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THE SHEAR STRESS DISTRIBUTION OF UNSTEADY INCOMPRESSIBLE BOUNDARY LAYER IN DIFFUSER REGION ON POROUS CONTOUR

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Abstract. *Through the porous contour in pependicular direction, the fluid of the same properties as incompressible fluid in basic flow, has been injected or ejected with velocity which is a function of the contour longitudinal coordinate and time. The corresponding equations of unsteady boundary layer, by introducing the appropriate variable transformations, momentum and energy equations and two sets of similarity parameters are transformed into generalized form. Generalized solutions obtained by numerical integration in the three once localized approximation, are used to calculate the laminar-turbulent transition based on zero skin friction criteria, and velocity distribution of unsteady boundary layer in diffuser region on wing aerofoil when center velocity changes with time as a degree function and when potential external velocity is measured in free flight. In diffuser region, as well as for both the accelerating and decelerating flows, the ejection of fluid increases the friction and postpones the boundary layer separation, and vice versa the injection of fluid reduces the friction and favours the separation of flow.*

MATHEMATICAL MODEL AND GENERALIZED SIMILARITY EQUATION

Generalized similarity method [1,2,3], is exposed to the problem of unsteady incompressible plane boundary layer on the porous surface [4,5,6,7,9], when the fluid of the same properties as fluid in basic flow has been injected or ejected through the surface in perpendicular direction with velocity v_w. The mathematical model of the noticed problem is described by the following equation:

$$\Psi_{ty} + \Psi_y \Psi_{xy} + (v_w - \Psi_x)\Psi_{yy} = U_t + UU_x + \upsilon\Psi_{yyy} \tag{1}$$

with boundary and initial conditions:

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$$y=0:\Psi=\Psi_y=0;\quad y\to\infty:\Psi_y\to U(x,t);\quad t=t_0:\Psi_y=u_1(x,y);\quad x=x_0:\Psi_y=u_0(t,y), \tag{2}$$

where the following notations are used: $\Psi(x,y,t)$ - stream function, $U(x,t)$ - free-stream velocity, υ - kinematic viscosity, $u_1(x,y)$ - thestreamwise velocity distribution in boundary layer in some determined point of time $t = t_0$, $u_0(t,y)$ - the streamwise velocity distribution in boundary layer in cross-section $x = x_0$, x - streamwise coordinate, y - crosswise coordinate, t - time. Introducing new variables in the form [3,6,7,8,9]:

$$x=x,\ t=t,\ \eta=yU^{b_0/2}(a_0\upsilon\int_0^x U^{b_0-1}dx)^{-1/2},\ \Phi(x,\eta,t)=\Psi U^{b_0/2-1}(a_0\upsilon\int_0^x U^{b_0-1}dx)^{-1/2} \tag{3}$$

where $a_0 = 0{,}4408$, $b_0 = 5.714$ [3], we transform the equation (1) to the new form. Afterwards, we introduce the group of parameters:

$$\begin{aligned} f_{k,n} &= U^{k-1}U^{(k+n)}_{x^{(k)}t^{(n)}}z^{**k+n} && (k,n=0,1,2,...;k\vee n\neq 0)\\ \lambda_{k,n} &= -\upsilon^{-1/2}U^k v^{(k+n)}_{w_{x^{(k)}t^{(n)}}}z^{k+n+1/2} && (k,n=0,1,2,...)\end{aligned} \tag{4}$$

as new independent variables, where:

$$z^{**}=\delta^{**2}/\upsilon,\ \delta^{**}=(a_0\upsilon U^{-b_0}\int_0^x U^{b_0-1}dx)^{1/2}B,\ B=\int_0^\infty \Phi_\eta(1-\Phi_\eta)d\eta\,. \tag{5}$$

