's constant μ and the densities ρ. These values were taken from table 1 [13], [14], which shows electromechanical constants of PZT ceramics used in the present experiments. In usual piezoelectric ceramics are $\sigma = -s_{12}^{E}/s_{11}^{E}$ and $\mu = 1/s_{44}^{E}$, where s^{E}'s are elastic compliances at constant electric field, using IEEE Standard notation [15].

Table 1. Electromechanical characteristics of the used ceramics

Type	s_{11}^{E} ($\times 10^{-12}$ m^2/N)	s_{12}^{E} ($\times 10^{-12}$ m^2/N)	s_{44}^{E} ($\times 10^{-12}$ m^2/N)	ρ (kg/m^3)
PZT8	11.5	–3.38	31.9	7600
PZT4	12.3	–4.05	39.0	7500
PZT2	11.6	–	45.0	7600
$BaTiO_3$	9.1	–	22.8	5700

The dispersion curves and frequency spectra for vibration of an isotropic elastic plate depend strongly on Poisson's ratio (σ) of the ceramic materials. It might be noticed on the basis figure 2, presenting the comparison the dispersion curves for σ=0.18 and σ=0.294. ξ is a propagation constant and $\Omega=(\omega\cdot a)/v_s$, normalized frequency, where a is disk radius, $\omega=2\pi f$, f is resonance frequency and $v_s=(\mu/\rho)^{1/2}$ is transverse (shear) wave velocity. Dispersion curves were calculated according literature [4].

Namely, the character of the dispersion curves change remarkably (relative position of Ω_1 to the Ω_2), according to the σ of a disk material is less than (figure 2a), equal to (approximately figure 2b) or larger than 1/3 (σ = 1/3 is the case without imaginary loop). For a practical material with σ almost equal 1/3, it would be interesting to see how the virtual absence of an imaginary loop influences the pattern of the spectra.

In this paper, theoretically and experimentally, the frequency spectra of piezoceramic disks is determined in detail with Poisson's ratio σ = 0.294 ($\sigma = -s_{12}^{E}/s_{11}^{E}$ for PZT8, because of the most number of PZT8 ceramics made experimental patterns), for range $d/2a$ from 0 to 3. Frequency spectra of extensional vibration are calculated according to Hutchinson's method [12]. His procedure is based on choosing a series of functions with unknown coefficients which satisfy the governing equations and boundary conditions.

The studies of vibrations in solid cylinders of finite length (disks) are often related to practical problems. Of practical interest are cylindrical resonators for which the diameter to length ratio is between zero and unity. When the lateral dimensions are no longer small as compared to the wavelength, the wave propagation is not uniform in a cross-section perpendicular to the direction of the wave propagation. This results in non-uniform output amplitudes when designing resonators.

