FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol.3, N° 11, 2001, pp. 159 - 166

NON-STATIONARY VIBRATIONS OF MULTILAYER PLATES AND CYLINDRICAL SHELLS. DIFFERENT THEORIES

UDC 624.073.132+624.074.43:624.042.1:539.32(045)

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Abstract. *The paper presents the comparison of outcomes of research of the stress-strained state (SSS) of multilayer plates and shells at impulse loading, which are obtained on the basis of the different two-dimensional theories and three-dimensional theories of an elasticity.*
For definition of SSS are applied the classical theory of Ambartsumyan, the first-order refined theory of multilayer plates and shells, and also non-classical refined theory of multilayer plates based on kinematic hypotheses, supposing cubic relation for tangential displacements and quadratic one for normal displacements from transversal coordinate in each of layers.
The possibilities of the theories and the validity of results obtained is illustrated by several examples of calculating vibration processes and the processes of propagation of elastic waves. The influence of a compliance of filler on stresses in carrying layers of plates and the variation of stresses on thickness of package at distributed and localized effects are investigated. The results of investigations allow to establish a class of structures and condition of their loading, at which the considered two-dimensional theories give reliable outcome.
Also, the vibration of multilayer plates of complicated form under impulse loading are considered. The approach offered is based on the elastic immersion method.

As a rule, two-dimensional theories are used for investigating the response of multilayer structures [1-5, 7]. Reduction of a three-dimensional problem to two-dimensional one can be realized by different ways. As noted in the papers of E.I. Grigoliuk [2, 3], the theory of multilayer shells and plates is being developed along two main lines. The first one is related to works in which the three-dimensional problem is reduced to a two-dimensional one on the basis of hypotheses applied to the entire package of layers as a whole [1]. It is accepted this direction to refer as continuous-structural. The second, more general line is related to works in which hypotheses for each

Received August 01, 2000
Presented at 5th YUSNM Niš 2000, Nonlinear Sciences at the Threshold of the Third Millenium, October 2-5, 2000, Faculty of Mechanical Engineering University of Niš

separate layers are used for deriving the equation [2-5, 7]. This approach is referred to as discrete-structural.

The selection of a model adequately describing the response of a structure is made on the basis of several criteria depending on the concrete problem. As a rule, the following criteria are used: relative thickness *h*/*l* (*h* is the thickness, *l* is the characteristic problem dimension), thickness ratio *h*/*R* (*R* is the shell radius), relationship of the layer elastic properties, type of loading.

In the paper some two-dimensional mathematical models of multilayer plates and shells concerning to different directions in the theories of multilayer structures are considered. Continuous-structural direction is represented by the classical theory S. A. Ambartsumyan (the non-deformable unified normal hypothesis for a package) [1], discrete-structural direction is presented by the theory E.I. Grigoliuk and P.P. Chulkov (hypothesis of the broken line for a package) [2] and refined high-order theory taking into account the effects of transverse shear deformation, transverse normal strain, and a nonlinear dependence of displacement on the transverse coordinate [7]. In further we designate theories S.A. Ambartsumyan as AT, E.I. Grigoliuk and P.P. Chulkov as GCT, and refined high-order theory as HORT. The hypotheses of GCT and HORT are particular cases of the generalized theory based on definition of the law of change of displacements for each layer as a power series on transversal coordinate

$$u_\alpha^i(x,y,z,t)=u_\alpha+\sum_{k=1}^{K}\left[\sum_{j=1}^{i-1}h_j^k u_{\alpha k}^j+(z-\delta_{i-1})^k u_{\alpha k}^i\right],\alpha=1,2,$$

$$u_3^i(x,y,z,t)=u_3+\sum_{\ell=1}^{L}\left[\sum_{j=1}^{i-1}h_j^\ell u_{3\ell}^j+(z-\delta_{i-1})^\ell u_{3\ell}^i\right], \tag{1}$$

where $h_j^k=(h_j)^k$, $\delta_{i-1}\le z\le\delta_i$, $\delta_i=\sum_{j=1}^{i}h_j$, $i=\overline{1,I}$,

u_α^i, u_3^i is displacements of the plate (or shell) point in the *i*th layer in the direction of coordinate axes *Ox*, *Oy*, *Oz* correspondingly; $u_\alpha, u_3, u_{\alpha k}^i, u_{3\ell}^i$ are sought for function depending on *x*, *y*, *t*; h_I is the thickness of the *i*th layer; *t* is time; *I* is the number of layers in a pack; *K*, *L* are numbers of the terms of a series. It is assumed that contact between the layers excludes their delamination and mutual slipping, the coordinate plane *Oxy* linked with the external surface of the first layer. At *K* = 1, *L* = 0 hypotheses (1) coincide with GCT hypotheses, and at *K* = 3, *L* = 2 coincide with hypotheses of the HORT. Thus, behavior of each layer of a plate in the GCT theory is described by equations S. P. Timoshenko, and in case of the HORT is described by of the K. H. Lo, R. M. Christensen and E. M. Wu hypotheses.