Now, previously transformed equation (1) is transformed to the new form:

$$\begin{aligned} &B^2\Phi_{\eta\eta\eta}+0.5[a_0B^2+(2-b_0)f_{1,0}]\Phi\Phi_{\eta\eta}+f_{1,0}(1-\Phi_\eta^2)+f_{0,1}(1-\Phi_\eta)+\\ &(0.5\eta T^{**}+B\lambda_{0,0})\Phi\eta\eta=\eta B^{-1}(\sum_{\substack{k,n=0\\k\vee n\neq 0}}^\infty C_{k,n}B_{f_{k,n}}+\sum_{k,n=0}^\infty R_{k,n}B_{\lambda_{k,n}})\Phi_{\eta\eta}+\\ &\{\sum_{\substack{k,n=0\\k\vee n\neq 0}}^\infty [C_{k,n}B_{\eta f_{k,n}}+A_{k,n}(\Phi_\eta\Phi_{\eta f_{k,n}}-\Phi_{f_{k,n}}\Phi_{\eta\eta})]+\\ &\sum_{k,n=0}^\infty [R_{k,n}\Phi_{\eta\lambda_{k,n}}+E_{k,n}(\Phi_\eta\Phi_{\eta\lambda_{k,n}}-\Phi_{\lambda_{k,n}}\Phi_{\eta\eta})]\},\end{aligned} \tag{6}$$

with corresponding boundary conditions:

$$\begin{aligned}&\eta=0:\ \Phi=\Phi_\eta;\quad \eta\to\infty:\ \Phi_\eta\to 1;\quad f_{k,n}=\lambda_{k,n}=0\\ &(k=0,1,...;k\vee n\neq 0):\ \Phi=\Phi_0(\eta)\end{aligned} \tag{7}$$

where $\Phi_0(\eta)$ is Blausius's solution for the problem of flat plate. In the equation (6) the following notations have been used:

$$\begin{aligned}&A_{k,n}=(k-1)f_{1,0}f_{k,n}+f_{k+1,n}+(k+n)f_{k,n}\overset{**}{F};\quad E_{k,n}=kf_{1,0}\lambda_{k,n}+\lambda_{k+1,n}+(k+n+0.5)\lambda_{k,n}\overset{**}{F};\\ &C_{k,n}=(k-1)f_{0,1}f_{k,n}+f_{k,n+1}+(k+n)f_{k,n}\overset{**}{T};\quad R_{k,n}=kf_{0,1}\lambda_{k,n}+\lambda_{k,n+1}+(k+n+0.5)\lambda_{k,n}\overset{**}{T};\\ &\overset{**}{T}=\overset{**}{z}_t;\quad \overset{**}{F}=U\overset{**}{z}_x\,.\end{aligned} \tag{8}$$

In order to take the equation (6) universal, the multipliers $\overset{**}{F}$ and $\overset{**}{T}$ have to be expressed by means of quantities which are explicit functions only of parameters (4). In the determination of this functions, one can use the momentum and energy equations of the considered problem:

$$(U\delta^*)_t + (U^2\delta^{**})_x + UU_x\delta^* - Uv_w - \tau_w / \rho = 0;$$
$$(U^2\delta^{**})_t + U^3\delta_{1\ x}^{**} + U^2(\delta_t^* + 3\delta_1^{**}U_x - 2\upsilon e) = 0 \tag{9}$$

where:

$$\delta^* = L^{1/2}\int_0^\infty (1-\Phi_\eta)d\eta;\;\; \tau_w = \rho\upsilon U^{b_0/2+1}L^{-1/2}(\Phi_{\eta\eta})_{\eta=0};$$
$$\delta_1^{**} = L^{1/2}\int_0^\infty \Phi_\eta(1-\Phi_\eta^2)d\eta;\;\; e = L^{-1/2}\int_0^\infty \Phi_{\eta\eta}^2 d\eta;\;\; L = a_0\upsilon U^{-b_0}\int_0^x U^{b_0-1}dx. \tag{10}$$

After certain transformations, the expressions for F^{**} and T^{**} have been obtained as universal, i.e. they do not depend on outer flow charasteristics. In the equation (6) the velocity at outer border of boundary layer and its derivatives, as well as the ejection or injection fluid velocity are not involved in explicit form, thus this equation can be called the generalized i.e. universal equation. The universal boundary conditions have the form as (7).

