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STRESS STATE AND STRAIN ENERGY DISTRIBUTION AT THE VICINITY OF ELLIPTICAL CRACK WITH COMPRESSION FORCES ACTING ON IT'S CONTOUR

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Abstract. *Model of elliptical crack with contour pressurized by continual uniform forces is of interest and has found application in mechanical and civil engineering, as well as in geomechanics and geology. At solving of this problem stress functions in elliptical coordinate system can be used (Timoshenko, Goodier), or the region outside the elliptical contour can be transformed at simpler shape and find appropriate stress function, accordingly by application of complex variable function and conformal mapping method (Muskhelishvili). Stress functions for the case of plane crack suggested by Westergaard and Sneddon can be applied. All this solutions are related on the problem of plane stress state in a plate, where three-dimensional stress state at the vicinity of crack is neglected. Analytical solutions for problem of elliptical shaped crack in an infinite plate by applying of complex variable function and conformal mapping method are presented in this paper. Crack is subjected to uniform pressure forces on it's contour, and plane stress state in all points of the plate is assumed. Comparable three-dimensional model of crack in the plate of finite dimensions is done. By application of finite element method, diagrams off stress and deformation distribution at the vicinity of crack, as well as at whole plate, are done. Diagrams of stress components, in selected sections are presented. Strain energy for characteristic directions* $y = 0,\ z = 0, \dots z_k$ *and* $z = 0,\ x = 0, \dots x_k$ *is calculated by using well-known relations of theory of elasticity. Then, surface of the strain energy for points in the middle plane* $z = 0$*, and plane perpendicular to it* $y = 0$*, in front of the crack tip, by using best fitting curve, and best fitting surface, and iteration procedure is reconstructed. Conclusion on three-dimensional stress state at the vicinity of crack tip, is derived from obtained stress diagrams, and estimation up to which distance from crack tip three-dimensional stress state exist is done. Also, from reconstructed strain energy surfaces, it's concentration and three-dimensional distribution is visible. On certain distance from the crack strain energy gets constant value, and at the most part of the plate is "undisturbed state".*

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1. INTRODUCTION. FRAMEWORK

Griffith theory assumed that cracks exist or they are initiated in solid body when the tensile stresses are reached the value when the strain energy realize rate is hire then the rate at which energy is gained by creation of new free surface. To consider the simplest physical model of crack based on Griffith theory, it is needed to determine the stress distribution in the vicinity of the crack and to determine the strain-energy distribution due to the crack existence. The most serious limitation of this theory is assumption that direction of crack growth is known as an a priori. Sih proposed the local strain energy density in a material element as the dominant criterion, which governs crack development. He proposed strain energy density factor S for material element at a finite distance r_o from the point of fracture initiation, where S is given by relation [1], [12]:

$$S = r_o \frac{dW}{dV} = r_o A'_{def} \tag{1}$$

where: $\frac{dW}{dV}$ is strain energy density per unit volume, or A'_{def} -specific strain energy.

When elastic deformation is predominant in a body, relation or linear elasticity can be used to describe strain-energy distribution [1], [2], [15]:

$$A'_{def} = \frac{1}{2\cdot E}[(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - 2\cdot \nu(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z) + 2\cdot(1+\nu)(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)] \tag{2}$$

Three-dimensional model of the plate, with crack of an elliptical contour is done in this paper Fig. 1. It is assumed that all -round constant pressure p = 1 N/mm^2 is applied at the direction perpendicular to the elliptical contour-surface of the crack. Finite element method is used for determining of the stress tensor components in the neighborhood of an elliptical crack. Three-dimensional mesh of finite elements is generated, as it is presented on Fig. 2. Elliptical hole at the middle of the plate is taken as model of Griffith crack [3], [4], [5], [6]. Elliptical crack has semi-major axis a = 5 mm and semi-minor axis b = 0.5 mm. As material of the plate is taken polyester Palatal P-6 with Young's elasticity modulus E = 4460 N/mm^2, and Poisson's modulus ν = 0.38.