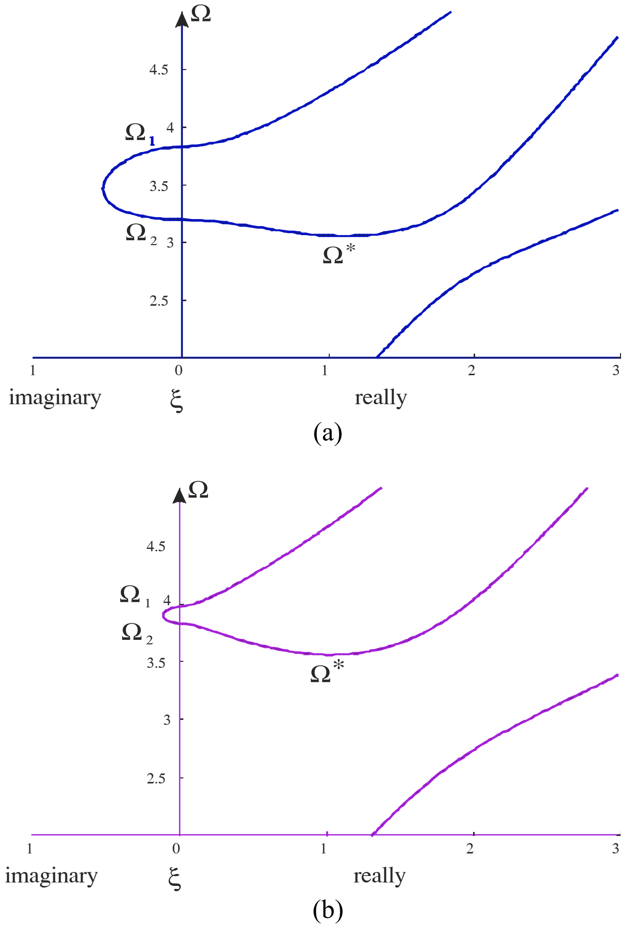

Fig. 2. Dispersion curves for disks with Poisson's ratio of 0.18 (a) and 0.294 (b) Ω^* is frequency minimum, Ω_1 cutoff frequency ($\xi = 0$) for the thickness extensional mode, Ω_2 cut off frequency ($\xi = 0$) for the thickness shear mode [3], [4]

Spectra solution approach is consisted in using the exact solutions of the general equations of linear elasticity (they are radial and axial displacements and axial, radial and shear stress), which are taken in the form of series which satisfy term by term the boundary conditions. This leads to an eigenvalue matrix the size of which is equal to the sum of the number of terms in each series [12]:

$$A = \begin{bmatrix} A_m & & B_{mn} \\ & \circ & \\ A_{mn} & & B_m \end{bmatrix} \tag{1}$$

The coefficients of eigenvalue matrix are the transcendental functions of frequency (J_i denotes ith-order Bessel function of the first kind):

$$A_m = \left(\frac{d}{2}\right)\left\{4\alpha^2\delta J_1(\delta)\left[J_0(\beta) - \frac{J_1(\beta)}{\beta}\right] + (\alpha^2 - \beta^2)J_1(\beta)\frac{[(\alpha^2-\beta^2)J_0(\delta) + 2\delta J_1(\delta)]}{\beta}\right\} \quad (2)$$

$$B_{mn} = 2\left[\frac{4\alpha^2\beta^2}{\alpha'^2 - \beta^2} + \frac{(\beta^2-\alpha^2)(2\delta^2-\Omega^2)}{\alpha'^2-\delta^2}\right]\delta\sin(\beta d/2)\sin(\delta d/2)/\beta \quad (3)$$

$$A_{mn} = \left[\frac{4\alpha^2\beta^2}{\alpha'^2 - \beta^2} + \frac{(\beta^2-\alpha^2)(2\delta^2-\Omega^2)}{\alpha'^2-\delta^2}\right]\delta J_1(\delta)J_1(\beta)/\beta \quad (4)$$

$$B_m = [4\alpha^2\delta\sin(\delta d/2)\cos(\beta d/2) + (\beta^2-\alpha^2)^2\sin(\beta d/2)\cos(\delta d/2)/\beta]/2 \quad (5)$$

with the exception that the A_0 is the twice general expression given above. In the preceding expressions, the subscripts have been omitted for simplicity. In Eqs. (2) and (5) the subscripts on α, β and δ are *m*. In Eqs. (3) and (4) the subscripts on α, β and δ are *n*, and the subscript on α' is *m*. The α's are defined as follows:

- for Eq. (2) $\alpha_m = 2m\pi/d$, $m = 0, 1, 2, \dots$,
- for Eq. (3) α_n = zeros of $J_1(\alpha_n)$, $\alpha_m' = 2m\pi/d$,
- for Eq. (4) $\alpha_n = 2n\pi/d$, α_m'=zeros of $J_1(\alpha_m')$,
- for Eq. (5) α_m = zeros of $J_1(\alpha_m)$, including $\alpha_m=0$.