APPROXIMATIVE UNIVERSAL EQUATION

The numerical integration of the equation (6), with the corresponding universal boundary conditions (7), can be performed "once and forever" only for its approximative form. It means, that the solution of universal equation in practice needs limitation of the number of the independent variables. It leads to the necesity of application of the "segment" method, in which all variables from someone have to be equal to zero. In such a way, the approximative universal equation is obtained. Having the above procedure in mind, the parameters $f_{1,0}, f_{0,1}$, $\lambda_{0,0}$ will be remained, while all others will be let to be equal to zero. Also, the derivative with respect to the first porous parameter $\lambda_{0,0}$ will be considered as equal to zero. The equation (6) in these approximation i.e. in three parametric "once localized" enviroment, has the form:

$$\begin{aligned} &B^2\Phi_{\eta\eta\eta} + 0.5[a_0B^2 + (2-b_0)f_{1,0}]\Phi\Phi_{\eta\eta} + f_{1,0}(1-\Phi_\eta^2) + f_{0,1}(l\Phi) + \\ &(0.5\eta T^{**} + B\lambda_{0,0})\Phi_{\eta\eta} = \eta B^{-1}[T^{**}(f_{1,0}B_{f_{1,0}} + f_{0,1}B_{f_{0,1}}) - f_{0,1}^2 B_{f_{0,1}}]\Phi_{\eta\eta} + \\ &[T^{**}(f_{1,0}B_{\eta f_{1,0}} + f_{0,1}\Phi_{\eta f_{0,1}}) - f_{0,1}^2\Phi_{\eta f_{0,1}} + f_{1,0}F^{**}(\Phi_\eta\Phi_{\eta f_{1,0}} - \Phi_{f_{1,0}}\Phi_{\eta\eta}) + \\ &f_{0,1}(F^{**} - f_{1,0})(\Phi_\eta\Phi_{\eta f_{0,1}} - \Phi_{f_{0,1}}\Phi_{\eta\eta})], \end{aligned} \tag{11}$$

and the corresponding boundary conditions (7) are reduced to the following:

$$\eta = 0:\ \Phi = \Phi_\eta = 0;\;\; \eta\to\infty:\ \Phi_\eta \to 1;\;\; f_{1,0} = f_{0,1} = \lambda_{0,0} = 0:\ \Phi = \Phi_0(\eta), \tag{12}$$

where the functions T^{**} and F^{**}, after the same approximation have the following forms:

$$T^{**} = \{2[2(f_{1,0}H_1^{**}{}_{f_{1,0}} + f_{0,1}H_1^{**}{}_{f_{0,1}}) + H_1^{**}][\zeta - 2f_{1,0} - H^{**}(f_{1,0} + f_{0,1}) - \lambda_{0,0} + f_{0,1}^2 H^{**}{}_{f_{0,1}}] +$$
$$+ 2[f_{0,1}(2 - f_{1,0}H_1^{**}{}_{f_{0,1}} - f_{0,1}H_1^{**}{}_{f_{0,1}}] + 6H_1^{**}{}_{f_{1,0}} - 4\alpha\}$$
$$\{[H^{**} + 2(f_{1,0}H^{**}_{f_{1,0}} - f_{0,1}H^{**}_{f_{0,1}})][2(f_{1,0}H_1^{**}{}_{f_{1,0}} + f_{0,1}H_1^{**}{}_{f_{0,1}}) + H_1^{**} - 1] - 1\}^{-1}; \tag{13}$$
$$F^{**} = 2\{\zeta - 2f_{1,0} - H^{**}(f_{1,0} + f_{0,1} + 0.5T^{**}) - \lambda_{0,0} + f_{0,1}H^{**}_{f_{0,1}}(f_{0,1} - T^{**}) - T^{**}f_{1,0}H^{**}_{f_{1,0}}\},$$

where is:

$$H^{**} = B^{-1}\int_0^\infty (1 - \Phi_\eta)d\eta = A/B; \quad H_1^{**} = B^{-1}\int_0^\infty \Phi_\eta(1 - \Phi_\eta^2)d\eta; \quad \zeta = B(\Phi_{\eta\eta})_{\eta=0}; \quad \alpha = B\int_0^\infty \Phi_{\eta\eta}^2 d\eta .$$

The numerical integration of the equation (11) with boundary conditions (12) has been performed by means of the difference schemes and by using Tridiagonal Algorithm method with iterations.The obtained results can be used in the withdrawing of general conclusions of boundary layer development and in calculation of particular problems.

