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NUMERICAL COMPUTATION OF UNSTEADY HEAT TRANSFER THROUGH POWDER GRAIN

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Abstract. *The interior ballistics problem of firing process in the small arms is considered. A mathematical model of a so called two-phase flow, which is created by gas - dynamic partial differential equations system, is used. Partial differential equation system has been solved numerically by finite difference method. Heat quantity which is crossing from powder gasses to powder grains, is determined, because this heat figures in flow energy equation. The equation of temperature distribution through deepness of powder grain is defined. As the example of numerical treatment of two-phase flow the numerical modeling of two-phase flow of powder grains and their combustion products in small arms barrel is shown. The operating conditions and shape of powder grains are variationed and their influence to the calculation results are numerically analyzed. Temperature distribution through deepness of powder grain is variationed and its influence to the calculation results is numerically analyzed. The whole procedure is included in a computer program, which results are verified through the comparasion with experimental results. The numerical results are analyzed and they compared with the experimental results and some conclusions for the future work on this problem are given.*

1. INTRODUCTION

Firing process in small-arms barrel is gas-dynamic process which is in volume between immovable barrel bottom and movable projectile characterized with the flow of two phases: solid - burning powders grains and gaseous - powders gases as product of combustion. Mathematical model is developed for any time of powder combustion. Firing process is considered from the moment when the powder gases pressure, as the product of powder combustion, becomes enough value for projectile envelope engraving in barrel grooves and so the projectile moving starts. It figures that all initial and

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boundary conditions in this moment are known. When the powder combustion is finished, two phase flow transforms to one phase flow, that is to the gases flow. Defined mathematical model is then transformed to the classical gas-dynamic model.

2. EQUATION SYSTEM IN LAGRANGE'S COORDINATES

The equation system is done in Euler's coordinates t (time) and x (any position in barrel from breechblock head to projectile bottom). Then it's transformed to equation system with Lagrange's co-ordinates t and s (powder grains and powders gases mixture at any position behind projectile). Starting suppositions and complete execution are given in [1], and here only finite equations are given:

1. Continuity equation for powder grains:

$$\frac{\partial \varepsilon}{\partial t} - \rho\varepsilon(u-u_b)\frac{\partial \varepsilon}{\partial s} - a_2(1-\varepsilon)\frac{\partial u_b}{\partial s} = b \tag{1}$$

2. Continuity equation for powder gases:

$$\frac{\partial \rho}{\partial t} + a_1\frac{\partial \rho}{\partial s} + a_2\frac{\rho(u-u_b)}{\varepsilon}\frac{\partial \varepsilon}{\partial s} + \rho a_2\frac{\partial u}{\partial s} + \rho a_2\frac{(1-\varepsilon)}{\varepsilon}\frac{\partial u_b}{\partial s} = \frac{b(\rho_b-\rho)}{\varepsilon} \tag{2}$$

3. Moving equation for powder gases:

$$\frac{\partial u}{\partial t} + a_1\frac{\partial u}{\partial s} + a_2\frac{(k-1)}{(1-\alpha\rho)}\frac{\partial e}{\partial s} + a_2\frac{e}{\rho}\frac{(k-1)}{(1-\alpha\rho)^2}\frac{\partial \rho}{\partial s} = f_1 \tag{3}$$

4. Moving equation for powder grains:

$$\frac{\partial u_b}{\partial t} - a_1\frac{\rho}{\rho_b}\frac{\varepsilon}{(1-\varepsilon)}\frac{\partial u_b}{\partial s} + a_2\frac{\rho}{\rho_b}\frac{(k-1)}{(1-\alpha\rho)}\frac{\partial e}{\partial s} + a_2\frac{e}{\rho_b}\frac{(k-1)}{(1-\alpha\rho)^2}\frac{\partial \rho}{\partial s} = \frac{f}{\rho_b(1-\varepsilon)} \tag{4}$$

5. Energy equation:

$$\frac{\partial e}{\partial t} + a_1\frac{\partial e}{\partial s} + a_3\frac{\partial \varepsilon}{\partial s} + \frac{pa_2}{\rho}\frac{\partial u}{\partial s} + a_4\frac{\partial u_b}{\partial s} = f_3 - \frac{pb}{\varepsilon\rho} \tag{5}$$

where are:

$$a_1 = \rho_b(1-\varepsilon)(u-u_b) \qquad a_2 = \rho\varepsilon + \rho_b(1-\varepsilon) \qquad a_3 = p(u-u_b) + \frac{pa_1}{\varepsilon\rho}$$

$$a_4 = a_2\frac{p}{\rho}\frac{(1-\varepsilon)}{\varepsilon} \qquad f_1 = \frac{1}{\rho\varepsilon}[b\rho_b(u_b-u) - f] \qquad f_2 = b\rho_b\left(e_b + \frac{p}{\rho_b} + \frac{u_b^2}{2}\right) - fu_b - \frac{bq}{u_z}$$

$$f_3 = \frac{1}{\varepsilon\rho}\left[f_2 - f_1\rho\varepsilon u - b\rho_b\left(e + \frac{u^2}{2}\right)\right] \quad \text{and} \quad b = (1-\varepsilon)\rho_b\frac{u_z S_z}{m_z}$$

