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TEMPERATURE, STRAIN AND STRESS FIELDS PRODUCED BY IMPULSIV ELECTROMAGNETIC RADIATION IN THE THIN METALLIC PLATE

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Abstract. *In the paper the case of transversal vibrations of the thin elastic metallic plate produced by impulsive electromagnetic radiation at the upper surface is considered. As each electromagnetic wave can be represented as a sum of simple plane waves, an analytical solution is given for only one plane wave. It is assumed that all field quantities of the wave vary with time as exp(jωt) and are represented in the complex form. As a result of time-changing electromagnetic field conducting currents are appearing. Using complex calculation we arrive to the distribution of the power of the eddy-current losses in the plate. That power can be treated as a volume heat source. Impulsive radiation can be represented mathematically as the sum of Heaviside functions. Using integral transform technique we can solve differential equations governing temperature field, transverse vibrations and stress field. In the case when the skin depth is small compared to the plate thickness, the problem can be treated as a thermal shock problem. The power which electromagnetic wave gives to the plate can be calculated by Pointing's vector. If we want to solve some geometrically complicated problem, we have to discuss Snellius laws. Using Fresnel coefficients of reflection and transmission the power of the interrupted wave can be found as a function of an interrupted angle. The calculation (for example, for radar and air space structures) can be performed using FEM.*

Key words: *electromagnetic field, temperature, plate, induction, heat, vibration, finite element.*

1. INTRODUCTION

Theory of electro-magneto-thermoelasticity investigates interaction between strain and electromagnetic field in a solid elastic body. It has received considerable attention because of the possible applications in detection of flaws in ferrous metals, optical acoustics, levitation by superconductors, magnetic fusion and many other electro-

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mechanical devices. On metallic deformabile solids subjected to electromagnetic fields two types of forces are exerted. The first type are forces between the stationary magnetic field and magnetized material (which are reacted as a moments). The second types are volume dynamic forces on a conducting currents appeared in electric conductors as a result of their motion or a time changing magnetic field. The influence of the elastic field on the magnetic field is described by modified Ohm's law.

In the paper it is assumed that the material of the plates is elastic, isotropic, soft ferromagnetic, which possesses a good electric conductivity. Many nickel-iron alloys used for motors, generators, inductors, transformers are of this type.

The vibrations of the thin metallic plates of soft ferromagnetic materials are described using three coupled systems of differential equations based on the classical theory of thin plates and linear theory of thermoelasticity.

The first system is a system of Maxwell's equations with the relations for slowly moving media and modified Ohm's low [4]:

$$rot\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}\,,\ rot\vec{K} = -\frac{\partial \vec{B}}{\partial t}\,,\ div\vec{D} = 0\,,\ div\vec{B} = 0\,, \tag{1.1}$$

$$\vec{D} = \varepsilon_0(\vec{K} + \dot{\vec{u}} \times \vec{B})\,,\ \vec{B} = \mu(\vec{H} - \dot{\vec{u}} \times \vec{D})\,,\ \vec{J} = \sigma(\vec{K} + \dot{\vec{u}} \times \vec{B})\,.$$

The following notations are applied: H – magnetic intensity, K – electric intensity, B – magnetic flux density (magnetic induction), D – electric induction, J – current density, u – deflection, μ_0 – permeability of free space, σ – electric conductivity, ε_0 – dielectric constant of free space.

In linear theory of thermoelasticity it is assumed that the temperature changes linearly across the thickness of the plate. Temperature field $\theta(x_1,x_2,x_3,t)$ can be described using two values, τ_0 and τ_1; τ_0 represents the temperature in the middle surface of the plate and τ_1 is the rate of temperature across the plate thickness [1]

$$\theta(x_1,x_2,x_3,t) = \tau_0(x_1,x_2,t) + x_3\tau_1(x_1,x_2,t)\,.$$

So, the second system of equations describes temperature field in a thin plate. It consists of two partial differential equations

$$\left(\nabla_1^2 - \beta_k - \frac{1}{\kappa_\eta}\frac{\partial}{\partial t}\right)\tau_k + \frac{\beta_k^k}{h}\left[x_3^k\frac{\partial\theta}{\partial x_3}\right]_{-\frac{h}{2}}^{\frac{h}{2}} - \eta^*\varepsilon_k = -\frac{\beta_k^k Q_k}{h\kappa}\,,\ (k = 0,1), \tag{1.2}$$

