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DYNAMIC MODEL OF GAS PRESSURE REGULATOR

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Abstract. *The procedure of forming of non-linear dynamic model of gas pressure regulator is presented. The non-linear dynamic model is necessary in number of cases, as in investigating the possibility of the occurence of self-exiting oscillations and conditions under which they are stable, for solution of tasks of optimal projecting of regulators, etc. Based on the assumption that dry friction is negligible, that the motion of the diaphragm and the valve disk is not constrained, and that the flow through restriction is laminar, a linear model sufficient for evaluation of stability and transient response analysis is obtained.*

INTRODUCTION

The determination of constructive parameters, which secure demanded accuracy in stationary regime represents the basic task of gas pressure regulator design. Concerning dynamic analyses of the regulator is most often limited to estimation of stability and evaluation of quality of regulation based on linear model. However, linear model is insufficient in the string of important cases: when is necessary to execute accurate analyses of dynamic behavioring, self-exiting oscillations and resonant appearances, with tasks of optimization and similar cases.

FORMING OF A MODEL

With forming of dynamic model of one stage regulator (Fig.1) working chamber 5 is considered as an object of regulation and control valve 2 as proportional regulator of direct action. Regulated parameter is gas pressure p in working chamber. To get non linear model it is essential to introduce following simplification: external disturbances are minor and momentary, the influence of dry friction, aerodynamic resistance in channels and variation of effective surface of the diaphragm 9 are neglected; processes of gas streaming are

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adiabatic and processes in working chamber are isotermic. It is assumed that there is no bringing up of the heath from the outside and that the gas temperature in working chamber is constant. The volume of working chamber scopes its own volume and outlet "gap".

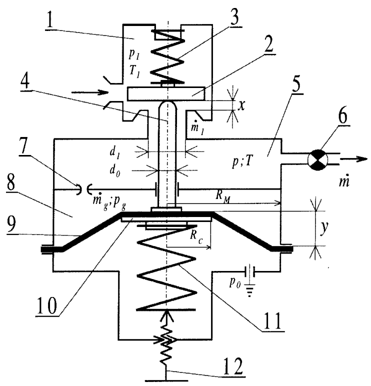

Fig. 1. One stage chamber gas pressure regulator.

1- chamber of high pressure; 2- control valve; 3- valve spring ; 4- valve pusher; 5- working chamber; 6-outlet cock (shut off valve); 7-restriction (damper); 8- damping chamber; 9- diaphragm; 10- disk (stiff centre of a diapraghm); 11- working spring; 12-the adjusting crew

The change of pressure in working chamber is given by equation:

$$\dot{p} = \frac{dp}{dt} = \frac{\chi RT}{V}\left(\dot{m}_1 - \dot{m} + \dot{m}_g\right) \tag{1}$$

where: V - the volume of working chamber;

T - gas temperature in working chamber;

$\dot{m}_1, \dot{m}, \dot{m}_g$ mass flows at the inlet, discharge and through restriction 7;

χ, R - exponent of adiabatic and gas constant for certain gas.

The volume of the damping chamber is changed by law:

$$Vg(t) = V_{g0} - A_M\, y(t)$$

where V_{g0}; A_M; - start volume of damping chamber and effective surface of the diaphragm, $y(t)$ - the bend of the centre of the diaphragm.

The change of pressure in damping chamber is:

$$\dot{p} = \frac{dp_g}{dt} = -\frac{\chi RT}{V_g}\left(T\,\dot{m}_g + \frac{p_g A_M\, \dot{y}}{R}\right) \tag{2}$$

Differential equation of the motion of the diaphragm is:

$$M\,\ddot{y} = \sum_i \Delta F_i \tag{3}$$

where: M - the mass of movable elements is reduced to the mass of the diaphragm;

$\sum_i \Delta F_i$ - the sum of forces increment that operates on diaphragm and control valve, in relation to their values in position "closed".

Reduced mass is:

$$M = \frac{M_2}{3} + M_0 + M_m + \left[\frac{M_1}{3} + M_V\right]^+$$

where M_2, M_0, M_m, M_1, M_V - individual mass of working spring, valve pusher, diaphragm, valve spring and valve.

Operator $[\ ]^+$ means that values in brackets are equal to zero at $y \le 0$.