6. From Lagrange's co-ordinates "s" is defined as:

$$\frac{\partial x}{\partial s} = \frac{1}{a_2} \tag{6}$$

where the flow variables are: u - the speed of powder gases; ub - the speed of powder grains; ρ - gas density; e - internal energy of gases; p - gas pressure; ε - porosity; x - place in barrel.

The coefficients a_1 to a_4, f, f_1, f_2 and f_3 are the functions of flow variables. The system (1) - (6) describes the flow in the real small-arms barrel. The equation system (1) to (6) joins all flow variables except the pressure of powder gases, which is defined by equation of powder gases state. This equation system is valid for powder grains combustion. When burning is finished next conditions are done:

$$\varepsilon = 0,\quad u_b = 0,\quad b = 0,\quad f_1 = f_2 = f_3 = 0,\quad a_1 = 0, a_2 = \rho \quad \text{and} \quad q = 0$$

The equation system (1) to (6) is transformed to the system which is valid until the moment when projectile exits the barrel. Additional equation system (one of them is the equation of state), initial and boundary conditions are appended [2].

Equation for heat quantity, which is crossing from powder gasses to powder grains is defined as:

$$q = \alpha_g (T - T_s) \tag{7}$$

where are:

α_g - coefficient of the heat transfer from powder gasses to powder grains surface
T_s - temperature on powder grain surface and
T - powder gases temperature.

Value of heat quantity figures in flow energy equation (5). One of the equations, which describes the thermodinamical state in small arms barrel, is equation of temperature distribution through deepness of powder grain, defined by K.K. Kuo [3], as a function of one parameter which is time varying.

$$T_b = T_{b0}\left(e^{\xi \frac{r_b - r_z}{r_b}} - \xi \frac{r_b - r_z}{r_b} \right) \tag{8}$$

respectively

$$T_s = T_{b0}(e^{\xi} - \xi) \tag{9}$$

where variable parameter is $\xi = \xi(t)$.

The numerical solutions of presented equation system (1) to (6) is based on the theory of finite-differences. Stability and convergence conditions are deduced. Program for personal computer is composed and used for influence analysis of powder grain shape and powder employment conditions to the calculation results.

3. UNSTEADY HEAT TRANSFER

Taking the elementary parallelopiped with dimension dx, dy, dz in optional body [5] and puting the energy heat balance, Furier's equation for unsteady heat transfer in isotropic solid body without heat spring is produced:

$$\frac{dT}{dt} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{10}$$

$$a = \frac{\lambda_b}{\rho_b c_p} \tag{11}$$

where are:

λ_b - coefficient of the heat transfer,
c_p - cpecific heat at constant pressure and
ρ_b - powder density.

The equation (10) solution gives the temperature disposition in space and time - temperature field $T = f(x,y,z,t)$. Equation (10) can be employed at any geometrical form of powder grain. In supposition that temperature has the same value on the all powder grain surface (what is realy in consideration of the grain dimension), it is sufficiently to opserve the change only in one dimension. For the smallest grain dimension (thickness $-2r_b$) is:

$$\frac{dT_b}{dt} = a\frac{\partial^2 T_b}{\partial r^2} \tag{12}$$

where are: T_b - powder grain temperature which is a function of t and r and
r - any powder grain depth.

Expresion (12) completely defines the temperature field of powder grain. Method of finite-differences gives the explicit numerical scheme for the equation (12) :

$$\frac{T_{i,k+1} - T_{i,k}}{\Delta t} = a\frac{T_{i-1,k} - 2T_{i,k} + T_{i+1.k}}{(\Delta r)^2} \tag{13}$$

From (13) is:

$$T_{i,k+1} = \frac{a\Delta t}{(\Delta r)^2}\left[(T_{i-1,k} + T_{i+1.k}) + \left(\frac{(\Delta r)^2}{a\Delta t} - 2\right)T_{i,k}\right] \tag{14}$$

Expression (14) gives a temperature distribution at axis r at any time if initial and boundary conditions are known. Initial conditions must contain a temperature distribution at axis r for $t = 0$, and boundary conditions must contain the temperature value on the body boundary $r = 0$.