$$\beta_k = \begin{cases} 0, & k=0 \\ \dfrac{12}{h^3}, & k=1 \end{cases},\ \varepsilon_k = \begin{cases} \varepsilon', & k=0 \\ \nabla_1^2\ddot{w}, & k=1 \end{cases},\ Q_k = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q(x_1,x_2,x_3,t)x_3^k dx_3\,,\ Q = -\frac{W + W_H + \dfrac{J^2}{\sigma}}{C_\varepsilon}\,,$$

where κ is coefficient of thermal intensity, η^* is representing the coupling between the temperature and the deformation fields, ε is the tensor of deformation, h is the plate thickness and ∇_1^2 is Laplace operator. Losses in a plate $Q(x_1,x_2,x_3,t)$ consist of three factors: volume heat source intensity, hysterisis losses and Joule's heat (eddy-current losses).

In the consideration of the vibrations of the plate, the assumption that the longitudinal vibrations are independent of the transverse vibrations is taken. Transverse vibrations can be obtained using the next differential equation [2]

$$D\nabla_1^4 w + D(1+\nu)\alpha_t \nabla_1^2 \tau_1 - \rho h \ddot{w} + \frac{\rho h^3}{12}\ddot{w} = (\sigma_{33}^+ - \sigma_{33}^-) + (T_{33}^+ - T_{33}^-) +$$
$$+\frac{h}{2}\frac{\partial}{\partial x_i}(\sigma_{i3}^+ + \sigma_{i3}^-) + \frac{h}{2}\frac{\partial}{\partial x_i}(T_{i3}^+ + T_{i3}^-) + \int_{-\frac{h}{2}}^{\frac{h}{2}} (X_{i,i} + f_{i,i}) x_3 dx_3 + \int_{-\frac{h}{2}}^{\frac{h}{2}} X_3 dx_3 \,, \tag{1.3}$$

and the bending stresses are given by the following relation

$$\sigma_{ij} = -\frac{Ex_3}{1-\nu^2}\{(1-\nu)w_{,ij} + [\nu w_{,kk} + (1+\nu)\alpha_t \tau_1]\delta_{ij}\} \,. \tag{1.4}$$

w denotes deflection of the plate in x_3-direction, ν is Poisson ratio, α_t is coefficient of thermal expansion, D is flexural rigidity of the plate, E is modulus of elasticity, X mechanical force, f Lorenz force, σ and T are mechanical and magnetic stresses, and ρ is plate density.

Of course, appropriate boundary and initial conditions have to be added to the presented system of equations.

2. ELECTROMAGNETIC WAVE

In the paper the case of transversal vibrations of the thin elastic metallic plate produced by impulsive electromagnetic radiation at the upper surface is considered.

Impulsive radiation can be displayed as a sum of pulsation functions (Fig. 2.1) and mathematically represented as a sum of Heaviside functions with delay.

$$\sum_{i=0}^{I}\left[H(t - it_i) - H(t - (it_i + t_0))\right]. \tag{2.1}$$

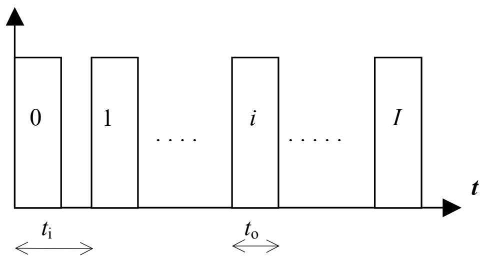


Fig. 2.1 Impulsive electromagnetic wave

On the basis of the fact that each electromagnetic wave with complex time changing field can be represented as a sum of the simple plane waves (using by methods of Fourie analysis), an analytical solution is given for only one plane wave with E_{xo} and H_{yo} components on the upper surface of the plate. It is assumed that all field quantities inside one impulse vary with time as exp(jωt) and are represented in the complex form.