UNSTEADY BOUNDARY LAYER ON POROUS WING AEROFOIL

Universal solutions of the equation (11) $\Phi''(0)$, A, B are used to calculate the charasteristic properties of unsteady boundary layer on wing aerofoil whose center velocity changes with time as a degree function. Substituting nondimensional coordinates: $\tilde{x} = x/l$ and $\tilde{t} = \tilde{U}t/l$, where is l - chord and U_∞ - endlessly velocity, nondimensional potentional external velocity becomes:

$$\tilde{U}(\tilde{x}, \tilde{t}) = \tilde{U}_1(\tilde{t})\tilde{U}_2(\tilde{x}) = (\tilde{B} + \tilde{A}\tilde{t}^{\,n})\tilde{U}_2(\tilde{x}) \tag{14}$$

with constant values for $\tilde{A}, \tilde{B}, n$. The Figure shows potential external velocity $\tilde{U}_2(\tilde{x}) = U/U_\infty$on wing aerofoil measured by J.Stueper in free flight [10], where lift coefficient is $c_1 = 0.4$, Reynolds number $R = 4{\cdot}10^6$ and chord $l = 1800$ mm. Substituting (14) in (4),(5) yields the following relations for the universal functions:

$$f_{1,0}/B^2 = a_0\tilde{U}^{-b_0}\tilde{U}_x Q; \quad f_{0,1}/B^2 = a_0\tilde{U}^{-(b_0-1)}\tilde{U}_t Q;$$
$$\lambda_{0,0}/B = -v_w(a_0 Q/\upsilon\tilde{U}^{b_0})^{1/2}; \quad Q = \int_0^{\tilde{x}} \tilde{U}^{b_0-1}d\tilde{x}. \tag{15}$$

Using (3) and (13) the expression for the dimensionless skin friction $\tilde{\tau}_w$ has the form

$$\tilde{\tau}_w = 2\tau_w R^{1/2}/(\rho U_\infty^2) = 2\tilde{U}^{(b\ /2)+1}(a_0\int_0^{\tilde{x}}\tilde{U}^{b_0-1}d\tilde{x})^{-1/2}\Phi''(0) \ . \tag{16}$$

With obtained universal quantities A and B one can determine on a same previous way the expressions for dimensionless displacement thickness $\tilde{\delta}^*$ and dimensionless momentum thickness $\tilde{\delta}^{**}$. Now, we select a given set of the constants $\tilde{A}, \tilde{B}, n$ and for particular point on contour $\tilde{x}_0$ and time $\tilde{t}_0$ searching by (15) the obtained universal functions $(f_{1,0}/B^2)_0$, $(f_{0,1}/B^2)_0$, $(\lambda_{0,0}/B)_0$, concerning $[\Phi''(0)]_0$, A_0, B_0 for different values of porous parameter $\lambda_{0,0}$. Afterwards, using (16) one can determine $\tilde{\tau}_w$ distribution on contour. Preliminary calculations of expressions (15) and (16) have been made for: $\widetilde{A} = \pm 1; \widetilde{B} = 1$; $n = 3$; $\widetilde{t} = 0.0,\ 0.1$; $\lambda_{0,0} = 0.0,\ \pm 0.2$. For all n values sufficient universal quantities could be found to cover the contour of wing aerofoil, and it means that $\widetilde{\tau}_w, \widetilde{\delta}^*, \widetilde{\delta}^{**}$ can be calculated for all variations of contour accelerating and decelerating through the fluid. It was not the case in reference [9] in which there were no porous parameter, and where the universal solutions are obtained using the approximative momentum and energy equation, so it was reason why we considered only very slow cylinder accelerating and decelerating.