Calculation scheme is stable [5] for condition:

$$(\Delta r)^2 / (a\Delta t) \geq 2 \tag{15}$$

The temperature value on the powder grain surface is necessary to define the heat transfer between gaseous and solid phase. Except the expression (8) and (9), the other expressions in dependence from adopted suppositions can be defined. Determination of surface grain temperature is conditioned by the fact that the deepness of heat transfer through grain is small because the time of flame spreading process is several milliseconds. As the deepness of the heat transfer is small in comparation with grain dimensions, heat transfer through grain can be considered one-dimensional. Temperature profile in grain has very steep gradient, and on the grain surface the conditions for kindling are realized before the termal wave comes to the powder grain center. That means (figure 1.) that T_s can be essentially changed and at the same time the middle grain temperature T_{bs} changes are very small.

Figure 1. shows powder grain with radius r_b and middle temperature T_{bs}, which is in gas with temperature T_g. Initial temperature in grain center is T_{b0}.

a) Temperature profile in powder grain can be presented as [6]:

$$\frac{T_b - T_{b0}}{T_s - T_{b0}} = \left[1 - \frac{r_z}{\delta}\right]^3 \tag{16}$$

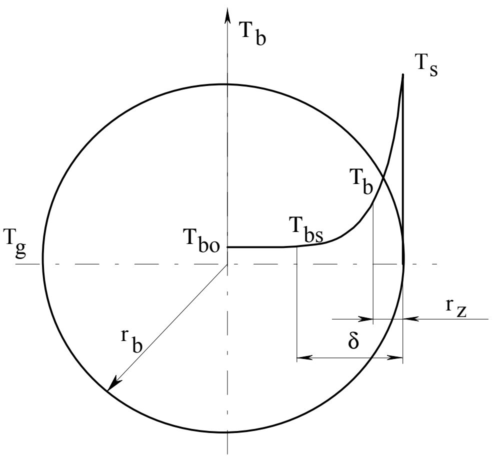

Fig. 1. Temperature profile in powder grain

b) According to the literature [7], temperature profile in solid phase is:

$$T_b - T_{b0} = (T_s - T_{b0})\exp\left(\frac{u_z r_z}{a}\right) \tag{17}$$

c) Russian literature [8] gives the temperature on the surface of powder burning layer:

$$T_s = T_{b0} + Q u_z / \lambda_b \tag{18}$$

where are: Q - heat quantity in overheats layer and u_z powder combustion speed.

Presented expressions are analyzed by the numerical calculation.

4. NUMERICAL ANALYSIS

Powder grains, which are used in small arms, artillery weapons and rocket's engine, are chosen for numerical analysis of unsteady heat transfer. Exploitation conditions, in which can be the powder grains are of different forms and dimensions, are analyzed. All calculation results are given in [2] and [10] and here - only the results for one arm for the reason of making comparative analysis of calculation and experimental results.

For small arms powder grains shape are used: plate with dimensions (0,85h0,85h0,09)mm, tubule with dimensions (0,55h0,13h1,15) and cylinder with dimensions (1,3h0,3)mm.

Figure 2 presents temperature change character in function of time and powder grain thickness for tubule shape, which is used in small arms. Time, which is necessary to realize powder heat homogeneity, is presented. These conditions represent the gas-dynamic flow in firing process when the temperature on the grain surface is 593K and in grain center 293K (ΔT = 300 K). Otherwise powder grains have the same function shape. Only the value orders are different and also function amplitude. Functions are presented in axsonometrical position, so that horizontal level of independent variables (base) is turned clockwise for an angle of 30^0, and the angle between the observation direction and the base is 25^0. The numbers on the vertical axis defines the drawing scale.

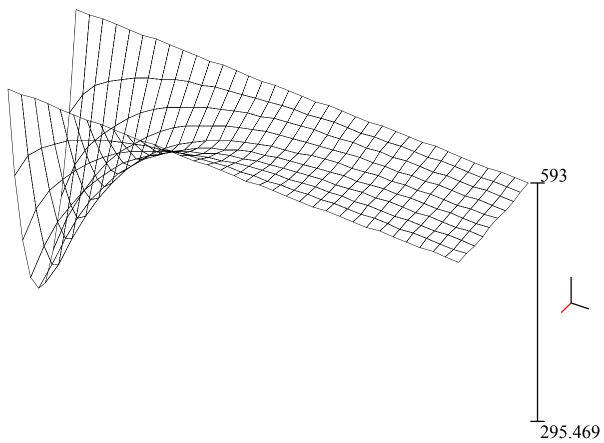

Fig. 2. Plate(ΔT = 300 K)

Time for powder heat homogeneity changes in dependence of applied temperature law and thickness of warmed zone. If the temperature profile in powder grain has steeper gradient and thinner warmed zone, the longer time for powder heat homogeneity is necessary.