The feasibility of the above mentioned theories is illustrated by several examples. It is further assumed that strains are small, materials of the layers are homogeneous isotropic, a transverse load is applied to the external surface of the first layer.

We shall consider a problem about influence of Young's modulus of a middle layer on maximum stresses in rectangular simply supported three-layer plate at transient loading. The carried layers are made of silica glass (SG), the mechanical characteristics of this material: E_i = 6,67·10^4 MPa, ρ_i = 2,5·10^3 kg m^{-3} , ν_i = 0,22 ($i=1,3$). Here E_i , ν_i

are Young's modulus and Poisson's ratio of the ith layer, ρ_i is the ith layer material density.

For the case $A = B = 0{,}5$ m (A, B - dimensions of a plate in the plan), $h_1 = h_2 = h_3 = 0{,}01$ m and impulse loading

$$p_3^e = P_0 H(t) \sin\frac{\pi x}{A} \sin\frac{\pi y}{B},$$

(here is accepted $P_0 = 0{,}1$ Mpa, $H(t)$ is Heaviside function) the relation of maximum stresses calculated in the middle of outside surfaces of a plate from Young's modulus of middle layer was shown. The Young's modulus of middle layer E_2, given in the following limits: $10^{-6}\,E \le E_2 \le E$ ($E = E_1 = E_3$, $\nu_1 = \nu_2 = \nu_3$, $\rho_1 = \rho_2 = \rho_3$). The results of calculation are presented in Fig. 1. The solid line corresponds to the three-dimensional theory [6], dashed line relates to the HORT [7], dash-dot line designates the AT [1], dots relate to the theory of GCT [2,4].

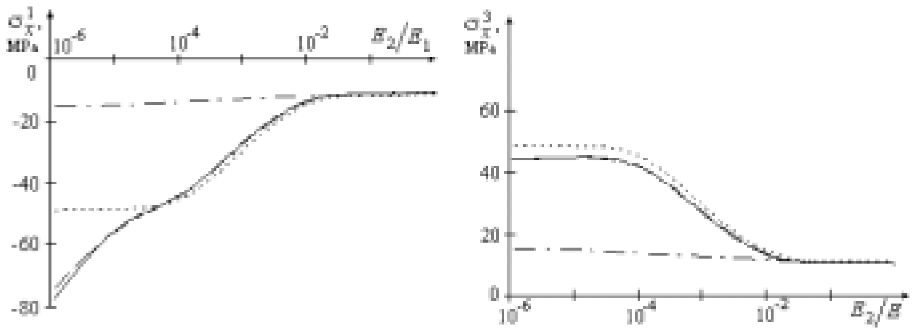


Fig. 1

As is easily seen, the good coincidence of results obtained on the basis of the considered theories is observed only for values E_2 close to E. So, the AT well agrees with the three-dimensional solution at values E_2 satisfying to an inequality $E \ge E_2 \ge 10^{-2}\,E$, the GCT – at $E \ge E_2 \ge 10^{-4}\,E$, and HORT can be used practically for all values E_2. Thus, the account of transversal shear and normal strains (stresses) of layers expands the boundaries of applicability of the theory. Note that in the considered problem the distribution of displacement (stress) in carrier layers remained practically linear for all values of Young's modulus of filler. Nonlinear character of distribution of the displacements (stresses) with respect to the thickness coordinate, which is taken into account in the HORT is displayed only for comparatively thick plates and shells, and also at a localized loading [6,7]. In the latter case, nonlinear character of relation of stresses from transversal coordinate is shown only in a small vicinity of loading area.

In spite of the fact that the HORT gives in all cases considerable degree of approximation, its application in real problems is inconvenient and ineffective. In most cases GCT suffices for description of non-stationary behavior of multilayer structures.

The functionability of a method based on the GCT and reliability of results for a multilayer cylindrical shell of length l, radius R, is illustrated on the following examples. The shell is referred to a right-hand system of orthogonal curvilinear coordinates x, φ, z.

The coordinate surface is connected to an outside surface of the first layer and has radius R.