3. CONDUCTING CURRENTS. JOULE'S HEAT

In the case of high conductivity dielectric current is negligible compared with the conducting current. So, for the homogeneous and isotropic medium (without free electric charges) system of Maxwell's equations can be presented in the next form (linear magnetic):

$$rot\vec{H} = \sigma\vec{K}\ ,\quad div\vec{K} = 0\ ,\quad rot\vec{K} = -\mu\frac{\partial\vec{H}}{\partial t}\ ,\quad div\vec{H} = 0\ . \tag{3.1}$$

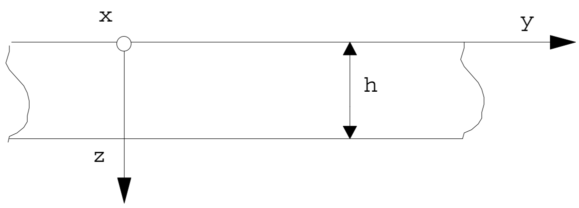


Fig. 3.1 Primordial coordinated system

Using symbolic-complex method ($\vec{A} = \underline{\vec{A}}e^{j\omega t}$) we arrive to the equations

$$rot\underline{\vec{H}} = \sigma\underline{\vec{K}}\ ,\quad div\underline{\vec{K}} = 0\ ,\quad rot\underline{\vec{K}} = -\mu(j\omega)\underline{\vec{H}}\ ,\quad div\underline{\vec{H}} = 0\ . \tag{3.2}$$

If the direction of the wave propagation is z-axe and if the field components are independent of x and y, from the equations of divergence we can conclude that the components $\underline{H}_z$ and $\underline{K}_z$ are zero. In the case of the plane wave, only normal components of the electric and magnetic field depend on each other. So, we have to make the analysis only for one wave with components $\underline{K}_x$ and $\underline{H}_y$. Let in the plane $z = 0$ they have next values

$$\vec{K} = K_x\vec{i} = K_0\cos(\omega t + \varphi)\vec{i}\ , \tag{3.3}$$

$$\vec{H} = H_y\vec{j} = H_0\cos(\omega t + \varphi)\vec{j}\ ,\quad H_0 = \sqrt{\frac{\varepsilon}{\mu}}K_0\ .$$

Maxwell's equations have next form

$$-\frac{\partial\underline{H}_y}{\partial z} = \sigma\underline{K}_x\ ,\quad \frac{\partial\underline{K}_x}{\partial z} = -\mu j\omega\underline{H}_y\ , \tag{3.4}$$

or

$$\frac{\partial^2\underline{H}_y}{\partial z^2} - \gamma\underline{H}_y = 0\ ,\quad \underline{K}_x = -\frac{1}{\sigma}\frac{\partial\underline{H}_y}{\partial z}\ , \tag{3.5}$$

where

$$\gamma^2 = j\sigma\mu\omega,\quad \gamma = \alpha + j\beta\ ,\quad \alpha = \beta = \sqrt{\frac{\sigma\mu\omega}{2}}\ . \tag{3.6}$$

If we want to find the solution only for one progressive wave, the basic solution of the equation (3.5) we can represent as

$$\underline{H}_y = \underline{C}e^{-\gamma z}, \quad \underline{K}_x = \frac{\gamma}{\sigma}\underline{H}_y, \tag{3.7}$$

and using boundary condition for *z=0*

$$\underline{C} = H_0 e^{j\varphi}.$$

Characteristic impedance is

$$\underline{Z}_c = \frac{\underline{K}}{\underline{H}} = \frac{\gamma}{\sigma} = \sqrt{\frac{\mu\omega}{2\sigma}}(1+j) = \sqrt{\frac{\omega\mu}{\sigma}}e^{j\frac{\pi}{4}}, \tag{3.8}$$

and the obtained result for the field components has the form

$$\underline{H}_y = H_0 e^{j\varphi}e^{-\alpha z}e^{-j\beta z}, \quad \underline{K}_x = H_0\sqrt{\frac{\omega\mu}{\sigma}}e^{j\frac{\pi}{4}}e^{j\varphi}e^{-\alpha z}e^{-j\beta z}, \tag{3.9}$$

or

$$H_y = \mathrm{Re}[\underline{H}_y e^{j\omega t}] = H_0 e^{-\alpha z}\cos(\omega t - \beta z + \varphi), \tag{3.10}$$

$$K_x = \mathrm{Re}[\underline{K}_x e^{j\omega t}] = H_0\sqrt{\frac{\omega\mu}{\sigma}}e^{-\alpha z}\cos\left(\omega t - \beta z + \varphi + \frac{\pi}{4}\right).$$