The dimensionless wavenumbers α, β and δ are related to the dimensionless frequency Ω as follows:

$$\alpha^2 + \beta^2 = \Omega^2 \quad (6)$$

and

$$\alpha^2 + \delta^2 = \Omega^2\frac{1-2\sigma}{2(1-\sigma)} \quad (7)$$

In equation (1) determinant of the matrice *A* must equal zero. The coefficient of the matrice for a fixed δ and σ are functions of the frequency Ω alone as expressed in (2)-(5), and the relationships in (6) and (7). The natural frequencies are found by searching for zeros of the determinant using the Matlab software.

In our case a sufficient accuracy was achieved by matrix 20×20. With an aim to complete the description of the cylindrical element frequency behavior, as the finite result the ceramic frequency spectra were determined in the function of $d/2a$ ratio. Expressions for the displacements include two types of the motion, one of which is symmetric about the central plane, whereas the other is antisymmetric. In this paper the analysis only "symmetric motion" was done.

On the fig. 3, for the case of the ceramic with $\sigma = 0.294$, shown the frequency spectra for five lowest resonance modes obtained by suggested method. The terrace-type spectra in the frequency minimum environment Ω^* and frequency Ω_1 and Ω_2 (figure 2), is particularly interesting, because of their closeness and coupling in the cause $\sigma = 0.294$.

3. EXSPERIMENTAL RESULTS

In order to show the possibilities of resonance frequencies determination of the piezoceramic disks by the proposed method, the piezoceramic disks resonance with the different ratio thickness/diameter frequencies was measured by network analyzer. In the table 2 it has been shown disks dimensions, which were used in the analysis.

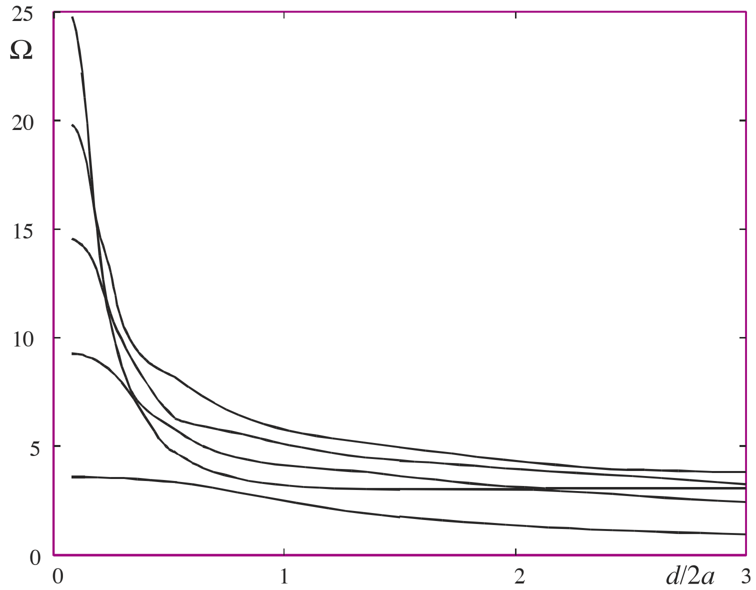

Fig. 3. Theoretical normalized frequency spectra of axisymmetric extensional vibration in disk plates with Poisson's ratio of 0.294

Table 2. Piezoceramic disks used in the analysis

	Type	$2a$ [mm]	d [mm]	$d/2a$
Disk I	PZT8	50	3	0.060
Disk II	PZT8	30.9	2.1	0.068
Disk III	PZT8	24.9	2.5	0.100
Disk IV	PZT8	38.4	6.35	0.165
Disk V	PZT2	8	7.6	0.950
Disk VI	PZT4	8	7.7	0.962
Disk VII	$BaTiO_3$	9.8	10	1.020

Piezoceramic disks from table 2 are poled along thickness and electroded on their flat surfaces. A frequency spectrum of the polarized disks is observed by automatic network analyzer HP 3042A, by measuring the dependency characteristics of attenuation a_Z from frequency, what is equal to the measurement of the impedance characteristic Z ($a_Z = 20\log(Z/50 + 1)$). The frequency where impedance of the specimen becomes lowest, is considered the "main" resonance frequency. For the other adjacent modes of vibration, the resonance frequency was determined as that of minimum impedance. In the complicated frequency response as shown in figure 4, it is difficult to determine, respectively, the intensity of each resonance. However, the resonance intensity itself is not essentially important for the present study, so that the defining of resonance dynamic range due to its simplicity was omitted. Dynamic range of each resonance might be, if is it necessary, defined as ratio of the maximal and minimal impedance for the corresponding resonance mode.