For the case an impulsive loading of a cylindrical shell, the results of calculation by using the GCT are compared to the analytical solution presented in the work [8]. The single-layer infinite cylindrical shell (h/R=1/10) is subjected to a concentrated impulse radially directed unit loading applied in the point $\varphi = 0$, $z = h$:

$$p^e(\varphi,t) = \delta(\varphi)\delta(t),$$

where $\delta(t)$, $\delta(\varphi)$ are Dirac delta-functions. Fig. 2 shows the relation $u_3(0,t)$ (Fig. 2a) and $\sigma_\varphi(0,h,t)$ (Fig. 2b) from time. The dimensionless time τ is normalized on wave travel time of membrane stresses along shell radius, τ=tV/R, $V = \sqrt{E/[\rho(1-\nu^2)]}$. Here it is accepted $E = 6{,}12\cdot10^4$ MPa, $\rho = 2{,}5\cdot10^3$ kg m^{-3}, $\nu = 0{,}3$. The solid line corresponds to the GCT, dash-dot line relates to the analytical solution.

Due to wave character of the GCT, the stresses remain close to zero so long as the wave extending from a point of loading, will not reach a considered point on the shell.

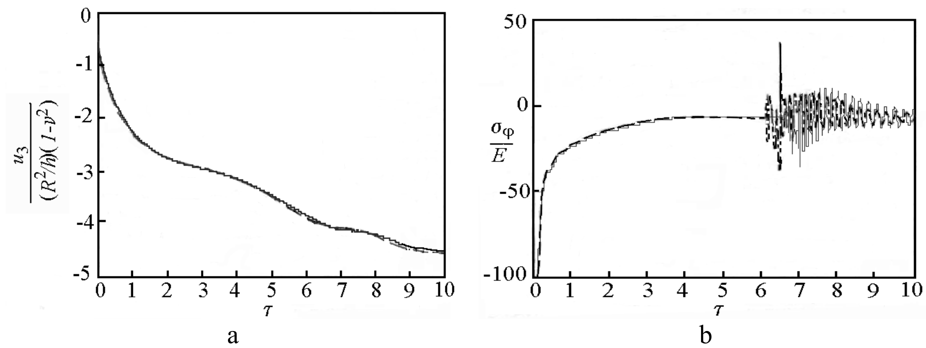


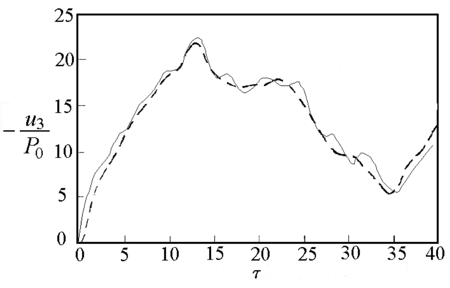
Fig. 2

For cylindrical shells of finite length the comparison with the exact solution introduced in [9] for $\nu = 0.3$; $h/R = 1/10$; $l/R = 5$ was conducted. The shell was subjected to external instantly applied to an outside surface radially directed loading

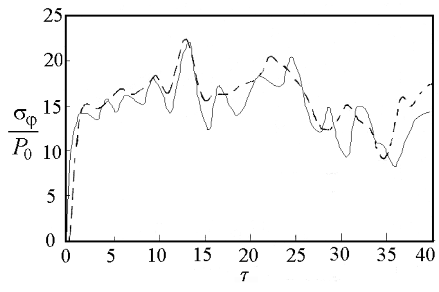
$$p^e(\varphi, x, t) = P_0 f(\varphi) g(x) H(t),\ \frac{L-\lambda}{2} \le x \le \frac{L+\lambda}{2};\ -\frac{\varepsilon}{2} \le \varphi \le \frac{\varepsilon}{2};\ \lambda{=}0.5R,\ \varepsilon{=}0.5.$$

Here P_0 is intensity of loading (0.1 MPa), $f(\varphi)$ and $g(x)$ are distributions of loading along coordinate φ and x, respectively.

In Fig. 3 (a, b) the response of a cylindrical shell to dynamic loading is shown. Fig. 3a displays va-riation of radial displacement normalized on R on median surface in time. In Fig. 3b the variation in time of hoop stresses on an external surface normalized on ρV^2, $V = \sqrt{E/[2\rho(1+\nu)]}$ is shown. The dimension-less time is normalized on travel time of shear wave along shell radius, $\tau = tV/R$. The material properties were selected as well as in the previous problem. The solid line corresponds to the GCT, dash-dot line relates to the analytical solution. Let us observe the good agreement between the theory GCT and theory of elasticity; the distinction between the appropriate results makes less than 0,5 % near to a maximum.



a



b

Fig. 3

Now we consider some application of GCT to research of vibration of multilayer plates of com-plicated form under impulse loading. The approach offered is based on the elastic immersion method [5].