Gas-dynamic calculation results for small arms show (table 2) that there is no difference in results obtained by the calculations with and without heat transfer if the time step is taken by convergence condition (small step). If the time step is taken by stability condition (big step) there are the differences. But to respect the theoretical suppositions about heat transfer through powder grain it is correct to make calculations with heat transfer.

Table 2. Gasdynamic calculation results in characteristic moments

Type powder in arms	Calculation with heat		Calculation without heat		End of combustion, ms
	p_m, bar	V_0, m/s	p_m, bar	V_0, m/s	
Plate	2150	535	2152	535,5	0,35
Tubule	3254	736,5	3257	736,8	0,65
Cylinder	3302	938	3304	938,5	0,9
Tubule	3144	731	3145	732	0,75

But for the solid propellant in rockets and big powder grain in artillery weapons [10] where tangential strain and deformation are defined in concordance with temperature profile, strain and deformations at propellants with bigger thickness and their biggest values are at the beginning of combusting. These strains are not ignorant.

Experimental research is realized to get real results for firing process in barrel and to compare experimental and calculations results. The curves of powder gases pressure in function of time are registered by experiments. Powder gases pressure is registered in cartridge case. Experiment is repeated at least 30 times.

Mathematical model and computer program are corrected with conditions for experimental barrel and the calculations are executed with such conditions, because firing conditions in experimental barrel are different from conditions in fighting barrel.

Experimental average value for maximum pressure (294.77 MPa*)* practically is identical to the model (295.1 MPa) by 20 elements for variable *s,* and all experimental curves are compatible with the calculations so that the correctness of given theory is validated.

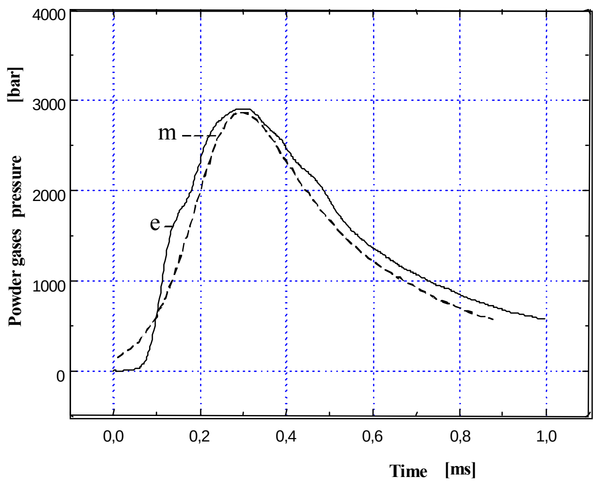

Fig. 3. Diagrams $p(t)$ model and average experimental value

5. CONCLUSION

This paper gives theoretic analysis of firing process in small-arms barrel based on the numerical computer modeling. The equation for distribution of temperature through deepness of powder grain is defined. By the analysis of gasdynamic calculation results for small arms it came to the knowledge that powder grains have small mass, quickly burnaway and heat transfer is small. For powder grains (rocket propellants) and great temperature differences, heat quantity has bigger value and it may not be ignored by the calculation.

Powder gases pressure in the small arms barrel in function of time is presented by comparison analysis. A comparison of experimental and calculation results for powder gases pressure shows their good approaching during the whole firing process time and so the mathematical model is validated.

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NUMERIČKI PRORAČUN NERAVNOMERNOG PRENOSA TOPLOTE KROZ PRAH BARUTA (BARUTNI PRAH)

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Razmatra se unutrašnji balistički problem procesa opaljivanja u oružju malih dimenzija. Koristi se matematički model takozvanog dvofaznog strujanja koje stvara gas, predstavljen sistemom dinamičkih parcijalnih diferencijalnih jednačina. Sistem parcijalnih diferencijalnih jednačina rešen je numerički primenom metoda konačnih razlika. Određena je količina toplote koja prelazi iz barutnog gasa u barutni prah, zbog toga što ova toplota figuriše u jednačini energetskog bilansa (toka energije). Jednačina raspodele temperature kroz barutni prah je definisana. Kao primer numeričkog rešavanja dvofaznog strujanja, prikazano je numeričko modeliranje dvofaznog strujanja barutnog praha i produkta njegovog sagorevanja u oružju malih dimenzija. Menjani su radni uslovi i oblik zrna baruta i numerički je analiziran njihov uticaj na rezultate proračuna. Menjana je i raspodela temperature kroz barutni prah i analiziran njen uticaj na rezultate proračuna. Čitav postupak je obuhvaćen kompjuterskim programom, čiji su rezultati verifikovani poređenjem sa eksperimentalnim rezultatima. Numerički rezultati su analizirani i upoređeni sa eksperimentalnim rezultatima, a dati su i neki zaključci za budući rad na ovom problemu.