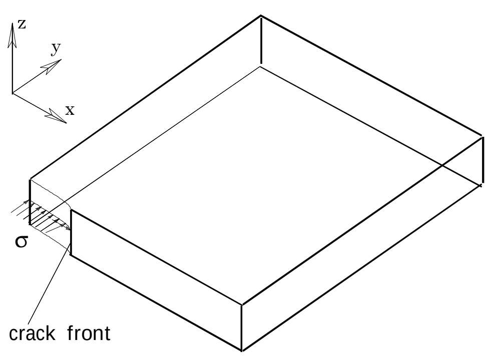


Fig. 1.

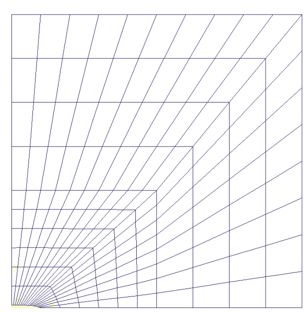


Fig. 2.

One quarter of square plate with dimensions 100×100 mm, and 10 mm thick, is taken for analyzing of stress state and strain energy state. Cartesian coordinates x, y and z with center at the middle of the plate are used. So, x is between 0 and +50 mm, y is between 0 and +50 mm, and z is −5 mm and up to +5 mm $+5$ mm .

2. Stress distribution

Distributions of stress components σ_x, σ_y, σ_z, τ_{xy} in the middle plane are presented on Fig. 3, 4, 5 and 6. Distributions of stress components σ_x, σ_y, σ_z, τ_{xy} at the plane section $y = 0$ of the plate are presented on Fig. 7, 8, 9, and 10 respectively. Diagrams of the stress components σ_x, σ_y, σ_z, τ_{xy}, τ_{yz} and τ_{zx} for $y = 0$, and $z = 0$ are presented on Fig. 11. Same stress components for $x = 0$, and $z = 0$, and on the crack tip for $x = 4.9$ mm, and $z = 0$ are presented on Fig. 12, and Fig. 13 respectively.

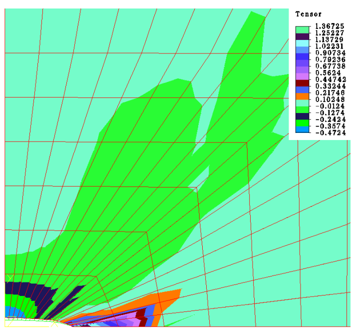
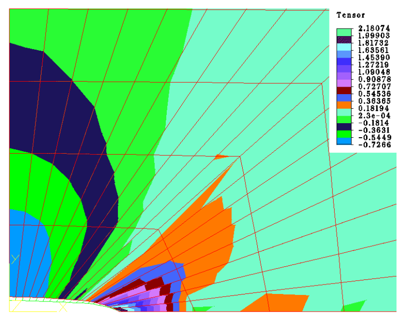


Fig. 3.



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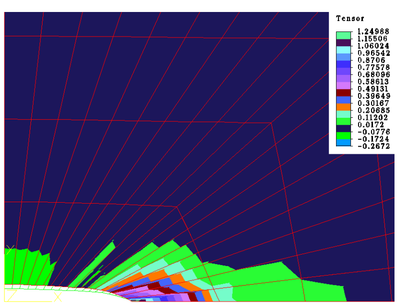

Fig. 5.

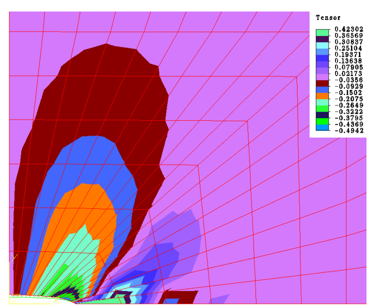

Fig. 6.