Electromagnetic wave is followed by the conducting currents, density

$$\underline{J}_x = \sigma\underline{K}_x = H_0\sqrt{\omega\mu\sigma}e^{-\alpha z}e^{-j\beta z}e^{j\left(\varphi+\frac{\pi}{4}\right)}. \tag{3.11}$$

Field amplitudes and current amplitudes decrease according to the exponential law $e^{-\alpha z}$ along the trajectory of the wave propagation. The constant of the penetration is proper to the decay of one Neper (0.368) and its value is

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\sigma\mu\pi f}}, \quad \left(\omega = \frac{2\pi}{T} = 2\pi f\right). \tag{3.12}$$

Skin depth is decided with increasing of frequency, conductivity and permeability. The reason for that phenomenon is heat losses in the metal. Active power which the conductor absorbs throw the part S of the surface can be obtained using the method of Pointing's vector (for $z = 0$, $\varphi = 0$)

$$\underline{\Gamma} = \frac{1}{2}\underline{K}_x\underline{H}_y^* = \frac{1}{2}\sqrt{\frac{\omega\mu}{\sigma}}H_0^2 e^{j\frac{\pi}{4}}, \mathrm{P} = \mathrm{Re}[\underline{\Gamma}]S = \frac{1}{2}\sqrt{\frac{\omega\mu}{2\sigma}}H_0^2 S. \tag{3.13}$$

Presented formulas are valid for the conductors with the skin depth negligible compared with the curvature diameter. If the skin depth is much less then the plate thickness, the whole absorbed power is converted in heat on the surface (the case of thermal shock).

Otherwise distribution of the Joule's heat has to be determined in the following:

$$\|J\|^2 = \underline{J}_x\underline{J}_x^* = H_0^2\sigma\omega\mu e^{2\alpha z}, \tag{3.14}$$

$$P_v = \frac{1}{2\sigma}\int_0^h H_0^2 \sigma\omega\mu e^{2\alpha z} dz = \frac{1}{2} H_0^2 \sqrt{\frac{\omega\mu}{2\sigma}} (1 - e^{-2\alpha h}) . \tag{3.15}$$

The whole power of the eddy-current is

$$P = \frac{H_0^2 S}{2} \sqrt{\frac{\omega\mu}{2\sigma}} (1 - e^{-2\alpha h}) , \; P_v(z) = \frac{1}{2} H_0^2 \omega\mu e^{-\sqrt{2\sigma\mu\omega} z} . \tag{3.16}$$

Using coordinate system shown on Fig. 3.2 last express can be presented as

$$P_v(x_3) = \frac{1}{2} H_0^2 \omega\mu e^{-h\sqrt{\frac{\sigma\mu\omega}{2}}} e^{x_3 \sqrt{2\sigma\mu\omega}} . \tag{3.17}$$

That power we can treat as a volume heat source with intensity $P_v(x_3)$.

In the case of nonlinear magnetic material the factor which involves heat losses of the hysterisis loop has to be added to presented calcualtion. For the most of the soft ferromagnetic materials the basic curve of the magnetization is nearly linear. This fact improves that the middle value for permeability μ_{sr} can be used in calculation.

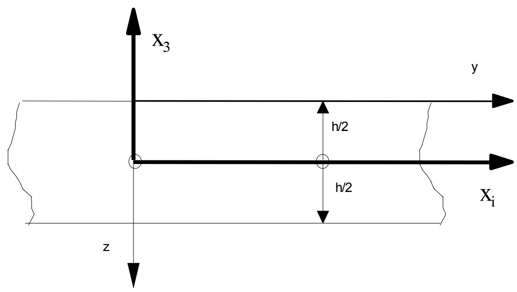


Fig. 3.2 Coordinate system (middle surface of the plate)

Hysterisis losses are proportional to the square of the frequency and field amplitudes

$$P_H \approx H_{max}^2 f , \; P_H(z) = k H_0^2 e^{-2\alpha z} f , \tag{3.18}$$

which improves that their distribution is the same as the distribution of the eddy-current losses. Coefficient k is the known material characteristic.

