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STABILITY OF THE FREELY SUPPORTED RECTANGULAR PLATE STRENGTHENED WITH THE TRANSVERZAL PROPPING IN THE PLASTIC RANGE

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Abstract. *Rectangular plate is loaded by uniformly distributed forces of pressure in the mid-surface. It is freely supported on all four sides and strengthened with one arbitrarily set transversal propping. Stability of the plate is considered under the assumption that under even distribution of pressure (N_y) it buckles in the direction O_x into (m) sinusoidal half-waves.*
In this composition frequency equation is determined for real roots of the characheristic differential plate equation.

1. INTRODUCTION

According to the small plasto-elastic deformation theory, relatively to so called "deformation theory", differential equation of mid-surface of rectangular plate in the plastic range, known in the literature as Stowell's equation, has a form

$$\left[1-\frac{3}{4}\left(1-\frac{\varphi_k}{\varphi_c}\right)\frac{\sigma_x^2}{\sigma_i^2}\right]\frac{\partial^4 w}{\partial x^4}+2\left[1-\frac{3}{4}\left(1-\frac{\varphi_k}{\varphi_c}\right)\frac{\sigma_x\sigma_y+2\tau^2}{\sigma_i^2}\right]\frac{\partial^4 w}{\partial x^2\partial y^2}+$$
$$+\left[1-\frac{3}{4}\left(1-\frac{\varphi_k}{\varphi_c}\right)\frac{\sigma_y^2}{\sigma_i^2}\right]\frac{\partial^4 w}{\partial y^4}-3\left(1-\frac{\varphi_k}{\varphi_c}\right)\frac{\tau}{\sigma_i^2}\left(\sigma_x\frac{\partial^4 w}{\partial x^3\partial y}+\sigma_y\frac{\partial^4 w}{\partial x\partial y^3}\right)+\frac{h}{D_c^{'}}\prod(\sigma,w)=0. \tag{1}$$

In papers [5], [6] and [7] problems of stability of unstrengthened rectangular plate loaded by uniformly distributed forces of pressure for different limiting conditions were solved. Supposed solutions satisfied given differential equation in simplified forms and given limiting conditions.

Stability of the rectangular plate freely supported on all four sides, strengthened with one arbitrarily set transverzal propping, loaded by uniformly distributed forces of pressure in the mid-surface will be considered in this paper.

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2. PROPOSITION AND SOLUTION OF THE PROBLEM

Rectangular plate strengthened with one arbitrarily set transverzal propping is presented in picture 1. It is loaded by forces of pressure N_y in O_y direction. Accept above mentioned loading, plate is also affected by pressure from propping, which has axial moment of inertia.

Propping divides plate into ranges 1 and 2 which each for itself presents un-strengthened plate.

Differential equation (1) under conditions of propping given in picture 1:

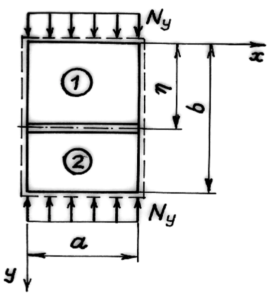

Fig. 1.

$$\sigma_x = \tau = 0\,,\ \sigma_i = \sigma_y\,,\ \Pi(\sigma, w) = \sigma_y \frac{\partial^2 w}{\partial y^2}\,,\ N_y = h\sigma_y = h\sigma. \quad (2)$$

reduces to next simplier form

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \left(\frac{1}{4} + \frac{3}{4}\frac{\varphi_k}{\varphi_c}\right)\frac{\partial^4 w}{\partial y^4} + \frac{h\sigma}{D_c^{'}}\frac{\partial^2 w}{\partial y^2} = 0. \quad (3)$$

Solution of this differential equation will be searched for in form

$$w_i = f_i(y)\sin\frac{m\pi x}{a}, \qquad i = 1, 2 \quad (4)$$

by which it is supposed that, under effect of uniformly distributed pressure (N_y), plate buckles in direction (x) into (m) sinusoidal half-waves. Plate buckling in direction (y) is defined by functions $f_i(y)$, (i=1,2), which will be determined later.

Supposed deflection function satisfies limiting conditions on unloaded sides $x = 0$ and $x = a$, on which deflections and moments are equal to zero.