It is visible on Fig. 11 that gradient of normal stress components σ_x, σ_y, σ_z is extremely high in front of the crack tip, in crack direction. Shear stress component τ_{xy} has values different than zero up to distance between one, and 1.3 plate thickness. Other shear stresses τ_{yz}, and τ_{zx} are equal to zero. It is visible on Fig. 12 that normal stress component σ_x is different than zero up to one plate thickness, σ_y is different than zero on distance up to 1.5 plate thickness, and σ_z is different than zero up to distance half of plate thickness. This figure presents stress state at the middle plane of the plate. It is evidently on Fig. 13 that normal stress σ_x is different than zero up to distance of about one plate thickness, and that change it's sign at same point close to the crack tip (calculated distance is y = 1.515 mm). Normal stress σ_y is different than zero up one plate thickness, and change it's sign from plus to minus on calculated distance y = 2.525 mm from the crack tip. Normal stress component σ_z is different than zero, and has high gradient up to distance of y = 0.2 mm y = 0.2 mm from the crack tip. Shear stress component τ_{xy} is different than zero up to distance of two plate thickness from crack tip. Other shear stresses τ_{yz}, and τ_{zx} are equal to zero, and this is consistent whit theory.

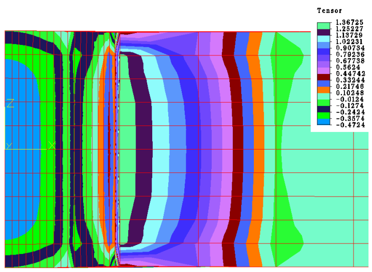

Fig. 7.

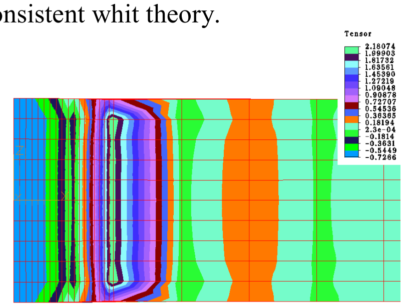

Fig. 8.

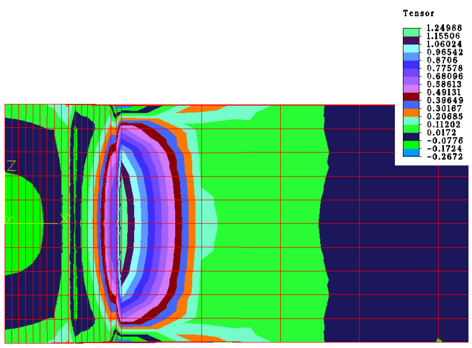

Fig. 9.

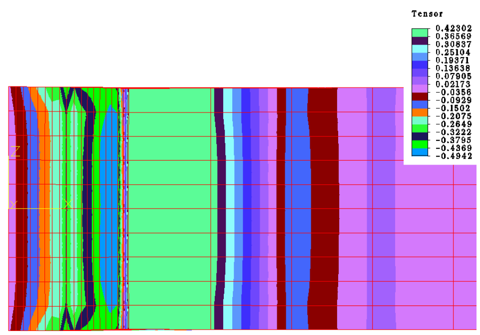

Fig. 10.

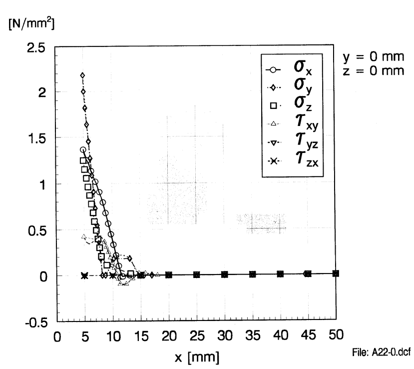

Fig. 11.

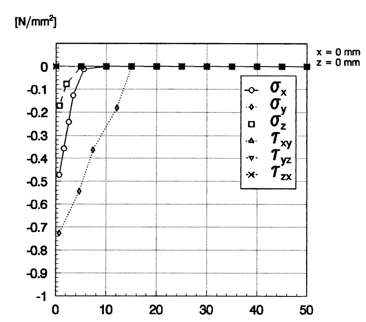

Fig. 12.

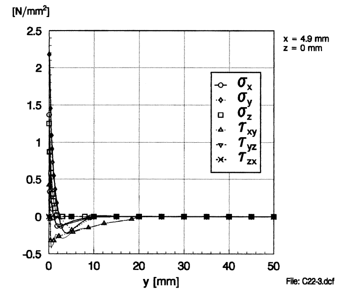

Fig. 13.