At the moment when valve touches the valve sitting, and the closure of control valve happens, the separation of the pusher from the valve happens and the diaphragm has the possibility to continue with the motion. Then the reduced mass of the diaphragm is reduced for the total of masses of valve spring and valve ($M_1/3 + M_V$) and in equation (1) is the flow $\dot{m}_1 = 0$

The sum of increment of the forces in equation (3) is:

$$\sum_i \Delta F_i = \Delta F_1 + \Delta F_A + \Delta F_M(\Delta p_g) + \Delta F_M(y) + \Delta F_2 + F_g(\dot{y}) \tag{3a}$$

where certain components are calculated by following terms:

- the force increment of valve spring $\Delta F_1 = -c_1 x$, $x = [y]^+$;
- the increment of aerodynamic force of the valve

$$\Delta F_A = \Delta p A_s + c_A x, \quad \Delta p = p - p^3, A_s = 0.25 d_s^2 \pi, \quad c_A = 2d_s(p_1 - 0.63p)[2]$$

- the force increment in the diaphragm under the influence of the pressure in damping chamber

$$\Delta F_M(\Delta p_g) = \Delta p_g A_M, \quad \Delta p_g = p_g - p^3;$$

- the force increment in the diaphragm caused by deflection y

$$\Delta F_M(y) = -c_M y;$$

- the force increment in working (operating) spring $\Delta F_2 = -c_2 y$

Here it should be mentioned that equation (3a) does not describe the motion of the valve after the impact with the valve sitting.

At calculation of damping force $F_g(\dot{y})$ assumption on laminar gas streaming through restriction (7) is often adopted. The gas streaming through mentioned restriction is in that case calculated by Poiselli′s term (2)

$$\dot{m}_L = \frac{\pi d_g^4 \bar{p}(p_g - p)}{128 \eta l_g RT} \tag{4}$$

where d_g, l_g - diameter and length of the restriction;

$\bar{p}$ - average value of gas pressure by the restriction length;

n - coefficient of the dynamic gas viscosity.

To achieve force dependence of laminar damping $F_L(\dot{y})$ out of constructive restriction parameters it is assumed that the motion of the diaphragm is done by constant speed ($\dot{y} = const.$) and that $(p_g - p)\dot{y} \succ 0$. Let's elaborate following equations of the diaphragm motions:

$$cy = -A_M(p - p_a) + \sum_i \Delta F_i;$$

$$F_L(\dot{y}) + cy = -A_M(p_g - p_a) + \sum_i \Delta F_i.$$

where p_a - atmosphere pressure (the pressure of the environment), $c = c_1 + c_2$. The first equation is valid under the assumption that there is no damping. Eliminating the first equation of the second we get the force of laminar damping

$$F_L(\dot{y}) = -A_M(p_g - p).$$

If the approximate expression for flow is used

$$\dot{m}_L \approx \gamma A_M \dot{y} = \frac{\bar{p} A_M \dot{y}}{RT} \tag{4a}$$

and if expressions get equal (4) and (4a), we come to expression for pressure drop $\Delta p_g = (p_g - p)$. By entering Δp_g into previous expression for $F_L(\dot{y})$ we achieve the force of laminar damping in the form of

$$F_L(\dot{y}) = -\frac{128\eta l_g A_M^2 \dot{y}}{\pi d_g^4} \tag{5}$$

In this way, the damping force is linear function of the speed of the motion of the centre of the diaphragm:

$$F_L(\dot{y}) = -\beta \dot{y} \tag{5a}$$

where coefficient of the damping is:

$$\beta = -\frac{128\eta A_M^2 l_g}{\pi d_g^4} \tag{6}$$

However, hypothesis on laminar flow is not always justified because the most frequent evaluation of the regime of the flow is done simplified - just according to the ratio l_g/d_g. Beside that, the sole fact goes out of the sight that in dynamic regime exists turbulent regime as well as transitional regime from laminar to turbulent (what can be cause of the regulator oscillations). The pressure drop Δp_{gr} in front and behind the position of the restriction, by which comes the transition from laminar to turbulent flow is determined, for the restriction with round cross section, by the term:

$$\Delta p_{gr} = \frac{32 R_{e.gr} g R T \eta^2 l_g}{\bar{p} d_g^3} \tag{7}$$

where $R_{e.gr}$ - maximum value of Reynold's number. The term (7) shows that at constant gas temperature T the pressure drop Δp_{gr} is the function of the relation $l_g/(\overline{p}_g d_g^3)$. If the interval of change R_{er} which characterizes transitional area from the regime of laminar flow (R_{eL}) to the regime of developed turbulence ($R_{e.\mathrm{T}}$), is narrow, approximately it can be assumed that mentioned transition is done in jumps on some average value R_e (e.g. $R_{e.gr}$ = 2300) and the flow trough restriction will be:

$$\dot{m}_g = \dot{m}_L \quad at \quad |\Delta p| \le \Delta p_{gr}$$

$$\dot{m}_g = \dot{m}_\mathrm{T} \quad at \quad |\Delta p| \succ \Delta p_{gr},$$

where $\dot{m}_L, \dot{m}_T$ = mass gas flow through damping at laminar and turbulent regime. The force of viscose damping is then determined by term:

$$F_g = F_L \quad at \quad |\Delta p| \le \Delta p_{gr};$$

$$F_g = F_T \quad at \quad |\Delta p| \succ \Delta p_g,$$

Here F_T - the force of turbulent damping which is calculated by term:

$$F_\mathrm{T} = -\frac{8A_M(V_g \dot{p}_g - p_g A_M \dot{y})^2 \, sign\, \dot{y}}{\mu_g gRT\pi^2 d_g^4 \overline{p}} \tag{8}$$

where μ_g - restriction flow coefficient. Term (8) is achieved by analogy as well as the expression (5). From achieved term it is seen that the force of viscose friction that is made by pneumatic restriction is non-linear function of drop pressure Δp_g on restriction and the speed of the motion of the diaphragm $\dot{y}$.

The solution for the real value of damping coefficient β of the regulator as a whole, is connected with certain difficulties. There is possibility that coefficient β is more accurately determined with the help of osscillogram of transitional processes of linear regulator model. If regulator as dynamic system is described by linear differential equations of the third order, then according to oscillogram of transitional process of the regulator by pressure, can be found the roots of its characteristical equations and through them damping coefficient β.

The important influence on operational accuracy and the stability of the regulator can have dry friction. By the rule, when taking into the consideration the function of the gas regulators it is assumed that dry friction is negligible small [2]. By examination of the influence of non linear dry friction on the appearance of self exiting oscillations it is noted that regulator at existence of dry friction can be in the state of stable balance or in the regime of self-exiting oscillation depending on start disturbances [2]. So, the complete non-linear model of gas pressure regulator has the form:

$$\left.\begin{aligned}
&\dot{p} = \frac{\chi RT}{V}\cdot\left(\dot{m}_1 - \dot{m} + \dot{m}_g\right)\\
&\dot{p}_g = -\frac{\chi RT}{V_g}\left(\dot{m}_g T + \frac{p_g A_M \dot{y}}{R}\right)\\
&M\ddot{y} = -(c_1 + c_A)x - \Delta p A_s - (c_M + c_2)y + A_M \Delta p_g + F_g(\dot{y})
\end{aligned}\right\} \quad (9)$$

$$x = [y]^+; \Delta p = p - p^3; \Delta p_g = p_g - p^3;$$
$$M = M(y)$$

Initial conditions at the opening of regulators are:

$$p(0) = p^3; \quad p_g(0) = p^3;$$
$$y(0) = 0; \quad \dot{y}(0) = 0$$

At describing of the process of the closure of the regulators it is essential to take that $\dot{M} = 0$ and to assign following initial conditions:

$$p(0) = p^0; \quad p_g(0) = p^0$$
$$y(0) = y^0; \quad \dot{y}(0) = 0.$$

The system of equations (9) is valid if the motion of the diaphragm is not limited by supporters whose coordinates are y^-, y^+, that is for the case when $y^- < y < y^+$. At the moment when the diaphragm touches to the supporters (without impact) the system of equations (9) must be fulfilled with the following condition of diaphragm immobility:

at $y = y^+, \dot{y} = 0;\ \ddot{y} = 0,$ if $A_M(p_g - p_g{}^+) - A_1(p - p^+) \le 0;$

at $y = y^-, \dot{y} = 0;\ \ddot{y} = 0,$ if $p_g \ge p_g^-.$

where $p_g{}^+$, $p_g{}^-$ are gas pressures in damping chamber at the moment diaphragm touches to the upper and lower supporter.