Needed derivatives of deflection function (4) are:

$$\begin{aligned} &\frac{\partial^4 w}{\partial x^4} = f(y)\frac{m^4\pi^4}{a^4}\sin\frac{m\pi x}{a}, && \frac{\partial^2 w}{\partial y^2} = f''(y)\sin\frac{m\pi x}{a}, \\ &\frac{\partial^4 w}{\partial x^2 \partial y^2} = f''(y)\frac{m^2\pi^2}{a^2}\sin\frac{m\pi x}{a}, && \frac{\partial^4 w}{\partial y^4} = f^{IV}(y)\sin\frac{m\pi x}{a}. \end{aligned} \quad (5)$$

By replacing derivatives (5) into differential equation (3) we get to a homogenous linear differential equation

$$\left(\frac{1}{4} + \frac{3}{4}\frac{\varphi_k}{\varphi_c}\right) f_i^{IV}(y) - 2\left(\frac{m^2\pi^2}{a^2} - \frac{h\sigma}{2D_c^{'}}\right) f_i^{''}(y) + \frac{m^4\pi^4}{a^4} f_i(y) = 0, \quad i = 1, 2. \quad (6)$$

Characteristic equation of differential equation (6) is

$$\lambda^4 - \frac{2}{\left(\frac{1}{4} + \frac{3}{4}\frac{\varphi_k}{\varphi_c}\right)}\left(\frac{m^2\pi^2}{a^2} - \frac{h\sigma}{2D_c^{'}}\right)\lambda^2 + \frac{m^4\pi^4}{\left(\frac{1}{4} + \frac{3}{4}\frac{\varphi_k}{\varphi_c}\right)a^4} = 0. \quad (7)$$

If we initiate a mark

$$\kappa = \frac{1}{4} + \frac{3}{4}\frac{\varphi_k}{\varphi_c} \tag{8}$$

roots of equation (7) can be written in the form

$$\lambda_{1,2,3,4} = \pm\sqrt{\frac{1}{\kappa}\left(\frac{m^2\pi^2}{a^2} - \frac{h\sigma}{2D_c^{'}}\right) \pm \sqrt{\frac{1}{\kappa^2}\left(\frac{m^2\pi^2}{a^2} - \frac{h\sigma}{2D_c^{'}}\right)^2 - \frac{m^4\pi^4}{ka^4}}}. \tag{9}$$

Here we initiate coefficient of proportionality to critical force of buckling

$$K = (N_y)_{cr}\frac{a^2}{D_c^{'}\pi^2} = \frac{h\sigma}{D_c^{'}}\frac{a^2}{\pi^2}. \tag{10}$$

a) Roots of the characteristic equation are real if fulfilled conditions

$$1 - \frac{K}{2m^2} > 0, \qquad \left(1 - \frac{K}{2m^2}\right)^2 - \kappa > 0,$$

and they can be written as

$$\lambda_{1,2} = \pm\frac{m\pi}{a\sqrt{\kappa}}\sqrt{\left(1 - \frac{K}{2m^2}\right) + \sqrt{\left(1 - \frac{K}{2m^2}\right)^2 - \kappa}} = \pm\alpha, \tag{11}$$

$$\lambda_{2,3} = \pm\frac{m\pi}{a\sqrt{\kappa}}\sqrt{\left(1 - \frac{K}{2m^2}\right) - \sqrt{\left(1 - \frac{K}{2m^2}\right)^2 - \kappa}} = \pm\beta. \tag{12}$$

Solutions of differential equation (3) for ranges (1) and (2) are

$$f_1(y) = A_1 ch\alpha y + A_2 sh\alpha y + A_3 ch\beta y + A_4 sh\beta y \tag{13}$$

$$f_2(y) = A_5 ch\alpha y + A_6 sh\alpha y + A_7 ch\beta y + A_8 sh\beta y \tag{14}$$

b) Roots are imaginary, if fulfilled conditions

$$\frac{K}{2m^2} - 1 > 0, \qquad \left(\frac{K}{2m^2}\right)^2 - \kappa > 0$$

so they can be written as

$$\lambda_{1,2} = \pm\frac{m\pi}{a\sqrt{\kappa}} i\sqrt{\left(\frac{K}{2m^2} - 1\right) - \sqrt{\left(\frac{K}{2m^2} - 1\right)^2 - \kappa}} = \pm\, i\alpha_1, \tag{15}$$

$$\lambda_{3,4} = \pm\frac{m\pi}{a\sqrt{\kappa}} i\sqrt{\left(\frac{K}{2m^2} - 1\right) + \sqrt{\left(\frac{K}{2m^2} - 1\right)^2 - \kappa}} = \pm\, i\beta_1, \tag{16}$$

Solutions of differential equation (3) for ranges (1) and (2) are

$$F_1 = B_1 \cos\alpha_1 y + B_2 \sin\alpha_1 y + B_3 \cos\beta_1 y + B_4 \sin\beta_1 y, \tag{17}$$

$$F_2 = B_5 \cos\alpha_1 y + B_6 \sin\alpha_1 y + B_7 \cos\beta_1 y + B_8 \sin\beta_1 y. \tag{18}$$

c) Roots are complex, if fulfilled conditions

$$\left(1-\frac{K}{2m^2}\right)^2 < 0, \qquad \left(1-\frac{K}{2m^2}\right) > 0$$

and they encircle short interval of practical problems.