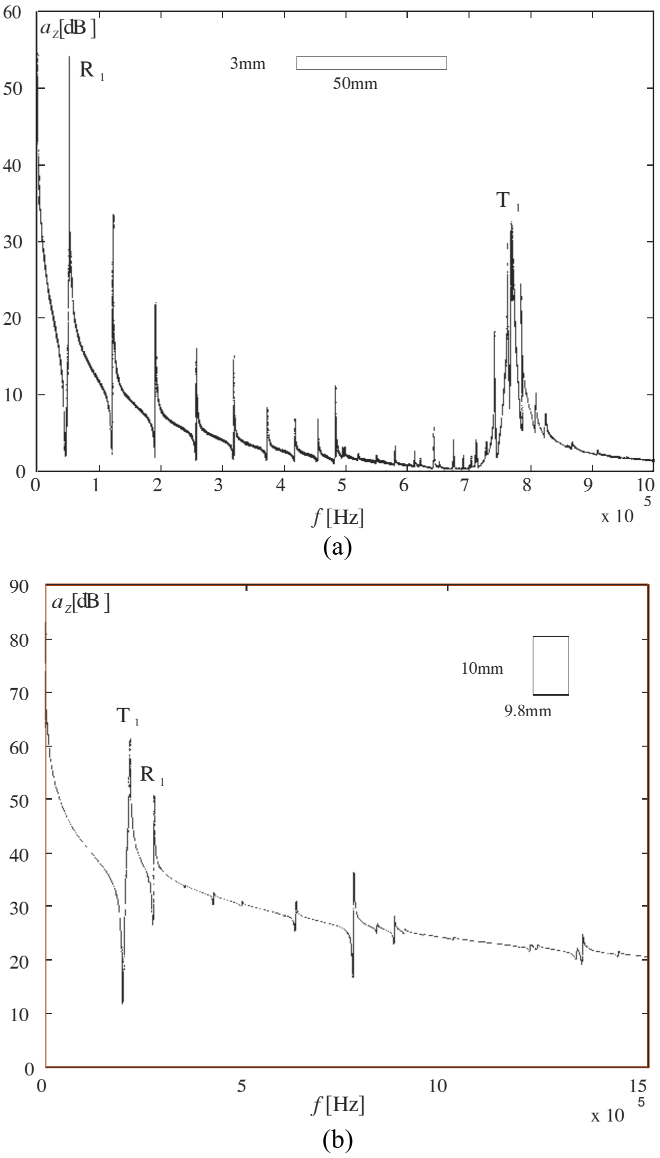

Fig. 4. Experimental impedance versus frequency for a disk I with dimensions $2a$ = 50 mm, d = 3 mm (a) and cylinder VII with dimensions $2a$ = 9.8 mm, d = 10mm (b) (R_1 is first radial mode, and T_1 first thickness mode)

4. COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL SPECTRA AND DISCUSSIONS

First of all, the normalized frequent spectra for the piezoceramic discs (table 2), obtained by 1D and 3D models in the literature, were shown in order to present the possibilities of resonance frequencies determination of the piezoceramic discs.

By the application of 1D model [7], the dependence characteristic of normalized resonance frequency (Ω) of the piezoceramic disks from the ratio thickness/diameter ($d/2a$) for the ceramic disks case in table 1, is shown in figure 5a. With the intention to estimate this kind of modeling, the same figure shows the resonance frequencies experimental values for the lowest modes of the disks in the table 2. It might be noticed that 1D model is inconvenient for determining the lowest radial resonance frequencies of piezoceramic disks.