So, density of the power of the heat losses is approximately

$$P_u(z) = H_0^2 e^{-2\alpha z} f(k + \pi\mu) , \; P_u(x_3) = \frac{1}{2} H_0^2 \omega \left(\mu + k \frac{\omega}{\pi} \right) e^{-h\sqrt{\frac{\sigma\mu\omega}{2}}} e^{x_3\sqrt{2\sigma\mu\omega}} = P e^{2\alpha x_3} . \tag{3.19}$$

Expressions (3.17) and (3.19) show that the heat source intensity increases on exponentially with the plate thickness. Gradient of the exponential curve increases with the increasing of the wave frequency, permeability and electric conductivity of the material.

The phenomenon of the conducting current concentration on the surface, valid for conductors with very high electric conductivity and magnetic permeability subject to high frequency wave, is known as skin effect.

4. TEMPERATURE FIELD

Let the plate is isolated on the upper and the lower surface and the temperature along the lateral sides is equal to initial temperature $T_0(\theta = 0)$. The initial and the boundary conditions have the form

$$\theta\big|_{t=0} = 0\,,\ \theta\big|_{x_1=0,a} = 0\,,\ \theta\big|_{x_2=0,b} = 0\,,\ \frac{\partial\theta}{\partial x_3}\bigg|_{x_3=\pm\frac{h}{2}} = 0\,. \tag{4.1}$$

Using (3.17) the power of the heat source is

$$W(x_3) = \frac{1}{2}H_0^2\omega\mu e^{-h\sqrt{\frac{\sigma\mu\omega}{2}}} e^{x_3\sqrt{2\sigma\mu\omega}} = Pe^{2\alpha x_3}\,, \tag{4.2}$$

$$W(x_3,t) = Pe^{2\alpha x_3}\sum_{i=0}^{I}[H(t-it_i) - H(t-(it_i+t_0))]\,.$$

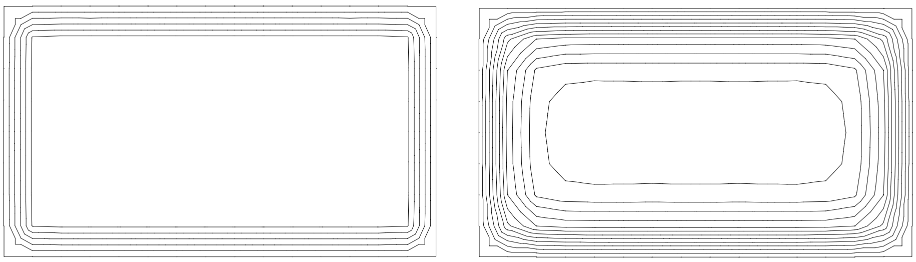
According to the presented boundary conditions, from equations (1.2), using integral transform technique we arrive to the solution for the temperature field in the form

$$\tau_k = \frac{4}{ab}\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty}\frac{4\beta_k^k C_k}{\lambda_0 h\alpha_n\alpha_m\Delta_{mn}}\sin\alpha_n x_1 \sin\alpha_m x_2 \tag{4.3}$$

$$\sum_{i=0}^{I}\{[1-e^{-\kappa(\beta_k+\Delta_{mn})(t-it_i)}[1-e^{-\kappa(\beta_k+\Delta_{mn})(t-(it_i+t_0))}]]H(t-it_i) - H(t-(it_i+t_0))\}\,,$$

$$C_k = \begin{cases} \dfrac{P}{2\alpha^2}[\alpha hCh(\alpha h) - Sh(\alpha h)], & k=1 \\ \dfrac{\mathrm{P}}{\alpha}Sh(\alpha h), & k=0 \end{cases}\ ;\ \Delta_{mn} = \alpha_n^2 + \alpha_m^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2.$$

Distribution of the isotherm lines in the middle surface of the plate for the case of one impulse of $t_0 = 10$s is presented in Fig. 4.1. Plate dimensions were $a = 30$cm, $b = 16$cm, $h = 1$mm.