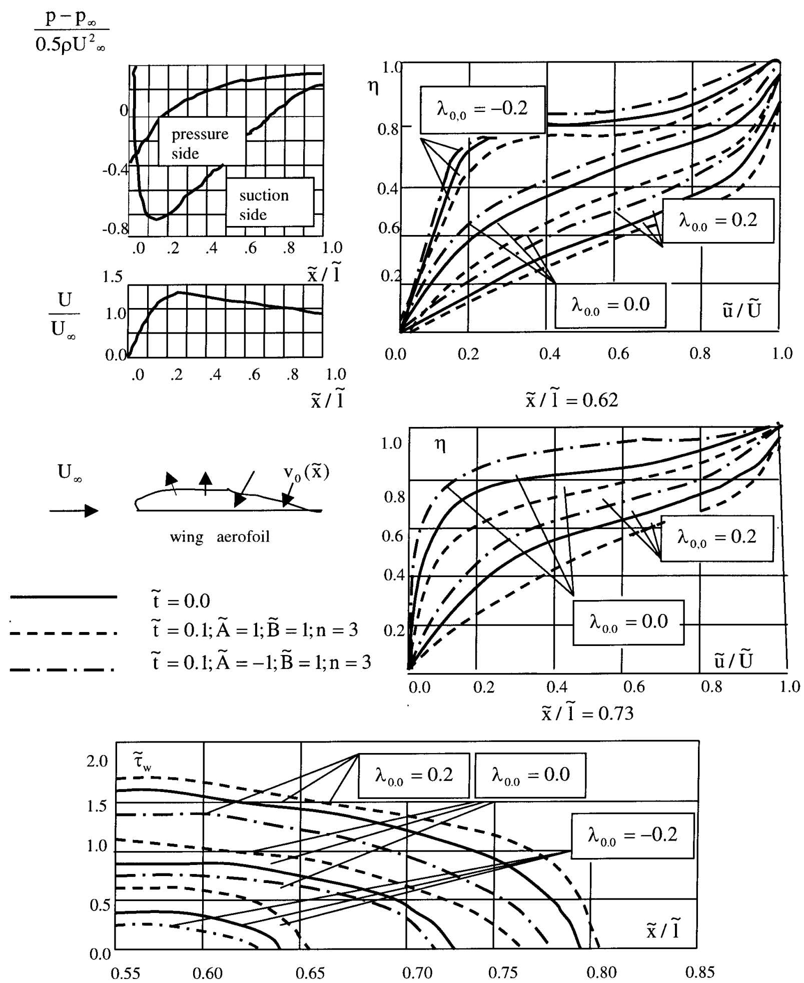

Fig. 1. Graphical presentation of the stress distribution of unsteady layer on porous wing aerofoil

For determination of the nondimensional streamwise velocity distibution $\widetilde{u}(\widetilde{x}, \widetilde{t})$ in some boundary layer cross-section $\widetilde{x}_0$ and for time $\widetilde{t}_0$, one can use the relation:

$$\frac{\tilde{u}(\tilde{x}_0,\tilde{t}_0)}{\tilde{U}(\tilde{x}_0,\tilde{t}_0)}=\frac{\partial}{\partial\eta}\Phi[\eta;f_{1,0}(\tilde{x}_0,\tilde{t}_0);f_{0,1}(\tilde{x}_0,\tilde{t}_0);\lambda_{0,0}(\tilde{x}_0,\tilde{t}_0)],\qquad(17)$$

where $\partial\Phi/\partial\eta$ is universal solution obtained from numerical integration of equation (11). There are the distributions of velocities in different cross-sections of porous contour, i.e. for $\tilde{x}/\tilde{l}$ =0.62; 0.73; (see Fig.) when porous parameter is $\lambda_{0,0} = -0.2$; 0.0; 0.2. In diffuser region for both the accelerating and decelerating flows and also for ejection and injection, velocity distribution receives the ridge point, so it is beginning the laminar-turbulent transition.