3D model [11] predicts with sufficient accuracy only the first radial and the first thickness modes of the cylinder-shaped piezoceramic element of arbitrary ratio $d/2a$, but the frequencies of higher radial modes don't agree with the measured results. It can be noticed from the figure 5b, for the disk resonance frequencies from table 2 determined by this model and experimentally.

Experimental frequency spectrum, i.e. ratio the resonance frequency and $d/2a$ for disks from table 2, is shown on the figure 5 for $d/2a$ in the range from 0.06 to 1.02. In order to carry out the comparison with theoretical spectra, the measured resonance frequencies were normalized in the previously mentioned way.

Comparing the normalized frequency spectra obtained by application of described method (figure 6) with the measurements of the disk resonance frequencies from table 2, it might be noticed that the large accuracy was realized in predicting all resonance modes. The frequency range, which is most frequently used in practical applications is shown here.

Large agreement of the theoretical curve and experimental results was noticed for PZT8 disks, for whose Poisson's ratio a calculation was done. Resonance frequencies PZT4 and PZT2 disks have a large deviation in relation to the theoretical frequency spectra because of their different Poisson's ratios (for instance for PZT4 is $\sigma = -s_{12}^{E}/s_{11}^{E}=0.33$). Dynamic ranges of some resonance frequencies are too weak in order to be registered (for instance, the third resonant mode of PZT2 disk). Disk VII of $BaTiO_3$ ceramic was used in the experimental measurements because of its Poisson's ratio $\sigma = 0.3$, which is close to the used ones $\sigma = 0.294$ due to this. It is obvious that its frequency spectrum is close to the calculated one.

The theoretical spectra have been calculated on the basis of the hypothesis that the disk is elastically isotropic. On the other hand, the piezoelectric ceramics used for measurements are not isotropic. Furthermore, in piezoelectric ceramics with high electromechanical coupling factor, the resonance frequency of piezoelectric disk changes from that of an elastically isotropic disk. Because of that the frequency differences exist on the figure 6.

It is possible, in the aim of agreement between the theoretical and experimental spectra, to correct insignificantly the piezoelectric coefficient s_{44}^{E}, which will influence on the frequency value Ω by which the experimental results are normalized. In this way, a large coincidence the compared spectrum is attained without value change σ as in [3]. On the figure 7, the experimental frequency spectrum was shown again only for PZT8 and $BaTiO_3$ discs, together with the theoretical frequency spectrum on the figure 6, for insignificantly corrective value s_{44}^{E} given in the table 3. On that way, the frequency spectrum was obtained for which the same ceramic material doesn't depend upon just a sample dimensions, but upon its dimensions ratio.