$t = 5$s $\qquad\qquad$ $t = 20$s

Fig. 4.1 Isotherm lines in the middle surface

5. TRANSVERSAL VIBRATIONS. STRESS FIELD

Let the plate be simply supported along the entire edge. Boundary conditions have the form

$$w\big|_{x_1=0,a}=0\,,\; M_{11}\big|_{x_1=0,a}=\left[\frac{\partial^2 w}{\partial x_1^2}+\nu\frac{\partial^2 w}{\partial x_2^2}+(1+\nu)\alpha_t\tau_1\right]D\Bigg|_{x_1=0,a}=0\,, \tag{5.1}$$

$$w\big|_{x_2=0,b}=0\,,\; M_{22}\big|_{x_2=0,b}=\left[\nu\frac{\partial^2 w}{\partial x_1^2}+\frac{\partial^2 w}{\partial x_2^2}+(1+\nu)\alpha_t\tau_1\right]D\Bigg|_{x_2=0,b}=0\,.$$

Initial conditions are responsible to the natural undeformed state

$$w\big|_{t=0}=0\;,\;\frac{\partial w}{\partial t}\bigg|_{t=0}=0\,. \tag{5.2}$$

Applying the integral transform technique to the equation (1.3) the solution for the transversal vibrations is represented in the form

$$w(x_1,x_2,t)=\frac{4}{ab}\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty}\frac{4\alpha_t(1+\nu)\beta_1 C_1\kappa}{\lambda_0 h\alpha_n\alpha_m\Delta_{mn}}\sin\alpha_n x_1\sin\alpha_m x_2 \tag{5.3}$$

$$\sum_{i=0}^{I}\left[I(m,n,t-it_i)H(t-it_i)-I(m,n,t-(it_i+t_0))H(t-(it_i+t_0))\right],$$

$$I(m,n,t)=\frac{1-e^{-\kappa(\Delta_{mn}+\beta_1)t}}{\kappa(\Delta_{mn}+\beta_1)}-\frac{\kappa(\Delta_{mn}+\beta_1)\cos\frac{\Delta_{mn}}{K}t+\frac{\Delta_{mn}}{K}\sin\frac{\Delta_{mn}}{K}t-\kappa(\Delta_{mn}+\beta_1)e^{-\kappa(\Delta_{mn}+\beta_1)t}}{\left(\frac{\Delta_{mn}}{K}\right)^2+\kappa^2(\Delta_{mn}+\beta_1)^2}\,.$$

Stress field is obtained using equation (1.4).

In Fig. 5.1 deformation of the plate during impulse is presented, and Fig. (5.2) shows appropriate stress field.

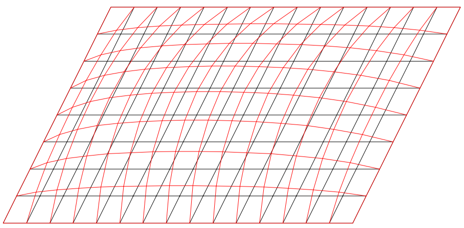


Fig. 5.1 Deformation of the plate

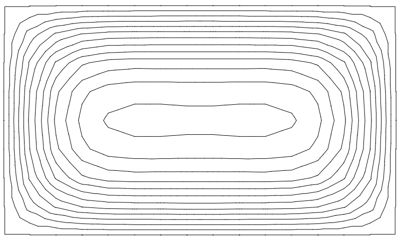


Fig. 5.2 Stress field

6. HIGH FREQUENCY WAVES. APLICATION ON THE GEOMETRICALLY COMPLICATED PROBLEMS

In the case of the high frequency wave skin depth is very small. It denotes that the hole Pointing's vector

$$\underline{\Gamma} = \frac{1}{2}\underline{K}_x \underline{H}_y^*, \; \mathrm{P} = \mathrm{Re}[\underline{\Gamma}]S = \frac{1}{2}\sqrt{\frac{\omega\mu}{2\sigma}} H_0^2 S, \tag{6.1}$$

converts to heat on the plate surface. Than we have thermal shock problem.

To obligate the power of the electromagnetic wave penetrated in the plate, we have to discus Snellius's lows. So, look at the plane of incidence of an interrupted, reflected and transmitted waves (Fig. 6.1).