The multilayer plate is assembled from I layers of constant thickness. In the coordinate plane $x0y$ the plate occupies a simply connected domain Ω having a boundary L of an arbitrary form: $x_L = x(\varphi)$, $y_L = y(\varphi)$, $\varphi_0 \le \varphi \le \varphi_1$. The plate is subjected to an external impulse load $\mathbf{P}^e = \{p_j^e(x,y,t)\}$ ($j = \overline{1,2I+3}$) distributed on domain $\Omega_p \subset \Omega$.

The original plate occupied domain Ω is immersed into an auxiliary enveloping plate having an identical layer composition. An auxiliary plate occupies domain $\Omega_E, \Omega \subset \Omega_E$ in the plane $x0y$. The enveloping plate is loaded within the limits of domain Ω the same way as the given plate. The contours L_E of enveloping plate and boundary conditions are selected so that it was possible to obtain a simple analytical solution. Here as the auxiliary plate we take a rectangular simply supported plate.

To ensure fulfillment of real boundary conditions to an auxiliary plate on a trace of boundary L some additional compensatory forces and moments $\mathbf{Q}^c = \{q^c_j(x,y,t)\}$ ($j = \overline{1,2I+3}$, $x,y \in L$) are applied. Hence, the initial problem of vibration of the plate Ω affected by a given impulse load is reduced to the problem of vibration of the rectangular plate Ω_E under the action of same impulse load and compensatory loads Q^c specified as curvilinear distributions $\mathbf{P}^c = \{p^c_j(x,y,t)\}$, $j = \overline{1,2I+3}$

$$\mathbf{P}^c(x,y,t) = \int_{\varphi_0}^{\varphi_1} \mathbf{Q}^c(\varphi,t)\delta(x - x_L, y - y_L)\Gamma(\varphi)\mathrm{d}\varphi,$$

where $\delta(x - x_L, y - y_L)$ is the two-dimensional Dirac function.

Compensatory and given loads as well as displacements are expanded into trigonometrical series in

domain Ω_E by functions satisfying simply supported conditions on the boundary L_E

$$p_j^e(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} p_j^e(t)B_{jmn}(x,y), p_j^c(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} p_j^c(t)B_{jmn}(x,y), j = \overline{1,2I+3},$$

where

$$B_{1mn} = \cos(\alpha_m x)\sin(\beta_n y), B_{2mn} = \sin(\alpha_m x)\cos(\beta_n y), B_{3mn} = \sin(\alpha_m x)\sin(\beta_n y),$$

$$B_{3+i,mn} = B_{1mn}, B_{3+I+i,mn} = B_{2mn}, \alpha_m = m\pi / A, \beta_n = n\pi / B, i = \overline{1, I}.$$

Compensatory loads are determined from the system of integral equations. For this purpose compensatory forces and moments Q^c are expanded into a single series along a trace of the boundary L

$$q_j^c = \sum_{\alpha=1,2} \sum_{\mu=0}^{\infty} f_{j\alpha\mu}(t) b_{\alpha\mu}(\varphi), j = \overline{1, 2I+3},$$

where $b_{1\mu} = \sin[\mu\gamma(\varphi)], b_{2\mu} = \cos[\mu\gamma(\varphi)], \gamma(\varphi) = 2\pi(\varphi - \varphi_0)/(\varphi_1 - \varphi_0), 0 \le \gamma(\varphi) \le 2\pi$.

A three-layer elliptical plate with parameters $\alpha = 0.2$ m, $\beta = 0.1$ m, $h_i = 5 \times 10^{-3}$ m (i = 1, 2, 3) is considered (Fig.4). The first and third layers are made of SG. The mechanical properties of material of the second layer are $E_2 = 2.8 \times 10^2$ MPa, $\nu_2 = 0.38$, $\rho_2 = 1.2 \times 10^3$ kg m^{-3}.