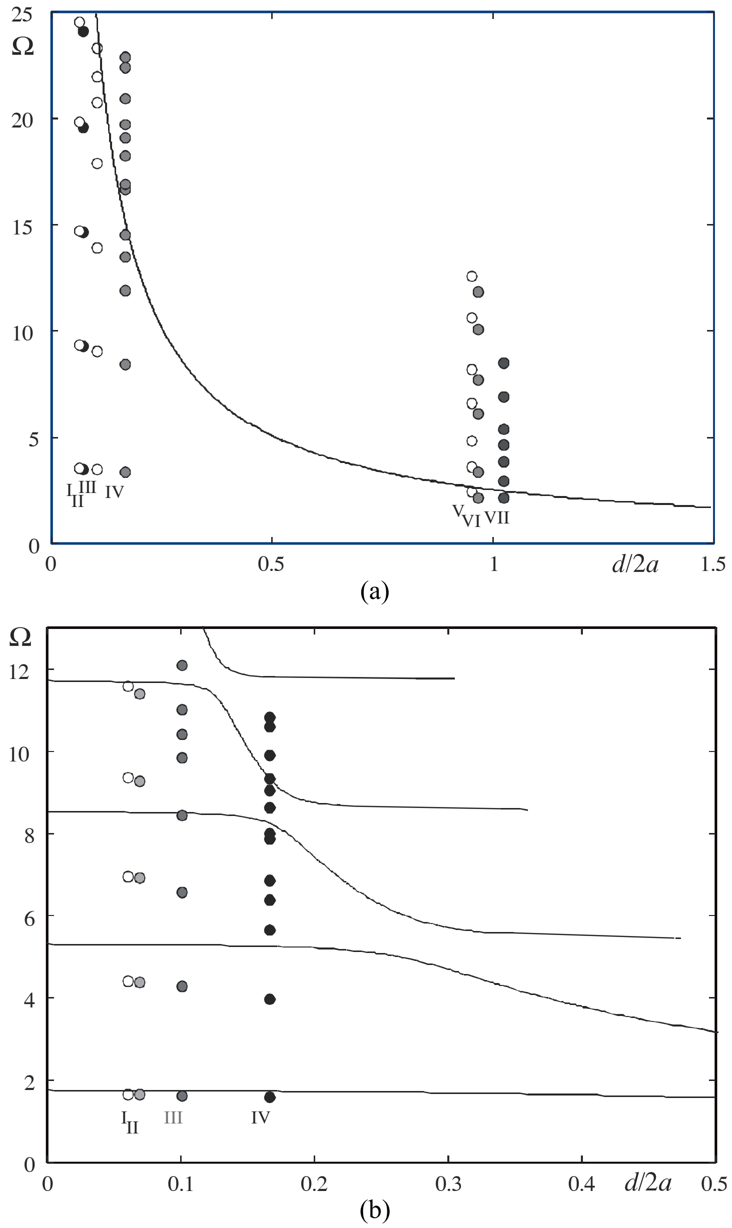

Fig. 5. Comparison between experimental (circles) and theoretical (lines) normalized frequency spectra in piezoceramic disks, computed with the 1D model (a) and 3D model (b)

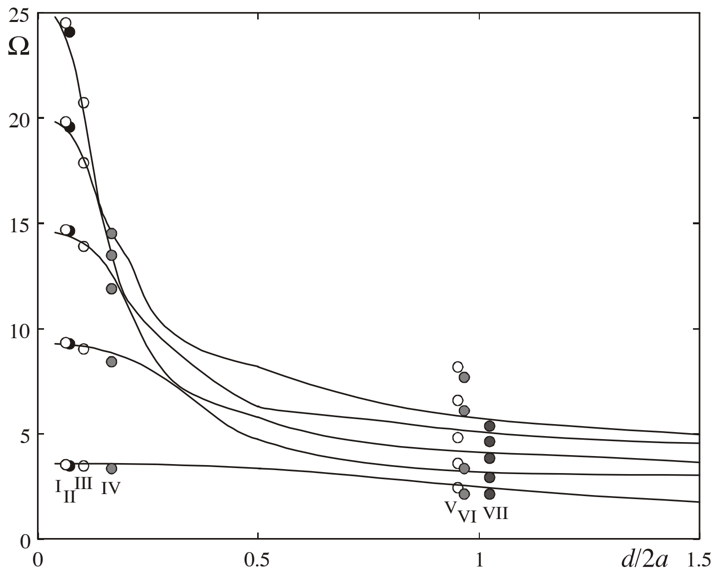

Fig. 6. Comparison between experimental (circles) and theoretical (lines) normalized frequency spectra for used disks, computed with proposed 3D method