Linearizing the system of dynamic equations (9) is possible if it is assumed that elastic characteristic of the diaphragm is close to linear; the motion of the diaphragm is not limited by supporters and it does not come to the impact of the valve sitting; gas flow through restriction is laminar and dry friction is negligible.

Then the linearized system of dynamic equations (9) has the form:

$$\begin{aligned}
&\Delta \dot{p} + a_{11}\Delta p + a_{12}\Delta p_g + a_{13}\Delta y = F(y) \\
&a_{21}\Delta p + \Delta \dot{p}_g + a_{22}\Delta p_g + a_{23}\Delta \dot{y} + a_{24}\Delta y = 0 \\
&a_{31}\Delta p_g + M\Delta \ddot{y} + \beta\Delta \dot{y} + a_{32}\Delta y = 0 \\
&F(t) = b_1\Delta p_1 + b_2\Delta T_1 + b_3\Delta S
\end{aligned} \tag{9a}$$

System coefficients are determined by relations:

$$a_{11} = \frac{\chi RT}{V}\left(\dot{m} - \dot{m}_g\right)_p,\ a_{12} = \frac{\chi RT\left(\dot{m}\right)_{p_g}}{V},\ a_{13} = -\frac{\chi RT(\dot{m}_1)_y}{V}$$

$$a_{21} = \frac{\chi RT\left(\dot{m}_g\right)_p}{V}, a_{22} = \frac{\chi RT\left[T(\dot{m}_g)p_g - \frac{A_M \dot{y}}{R}\right]}{V_g},$$

$$a_{23} = -\frac{\chi p_g A_M}{V_g}, a_{24} = \chi R(\frac{T\dot{m}_g - p_g A_M \dot{y}}{R})\left(\frac{1}{V_g}\right)_y$$

$$a_{31} = A_{M,}\ a_{32} = c_\Sigma = c_1 + c_2 + c_M$$

$$b_1 = \frac{\chi R(\dot{m}_1)_{p_1}}{V},\ b_2 = \chi R(\dot{m}_1)_{T_1},\ b_3 = -\frac{\chi R(\dot{m})_S}{V}$$

where index of variable by the brackets determine partial derivative of expression in brackets by that variable. Partial derivatives are calculated for the values of variables in stationary regime of the flow. The size S represents the surface of the opening of the outlet cock, and c_Σ total stiffness of the springs and diaphragm.

CONCLUSION

The formation of mathematical model of dynamic of gas pressure regulator represents step further in perception of the whole problem connected to optimal synthesis of devices like this one. This model enables analysis of self-exiting oscillations of systems (stable oscillations of certain amplitude and frequency without the presence of outside disturbances) and using that we should get answers to many questions: by what values of constructive regulator parameters self-exiting oscillating regime is possible; how each of the parameters influences self-exiting oscillations; how can self-exiting oscillation parameters be corrected or how they can be annulled, if necessary. Non-linear model, with certain simplifying, is used at modeling impulse regime, what enables optimization of regulators by criteria of quality of impulse regime.

REFERENCES

1. *Беляев Н.М., Уваров Е.И., Степанчук Ю.М.*: Пневмогидравлические системы-расчёт и sпроектирование, Москва,1988.
2. *Ермилов В.А., Нестеренко Ю.В., Николаев В.Г.*: Газовые редуктоы, Ленинград,1981.
3. *Vujić D., Radojković S.*: Matematički model statičke karakteristike regulatora pritiska, Zbornik radova HIPNEF 98, Beograd, 1998.

DINAMIČKI MODEL REGULATORA PRITISKA GASA

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Izložen je postupak formiranja nelinearnog dinamičkog modela regulatora pritiska gasa. Nelinearni model je neophodan u slučajevima kao što su ispitivanje mogućnosti pojave samooscilacija i uslova pod kojima su one stabilne, za rešavanje zadataka optimalnog projektovanja regulatora itd. Na osnovu pretpostavke da je suvo trenje zanemarljivo, da kretanje membrane i zatvarača ventila nije ograničeno, a strujanje kroz prigušnicu laminarno, dobijen je linearni model dovoljan za proveru stabilnosti ili za ocenu kvaliteta prelaznog procesa.