For controling the boundary layer separation point, shear stress and drag, the expression for nondimensional ejection and injection velocity distribution $\tilde{v}_w$ can be applicable in form:

$$\tilde{v}_w=-\lambda_{0,0}B^{-1}(\upsilon\tilde{U}^{b_0})^{\frac{1}{2}}\left(a_0\int_0^{\tilde{x}}\tilde{U}^{b_0-1}d\tilde{x}\right)^{-1/2}\qquad(18)$$

obtained by (4) for $k = n = 0$, from which for wanted separation point $\tilde{x}_0$ in time $\tilde{t}_0$, one can determine the needful value for $\tilde{v}_w$. In that way we can control the boundary layer separation.

It's found that for both in cofuser and in diffuser contour regions the accelerating flow ($\tilde{A} = 1$) increases the shear stress and postpones the separation of boundary layer i.e. laminar-turbulent transition section, and vice versa the decelerating flow reduces the shear stress and favours the separation of flow. It can be noted that the unsteady parameter has a significant influence on a shear stress distribution and especially on the laminar-turbulent transition location obtained by zero skin friction criteria. When this parameter is increasing ($\tilde{t} = 0.1$; 0.2) the shear stress magnitude is increasing on whole contour and the separation point is removing along the surface. It means, that the acceleration leads to the postponing of the boundary layer in diffuser region from 73.8% for steady flow i.e. $\tilde{t} = 0.0$ to the 78.7% of contour for $\tilde{t} = 0.1$, and it is when there are no fluid injection or ejection through the porous contour. It is important fact, because the achivement of laminar flow on 73.8%–78.7% of contour significantly reduce the contour drag. The decelerating flow ($\tilde{A} = -1$) favours the occurring of the separation and for steady flow the separation is occurring at lower contour values i.e. on 72.2% of contour for $\tilde{t}$ =0.1, also when there is no fluid injection and ejection. As well as for both the accelerating and decelerating flows, the ejection of fluid increases the shear stress, especially in confuser region about stagnation point, where shear stress is dramatically increased in time. It's not good for drag, so one can control this great shear stress with fluid injection, when his value is noticeably reduction. Also, the ejection of fluid postpones the boundary layer separation, and vice versa the injection of fluid reduces the shear stress and favours the flow separation. The great fluid ejection, i.e. when the porous parameter is $\lambda_{0,0} = 0.2$, leads to the postponong of separation to 81.4% of contour for $\tilde{t} = 0.1$. And for a great fluid injection ($\lambda_{0,0} = -0.2$) the separation is occurring at lower contour values, about 63% of contour for a decelerating flow and for $\tilde{t} = 0.1$

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RASPODELA SMIČUĆEG GRANIČNOG ODVAJAJUĆEG NAPONA NESTACIONARNOG NESTIŠLJIVOG GRANIČNOG SLOJA U OBLASTI DIFUZORA NA POROZNOJ KONTURI

Dečan Ivanović

Kroz poroznu konturu u normalnom pravcu, fluid istih svojstava kao nestišljivi fluid u osnovnom toku se ubrizgava ili izbrizgava brzinom koja je funkcija uzdužne koordinate konture i vremena. Odgovarajuće jednačine nestacionarnog graničnog sloja transformišu se u generalizovani oblik uvođenjem odgovarajuće transformacije promenljivih jednačina momenta i energije i skupa parametara sličnosti. Generalizovana rešenja dobijena numeričkom integracijom u lokalnoj aproksimaciji, koriste se za izračunavanje laminarno-turbulentnog prelaza zasnovanog na zeko skin funkciji kriterijuma, a raspodela brzine nestacionarnog graničnog sloja u oblasti difuzora na krilu aerofol kada se centralna brzina menja sa vremenom kao stepena funkcija i kada se rotacijalna spoljašnja brzina meri pri slobodnom letu. U oblasti difuzora, kao i pri ubrzanom i usporenom toku, izbrizgavanje fluida povećava trenje i odlaže separaciju graničnog sloja i obrnuto, ubrizgavanje fluida smanjuje trenje i ubrzava razdvajanje toka.