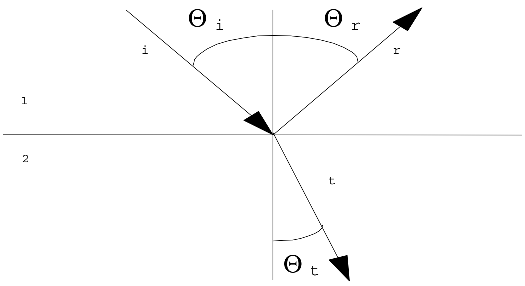


Fig. 6.1 Plane of incidence

Snelius's low of transmission is [5]

$$\underline{\gamma}_1 \sin\Theta_i = \underline{\gamma}_2 \sin\Theta_t. \tag{6.2}$$

Fresnel's coefficients of reflection and transmission can be determined using boundary conditions, which for the case of the electric field normal to the plane of an incidence have the next form (n)

$$\underline{K}_{0i} + \underline{K}_{0r} = \underline{K}_{0t}, \; (\underline{H}_{0r} - \underline{H}_{0i})\cos\Theta_i = -\underline{H}_{0t}\cos\Theta_t, \tag{6.3}$$

and in the case of the electric field parallel to the plane of an incidence (p)

$$\underline{H}_{0i} + \underline{H}_{0r} = \underline{H}_{0t}, \; (\underline{K}_{0i} - \underline{K}_{0r})\cos\Theta_i = \underline{K}_{0t}\cos\Theta_t. \tag{6.4}$$

Relation $\underline{T} = \dfrac{\underline{K}_{0t}}{\underline{K}_{0i}}$ represent Fresnel's coefficients, which in those two cases are [5]

$$\underline{T}_n = \frac{2\underline{Z}_2\cos\Theta_i}{\underline{Z}_2\cos\Theta_i + \underline{Z}_1\cos\Theta_t}, \; \underline{T}_p = \frac{2\underline{Z}_2\cos\Theta_i}{\underline{Z}_1\cos\Theta_i + \underline{Z}_2\cos\Theta_t}. \tag{6.5}$$

When medium 1 is an are and medium 2 metal conductor, appropriate complex impedance and complex angles are

$$\underline{Z}_1 = \sqrt{\frac{\mu}{\varepsilon}}, \quad \underline{\gamma}_1 = j\omega\sqrt{\varepsilon\mu}, \tag{6.6}$$

$$\underline{Z}_2 = \sqrt{\frac{\mu\omega}{2\sigma}}(1+j), \; \underline{\gamma}_2 = \sqrt{\frac{\omega\mu\sigma}{2}}(1+j).$$

For $k = \sqrt{\dfrac{\omega\mu}{2\sigma}}$ we have

$$\underline{T}_n = \frac{2k(1+j)\cos\Theta_1}{k(1+j)\cos\Theta_1 + \sqrt{\frac{\mu}{\varepsilon\sigma}(\sigma - j\omega\varepsilon\sin^2\Theta_1)}},$$

$$\underline{T}_p = \frac{2k(1+j)\cos\Theta_1}{\sqrt{\frac{\mu}{\varepsilon}}\cos\Theta_1 + \frac{k(1+j)}{\sqrt{\sigma}}\sqrt{\sigma - j\omega\varepsilon\sin^2\Theta_1}}. \quad (6.7)$$

Using presented relations diagrams between coefficients T_p and T_n and the interrupted angles can be formed. As for the solving of the problem we have to know only the power of an interrupted wave, finding only absolute values $|T_p|$ and $|T_n|$ is necessary.

This is very important for the geometrically complicated constructions, which can be approximate with the system of the thin plates.

For example, in the case of the calculation using finite element method, parabolic emission antenna can be represented as a system of thin plates. The requested value for the calculation is the power of the interrupted wave. It can be obtained by determining the interrupted angle. For example, for observation radar, width of one impulse is 6.5μs, frequency over one impulse 2.9-3.1GHz and it's highest power 2.5MW. Because of the skin effect, using Pointing's vector, we have possibility for very efficient appliance of FEM method. On Fig. 6.2 deformation and stress fields are presented for one spherical and one cylindrical antenna system.

Described effects enable simple calculation in the case of the moving laser heat sources [7].