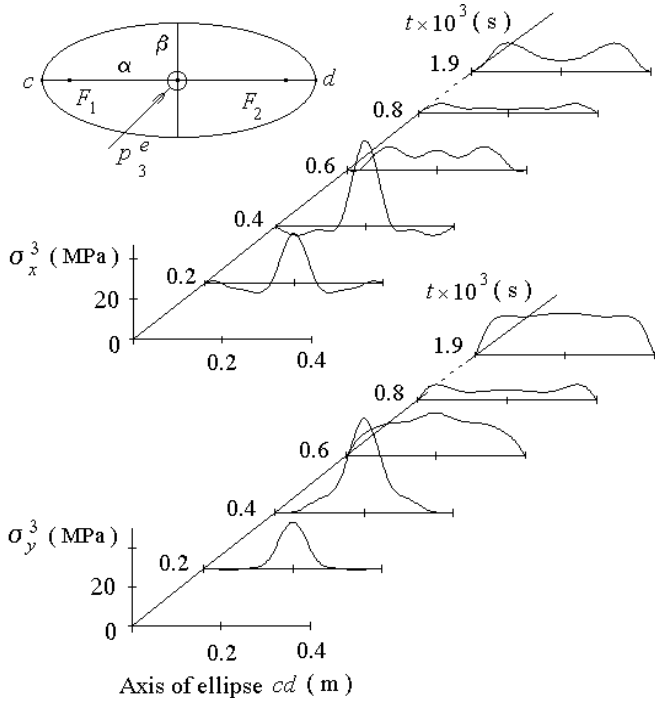


Fig. 4

An impulse load is uniformly distributed over a circular area with radius r and changes in time and coordinates according to the law

$$p_1^e = p_2^e = p_{3+i}^e = p_{3+I+i}^e = 0, \ x, y \in \Omega_p, \ \Omega_p : (x - x_0)^2 + (y - y_0)^2 \le r^2,$$

$$p_3^e(x,y,t) = 0.5P_0[1+\mathrm{sgn}(t_1-t)]\sin^2(\pi t/t_1)H(x)H(y),$$

where P_0 is load intensity, t_1 is impulse duration, $H(x)$, $H(y)$ are Heaviside functions. The characteristics of impulse load have the following values: $r = 10^{-2}$ m, $P_0 = 10$ MPa, $t_1 = 7 \times 10^{-4}$ s. The central point of loaded area coincides with the central point of the ellipse $x_0 = A/2$, $y_0 = B/2$, $z_0 = 0$. Dimensions of the enveloping plate are taken $A = B = 1$ m.

The distribution of normal stresses along an axis of the elliptical plate at different instants of time was studied: $\sigma_x^3 = \sigma_x^3(x,y,z,t)$, $\sigma_y^3 = \sigma_y^3(x,y,z,t)$, $A/2 - \alpha \le x \le A/2 + \alpha$, $y = B/2$, $z = \delta_3$, $t = t_k$. The calculation results are presented in Fig. 4. The stresses reach to maximum values in the instant of time $t_1 = 4 \times 10^{-4}$ s. Further the effect of stress concentration in both focal points F_1 and F_2 of the plate at $t_1 = 1.9 \times 10^{-3}$ s is observed.

The offered method allows to examine transient vibrations of multilayer plates of an arbitrary plan form with different boundary conditions.

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NESTACIONARNE VIBRACIJE VIŠESLOJNIH PLOČA I CILINDARSKIH LJUSPI. RAZLIČITE TEORIJE

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Rad predstavlja poređenje ishoda istraživanja naponsko deformacionih stanja višeslojnih ploča i ljuspi pri impulsnom opterećenju koji se dobijaju na osnovu ralićitih dvodimenzionalnih teorija i trodimenzionalnih teorija elastičnosti.

Za definiciju naponsko-deformacionog stanja (NPS) se primenjuje klasična teorija Ambartsu-myan-a, prvog reda refinisana teorija višeslojnih ploča i ljuspi, kao i neklasična refinisana teorija višeslojnih ploča zasnovanim na hipotezama kinematike, pretpostavljajući kubnu relaciju za tangencijalna pomeranja i kvadratnu relaciju za normalna pomeranja od transverzalne koordinate u svakom od slojeva.

Mogućnosti ovih teorija kao i važnost rezultata koji su dobijeni, prikazani su sa nekoliko primera procesa proračunavanja vibracija i procesa širenja elastičnih talasa. Uticaj savitljivosti ispune na napone u nosećim slojevima ploča i promena napona na debljinu paketa pri raspoređenim i lokalizovanim efektima se istražuje u ovom radu. Rezultati istraživanja dopuštaju da se utvrdi jedna klasa struktura i uslova njihovog opterećenja, kod kojih razmatrane dvodimenzijske teorije daju pouzdane rezultate.

Takođe, vibracija višeslojnih ploča komplikovanih oblika se razmatra pod dejstvom impulsnog opterećenja. Pristup koji se nudi je zasnovan na metodi elastičnog zadubljenja.