3. STRAIN ENERGY DISTRIBUTION

Specific strain energy or strain energy density per unit volume is calculated by using data from stress diagrams presented on previous figures, where interpolation and smoothing of stress function is previously done. For calculation of strain energy in characteristic directions FORTRAN program named "Defen" is created. Equation (2) well known in theory of elasticity [2] is used for calculation.

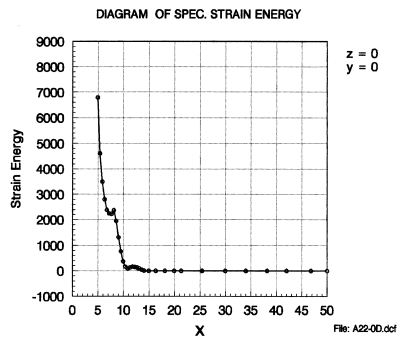

Fig. 14.

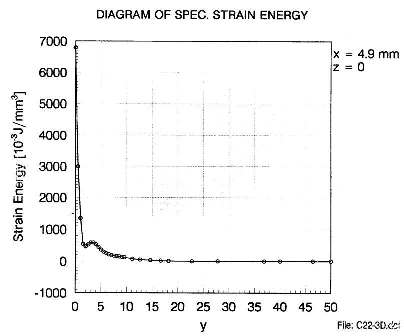

Fig. 15.

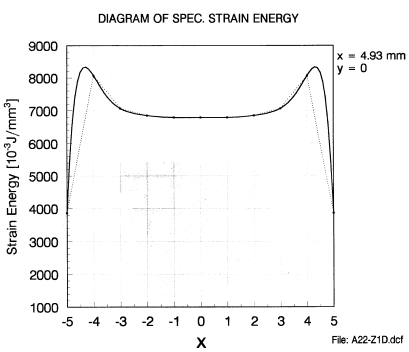

Fig. 16

Diagram of specific strain energy for direction $z = 0$, and $y = 0$ is presented on Fig. 14. Diagram of specific strain energy for direction $x = 4.9$ mm, and $z = 0$ is shown on Fig. 15 and diagram of specific strain energy at the crack tip front $x = 4.93$ mm and $y=0$ is shown on Fig. 16.

By using data from diagrams of the specific strain energy for various directions and by gridding and smoothing the surfaces which are presenting distribution of specific deformation energy in planes $z = 0$, $F = F(x,y)$, and $y = 0$, $F = F(x,z)$ are reconstructed. Surface of the specific deformation energy $F = F(x,z)$ for plane $y = 0$ is shown on Fig. 17. Surface of the specific strain energy $F = F(x,y)$ for plane $z = 0$ is presented on Fig. 18. It is visible on Fig. 18 that distribution of strain energy resulted with high value of energy in front of crack tip, where the peak of energy stored in material exists. It is visible on Fig. 16 as well as on Fig. 17 that specific strain energy is increasing from middle plane $z = 0$ to front and back free surface of the plate, and that peaks of energy are close to free surfaces off the plate. This is observed only in region close to the crack front, up to distance of about 0.25 of plate thickness, from the crack front.

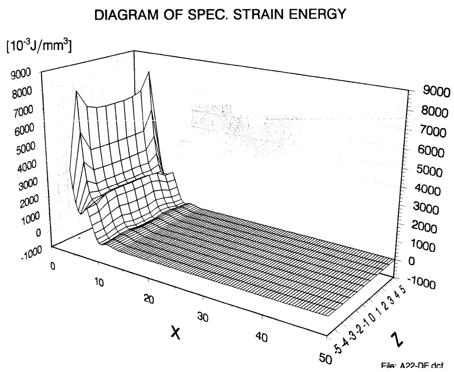

Fig. 17.

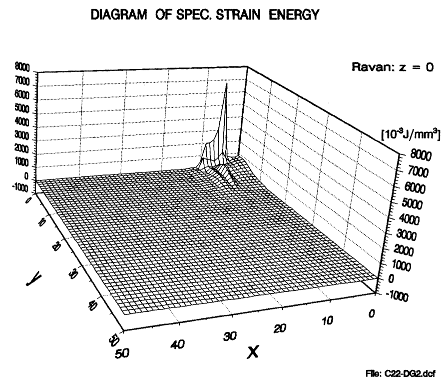

Fig. 18.