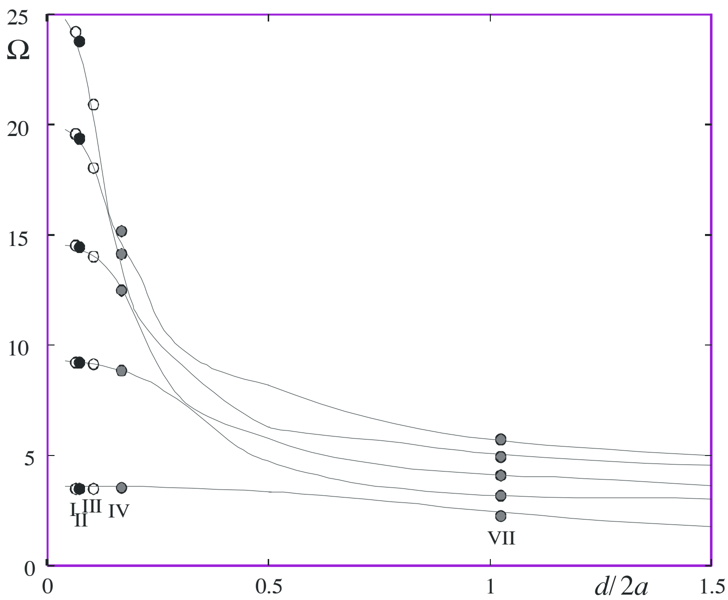

Fig. 7. Comparison between experimental and theoretical normalized frequency spectra for fitting value of the coefficient s_{44}^{E} from table 3

Table 3. Catalogue and fitting values of the piezoelectric coefficient s_{44}^{E}

	s_{44}^{E} ($\times 10^{-12}$ m^2/N)	
	Data catalogue	Fitting value
Disk I	31.9	31.2
Disk II	31.9	31.2
Disk III	31.9	32.5
Disk IV	31.9	35.1
Disk VII	22.8	26

5. CONCLUSION

It is very interesting to know whether frequency spectra of a ceramic plate having such a large elastic anisotropy can be approximated by the theory for an isotropic plate.

In this paper, the systematization of the existing models of piezoelectric transducer in the disk form, was done. The comparison of the suggested method was done with known piezoelectric approaches and models for determining the resonance frequencies. The characteristics of the one- and three-dimensional models of piezoelectric ceramic disks were shown and their main advantages and disadvantages were emphasized.

Frequency spectra of the axisymmetric extensional vibrations of the piezoelectric PZT (lead-zirconium-titanate) disks with Poisson's ratio 0.294, were examined theoretically and experimentally. A theoretic frequency spectrum was determined by application of the approximation for elastic isotropic disks and cylinders, with thickness/diameter ratios from 0 to 3. The determination way of the resonance frequencies of stress-free piezoceramic disks was shown, which may be applied to any ratio of the disk thickness and diameter. During the experiment the piezoelectric properties of ceramics were not taken into consideration. On that way, it is possible to take the coupling between thickness and radial modes of disk into consideration. And in the boundary cases of the thin disk, i.e. thin cylinder, the results are in good agreement with such existing (corresponding) models. Experimental verification of this method was done by comparison of the calculated frequency spectra with the measurements on the concrete samples with thickness/diameter ratio from 0.06 to 1.02. Although the spectra are very complicated in the arrays where the comparison was done, the common properties of the experimental and theoretical spectra are in good agreement.

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REZONANTNE FREKVENCIJE PZT PIEZOKERAMIČKIH DISKOVA: NUMERIČKI PRISTUP

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U ovom radu teorijski i eksperimentalno je istraživan frekventni spektar osnosimetričnih ekstenzionih vibracija u PZT piezokeramičkim diskovima sa Poisson-ovim odnosom (σ) 0.294. Teorijski frekventni spektar je izračunat primenom Hutchinson-ove teorije oscilovanja za izotropan elastičan disk (odnosno cilindar) sa odnosom debljine i prečnika (d/2a) od 0 do 3, bez razmatranja piezoelektričnih svojstava diskova. Eksperimentalni spektar je posmatran za diskove sa odnosom d/2a od 0.06 do 1.02. Eksperimentalni spektar se dobro poklapa sa teorijskim spektrom i svi posmatrani delovi spektra mogu biti kompletno predviđeni na osnovu teorije oscilacija izotropnog diska.