Limiting conditions on loaded sides $y=0,\ y=\mathrm{b}$ are such that deflections and moments are equal to zero, that is

$$\text{for } y=0,\quad w=0,\quad \text{in other words} \quad f_1(0)=0, \tag{19}$$

$$\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}=0,\quad \text{in other words} \quad f_1''(0)=0, \tag{20}$$

$$\text{for } y=b,\quad w=0,\quad \text{in other words} \quad f_2(b)=0, \tag{21}$$

$$\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}=0,\quad \text{in other words} \quad f_2''(b)=0, \tag{22}$$

On the boundary of ranges (1) and (2) of the plate strengthened with one transverzal propping, deflections, inlinations and bending moments for ranges (1) and (2) are equal at arbitrary distance :

$$f_1(\eta)=f_2(\eta), \tag{23}$$

$$f_1'(\eta)=f_2'(\eta), \tag{24}$$

$$f_1''(\eta)=f_2''(\eta). \tag{25}$$

Last, eight, equation for determining A_i (i=1,2,...,8) integration constants is obtained from condition that the difference between transverzal forces for ranges (1) and (2) along the propping represents the loading by which propping takes effect on plate

$$f_2^{III}(\eta)-f_1^{III}(\eta)+\phi_1(\eta)=0, \tag{26}$$

where mark represents the next term

$$\phi_1 = E_r I_r \frac{m^4\pi^4}{a^4 D_c'}, \tag{27}$$

where E_r is Young's modulus of elasticity of propping, I_r axial moment of inertia of transverse section of propping.

From the conditions (19)–(22) we have strengthened plate frequent equation for the case of real roots, that is case a)

$$\phi_1\left[-\frac{\beta}{\alpha}\left(\frac{sh^2\alpha\eta}{th\alpha b}-sh\alpha\eta\, ch\alpha\eta\right)-\frac{sh^2\beta\eta}{th\beta\eta}-sh\beta\eta\, ch\beta\eta\right]+\beta(\beta^2-\alpha^2)=0. \tag{28}$$

For propping positions $\eta = b/2,\ b/3,\ b/4$ from (28) we get values

$$\phi_1\left(\frac{b}{2}\right)=\frac{2\beta(\alpha^2-\beta^2)}{\frac{\beta}{\alpha}th\frac{\alpha b}{2}-th\frac{\beta b}{2}},\tag{29}$$

$$\phi_1\left(\frac{b}{3}\right)=\frac{\beta(\alpha^2-\beta^2)}{\frac{\beta}{2}\frac{1}{cth\frac{\alpha b}{3}+cth\frac{2\alpha b}{3}}-\frac{1}{cth\frac{\alpha b}{3}+cth\frac{2\alpha b}{3}}},\tag{30}$$

$$\phi_1\left(\frac{b}{4}\right)=\frac{4\beta(\alpha^2-\beta^2)}{\frac{\beta}{2}\left(th\frac{\alpha b}{2}+th\frac{\alpha b}{4}\right)-\left(th\frac{\beta b}{2}+th\frac{\beta b}{4}\right)}.\tag{31}$$

We can similarly get frequent equation in case b) when roots of characteristic equation are imaginary.

3. Conclusion

Derivated frequent equation for real roots of the characteristic equation of basic differential equation with determined terms (29)–(31) along with using the results of solutions for unstrengthened plate dependence of critical force of buckling from relation of sides b/a could be determined.

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STABILNOST SLOBODNO OSLONJENE PRAVOUGAONE PLOČE OJAČANE POPREČNIM UKRUĆENJEM U PLASTIČNOJ OBLASTI

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Pravougaona ploča je opterećena ravnomerno raspoređenim silama pritiska u srednjoj površini. Slobodno je oslonjena na sve četiri strane i ojačana jednim proizvoljno postavljenim poprečnim ukrućenjem. Razmatra se stabilnost ploče pod pretpostavkom da se pod dejstvom

ravnomerno raspoređenog pritiska (N_y) izvija u pravcu Ox u (m) sinusoidalnih polutalasa.
U ovom radu se određuje frekventna jednačina za realne korene karakteristične diferencijalne jednačine ploče.