4. Local effect of the crack

Analysis in this paper shows that crack and loads on it's surface are generating local stress state and local strain energy distribution up to the one and half plate thickness from the crack, and it is visible on the presented figures. It is evidently that all strain energy for analyzed case of crack shape, and loading conditions, is stored at region of the plate near by the crack surface up to distance of one plate thickness.

Results calculated by analytical solutions well known in literature [1], [9], [11], [14], [15], and in the papers [3], [4], [5], [6], are comparable with analyzed problem, but these analytical solutions are clearly two-dimensional and their framework is plane problem of theory of elasticity.

5. Conclusions. Discussion

Strain energy density is accepted criterion for crack development [1], [7], [11], [12], [13], [14]. It is consequently important to know distribution of strain energy in the crack vicinity. Strain energy density factor proposed by G. C. Sih is representing strain energy per unit of volume and it is calculated by using asymptotic formulas in papers of Sih [1], [12]. In this paper strain-energy surfaces are reconstructed. They are presenting the strain energy distribution in middle plane $z = 0$, and in plane $y=0$, of the plate with an elliptical crack subjected to uniform internal pressure in it's contour surface.

From diagrams of the stress components is visible that stress state is three-dimensional at the vicinity of the crack tip. Normal stress σ_z is different then zero on distance up to one plate thickness from the crack front at direction of crack propagation, and up to half of plate thickness at direction perpendicular to the crack. It is also visible that stresses σ_x, σ_y, σ_z are depending on coordinate z up to one plate thickness from the crack front at crack direction, and at direction perpendicular to the crack. Obtained results and diagrams are consistent with experimental evidence [8]. Stress distribution, and strain energy behaviour is analyzed in references [10], [13], [16], but in those papers, specific strain energy distribution for analyzed problems was not given in the form of three-dimensional surface.

REFERENCES

1. Gdoutos E. E., *Fracture Mechanics,* Kluwer Academic Publishers, Dordrecht, 1993
2. Hedrih K., (1988), *Izabrana poglavlja Teorije elastičnosti*, Mašinski fakultet, Niš
3. Hedrih K., Jovanović D., (1990), *Primena funkcije kompleksne promenljive i konformnof preslikavanja za određivanje eliptično-hiperboličkih koordinata tenzora napona ravno napregnutih ploča*, XIX Jugoslovenski kongres teorijske i primenjene mehanike, Zbornik radova, pp 13-18, Ohrid
4. Hedrih K., Jovanović D., *The stress state of the elliptical-annular plate by the complex variable function and conformal mapping method*, 15 str., 73-87, časopis, Teorijska i primenjena mehanika, br. 17, Jugoslovensko društvo za mehaniku, Beograd, 1991.
5. Hedrih K., Jovanović D., *The strain state of the elliptical - annular plate by the complex variable function and conformal mapping method,* 17 str., 29-45, časopis, Teorijska i primenjena mehanika, br. 21, Jugoslovensko društvo za mehaniku, Beograd, 1995.
6. Hedrih K., Jovanović D., *Stress state around elliptical cracks in elastic plates*, International minisymposium MF Recent Advances in Fracture and Damage Mechanics, Zbornik radova, Str. 153-157, YUCTAM Vrnjačka Banja, 1997
7. Hedrih K., Jovanović D., *Fenomeni nelinearne dinamike, saznanja mehanike loma i teorije slučajnih oscilacija i projektovanje mašinskih sistema sa sigurnošću od rizika i loma*, 26. JUPITER KONFERENCIJA sa međunarodnim učešćem, Zbornik radova, Str. 3.57-3.65, Beograd, februar, 2000
8. Jovanović D., (1990), *Primena metode fotoelastičnosti za ispitivanje naponskog stanja eliptično-prstenaste ploče opterećene koncentrisanim silama*, XIX Jugoslovenski kongres teorijske i primenjene mehanike, Zbornik radova, Ohrid
9. Mushelishvili N.I., (1953), *Some Basic problems of the Mathematical Theory of Elasticity*, P. Noordhof LTD., Groningen - Holland.
10. Parsons I. D., Hall J. F., Rosakis A. J., *A Finite Element Investigation of the Elastostatic State Near a Three Dimensional Edge Crack,* SM Report 86-29, Division of Engineering and Applied Science, California Institute of Technology, Pasadena, California, 1986
11. Sedmak S. *Uvod u mehaniku loma i konstruisanje sasigurnošću od loma,* monografija, Smederevska Palanka, 1980
12. Sih G. C., *Mechanics of Fracture Initiation and Propagation,* Kluwer Academic Publishers, Dordrecht, 1990
13. Sih G. C., Lee Y. D., *Review of Triaxial Crack Border Stress and Energym Behavior,* Theroretical and Applied Fracture Mechanics, No. 12, pp. 1-17, Elsevier Science Publishers, B. V., 1989
14. Sneddon, I. N., Lowengrub, M., *Crack Problems in the Classical Theory of Elasticity* John Wiley & Sons, Inc., New York, 1969
15. Timoshenko S. P. and Goodier J. N., (1951), *Theory of Elasticity*, New York
16. Theocaris P. S., *The Minimum Elastic Strain Energy Criterion for Mode I and II Fractures, Proc. of Symp. on Absorbed Spec. Energy / Strain Energy Density Criterion*, pp. 17-32, Budapest, Hungary, 1980