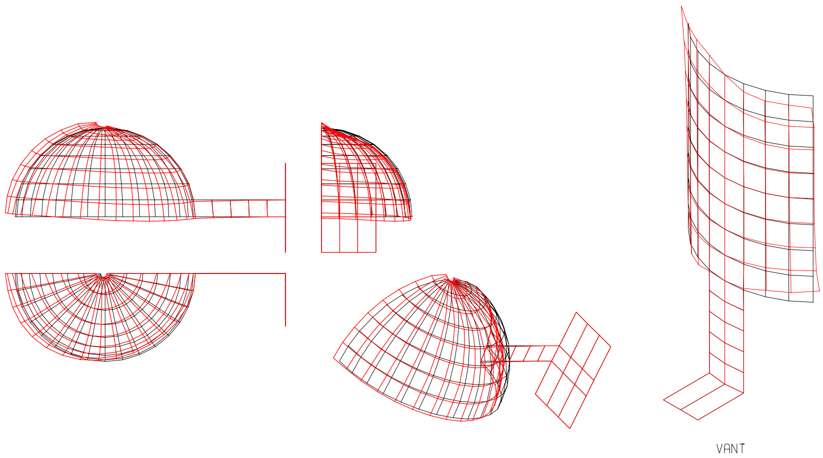


Fig 6.2a Deformation

For the problems with low frequency and conductivity, real temperature distribution across the plate thickness is of exponential type. For that nonlinear case of temperature loading it is very difficult to find vibrations in analytical form. So, finite element method (FEM) has to be involved in calculation. The stiffness matrix and the load matrix for plate element can be formed using analogy with the finite element of composite plate.

Corresponding values of the deflection obtained from the analytical solution and the numerical solution (real and reduced model) are the same, but there is a large dissipation between the stress distributions based on the real and the reduced FEM model. Reduced model can be used for dynamic modeling of the problem because of small computation time [6].

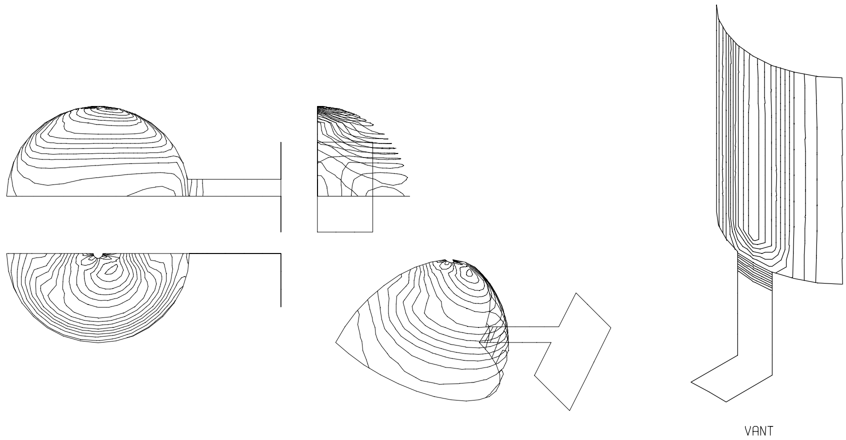


Fig 6.2b Stress field

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POLJA TEMPERATURE, NAPONA I DEFORMACIJE TANKE METALNE PLOČE IZAZVANA IMPULSNIM ELEKTOMAGNETSKIM TALASOM

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U radu su razmatrane poprečne vibracije tanke elastične metalne ploče izazvane delovanjem impulsnog elektromagnetskog talasa (na gornjoj površini ploče). Kako svaki elektromagnetski talas može da se prikaže kao zbir prostih ravanskih talasa, analitičko rešenje je dato samo za jedan ravanski talas. To podrazumeva da se sve karakteristike polja menjaju po eksponencijalnom

zakonu exp(jωt), pa se prikazuju u kompleksnom obliku. Kao proizvod vremenski promenljivog elektromagnetskog polja pojavljuju se kondukcione struje. Primenom kompleksnog računa dolazimo do raspodele snage toplotnih gubitaka u ploči, koja se može tretirati kao zapreminski izvor toplote. Impulsni talas može se matematički prikazati preko sume Hevisajdovih funkcija. Primenom tehnike integralnih transformacija možemo rešiti diferencijalne jednačine koje opisuju polje temperature, napona i pomeranja. U slučaju kada je dubina prodiranja mala u poređenju sa debljinom ploče problem može da se tretira kao termički udar. Energija koju elektromagnetski talas predaje ploči može da se odredi pomoću Pointingovog vektora. Ako želimo da rešimo geometrijski komplikovaniji problem moramo da razmotrimo i Snelijusove zakone prelamanja. Primenom Frenelovih koeficijenata refleksije i transmisije snaga upadnog talasa može se odrediti na osnovu upadnog ugla. Račun (na primer za radare i aero strukture) može se izvesti primenom metode konačnih elemenata