STANJE NAPONA I STANJE ENERGIJE DEFORMACIJE U OKOLINI ELIPTIČNE PRSLINE PRI DEJSTVU SILA PRITISKA NA NJENOJ KONTURI

Dragan B. Jovanović, Milena B. Jovanović

Model prsline eliptičnog oblika na čijoj konturi dejstvuju kontinualne sile pritiska, od interesa je i ima primenu u mašinskom, građevinskom inženjeringu, kao i u mehanici tla i geologiji. U rešavanju ovog problema mogu se koristiti naponske funkcije u eliptičnom koordinatnom sistemu (Timoshenko, Goodier) ili se može transformisati oblast izvan eliptične konture u jednostavniji oblik i pronaći odgovarajuća naponska funkcija, odnosno primeniti metoda funkcije kompleksne promenljive i konformnog preslikavanja (Muskhelishvili). Mogu se primeniti i naponske funkcije predložene od Westergaard-a i Sneddon-a za slučaj ravne prsline. Sva ova rešenja odnose se na problem ravnog stanja napona i ploči, pri čemu je zanemareno lokalno trodimenzionalno stanje napona u okolini prsline.

U radu su prikazana analitička rešenja problema prsline eliptičnog oblika u beskonačnoj ploči, primenom funkcije kompleksne promenljive i konformnog preslikavanja. Prslina je izložena kontinualno jednako raspodeljenim silama pritiska po konturi i pretpostavljeno je ravno stanje napona u svim tačkama ploče. Načinjen je takođe uporedni prostorni model prsline u ploči konačnih dimenzija i primenom metode konačnih elemenata dobijeni su dijagrami rasporeda napona i deformacija u okolini prsline, kao i u čitavoj ploči.

Prikazani su dijagrami komponentnih napona u izabranim ravnim presecima ploče. Specifična energija deformacije je izračunata preko poznatih relacija iz teorije elastičnosti za karakteristične pravce $y = 0,\ z = 0, \ldots z_k$ *i* $z = 0,\ x = 0, \ldots x_k$*. Zatim je korišćenjem best- fitting krivih i best-fitting površina, kroz postupak uzastopnih iteracija, izvršena rekonstrukcija površine energije deformacije za tačke u središnjoj ravni ploče* $z = 0$ *i za ravan upravnu na nju* $y = 0$*. Na osnovu dobijenih dijagrama napona, donet je zaključak o trodimenzionalnom stanju napona u blizini vrha prsline, sa procenom do kog rastojanja od vrha prsline se javlja trodimenzionalno stanje napona. Takođe se iz rekonstruisanih površina energije deformacije vidi njena koncentracija i njen trodimenzionalni raspored ispred vrha prsline. Na određenom rastojanju od prsline ona dobija konstantnu vrednost i u najvećem delu ploče vlada "neporemećeno stanje".*