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ON CONDITIONS OF EXISTENCE OF PERMANENT ROTATIONS OF THE CONNECTED RIGID BODIES SYSTEM ABOUT THE VERTICAL VECTOR

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Abstract. *In the classical problem of motion of a heavy rigid body about a fixed point the permanent rotations are well known and completely investigated as the most simple and good visually demonstrated type of motions. Numerous properties of these motions are established and their theoretical and applied significance is commonly known (here the list of scientific references is so extensive that O. Staude's paper [1] must be singled out at first). In multibody mechanics, where under a increasing of the quantity of the system bodies the quantity of mechanical parameters and the order of differential motion equations are increasing too, the studying of conditions of existence of such motions is a complicated problem. This, apparently, is a reason in a view of which the problem on permanent rotations of coupled rigid bodies system does not have a exhaustive solution up to present time.*

The success of analytical investigations in different mechanics problems, especially in multibody system dynamics, is often caused by a good choice of a form of motion equations for studied object. In 1st section of this paper the new form of motion equations of the considered mechanical system is suggested. It is derived from P.V. Kharlamov's equations [2,3] under the using of the mechanical parameters of the augmented bodies [4-6] in these equations. The obtained equations have a more compact form suitable for its studying.

In second section for the system of n heavy rigid bodies which are sequentially jointed in a chain by ideal spherical joints the conditions of existence for the motions are determined when the each of the bodies permanently rotates about the vertical vector. Section 4 contains the analysis of these conditions in a general case when the bodies angular velocities are different. Under the investigation a prior conditions on the mass distribution of the bodies and a way of their jointing are not used. The most simple case of two bodies is studied in 3rd section in detail.

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1. MOTION EQUATIONS OF A CONSTRAINED SYSTEM OF N CONNECTED RIGID BODIES

We will consider a system of n heavy rigid bodies B_1, B_2, …, B_n. The bodies B_i and B_{i+1} (i=1,2,…,n−1) are coupled in a common point O_{i+1}. The last link of the bodies chain, B_1, is fixed in a point O_1 on the immovable base.

Let us assume that an external influence on the system is expressed by a force $\boldsymbol{F}_i$ and a moment $\boldsymbol{M}_i$ which are acting on B_i and applied in the suspension point O_i (i=1,2,…,n). In addition, we suppose that the body B_{i-1} acts on B_i with a force $\boldsymbol{R}_i$ and a moment $\boldsymbol{L}_i$, applied in O_i, and the body B_{i+1} affects on B_i with a force $-\boldsymbol{R}_{i+1}$ and a moment $-\boldsymbol{L}_{i+1}$, applied in O_{i+1}.

Among different forms of motion equations of n connected rigid bodies system, known up to present time, on our opinion P.V. Kharlamov's equations [2] are the most useful for analytic investigations of its dynamical properties. These equations are:

$$(\boldsymbol{A}_i\boldsymbol{\omega}_i)^{\bullet} + m_i\mathbf{c}_i \times \sum_{j=1}^{i-1}(\boldsymbol{\omega}_j \times \mathbf{s}_j)^{\bullet} + \mathbf{s}_i \times \sum_{j=i+1}^{n} m_j\left(\boldsymbol{\omega}_j \times \mathbf{c}_j + \sum_{k=1}^{j-1}\boldsymbol{\omega}_k \times \mathbf{s}_k\right)^{\bullet} = \\ = \boldsymbol{M}_i + \boldsymbol{L}_i - \boldsymbol{L}_{i+1} + \mathbf{s}_i \times \sum_{j=i+1}^{n}\boldsymbol{F}_j, \tag{1.1}$$

where $\boldsymbol{A}_i$ is an inertia tensor of the body B_i constructed in the point O_i; $\boldsymbol{\omega}_i$ is an absolute angular velocity of B_i; m_i is a mass of B_i; $\boldsymbol{c}_i = \boldsymbol{O}_i\boldsymbol{C}_i$, C_i is the mass center of B_i; $\boldsymbol{s}_i = \boldsymbol{O}_i\boldsymbol{O}_{i+1}$; the point in these and other equations of this section designates the absolute derivative. In the equations (1.1) and below we suppose that the index i takes all the values from the set, $N = \{1,2,\ldots,n\}$,of indexes of the system bodies.

In order to write the equations (1.1) in a more compact form, for the each B_i we consider the body B_i^* which represents the body B_i with the apparent additional mass $m_i^* = \sum_{j=i+1}^{n} m_j$ in the point O_{i+1}. This mechanical object had been successfully used for a description of mass characteristics of the considered bodies chain [7, 8]. Below, abiding by [4,5], where analogous objects had been introduced for bodies systems with a tree-like structure, we will call B_i^* as the augmented body for B_i in contrast to the same J. Wittenburg's definition [6].

In such a case the equality of the absolute angular velocities of B_i and B_i^* is the evident fact. Let us denote that $\boldsymbol{A}_i^*$ is an inertia tensor of B_i^* relatively to O_i; $\mathbf{c}_i^* = \mathbf{O}_i\mathbf{C}_i^*$, where C_i^* is the mass center of B_i^* named as the barycenter [4]; $\boldsymbol{a}_i = m_i\mathbf{c}_i + m_i^*\mathbf{s}_i$ is the static moment of B_i^* with respect to O_i. Hence

$$\boldsymbol{c}_i^* = \boldsymbol{a}_i/(m_i + m_i^*), \qquad \boldsymbol{A}_i^*\boldsymbol{\omega}_i = \boldsymbol{A}_i\boldsymbol{\omega}_i + m_i^*\boldsymbol{s}_i \times (\boldsymbol{\omega}_i \times \boldsymbol{s}_i)\,. \tag{1.2}$$

By means of the relations

$$(\boldsymbol{A}_i^*\boldsymbol{\omega}_i)^{\bullet} = (\boldsymbol{A}_i\boldsymbol{\omega}_i)^{\bullet} + m_i^*\boldsymbol{s}_i \times (\boldsymbol{\omega}_i \times \boldsymbol{s}_i)^{\bullet},$$

$$\sum_{j=i+1}^{n} m_j \sum_{k=i+1}^{j-1}(\boldsymbol{\omega}_k \times \boldsymbol{s}_k)^{\bullet} = \sum_{j=i+1}^{n-1}(\boldsymbol{\omega}_j \times \boldsymbol{s}_j)^{\bullet}\sum_{k=j+1}^{n} m_k = \sum_{j=i+1}^{n-1}(\boldsymbol{\omega}_j \times m_j^*\boldsymbol{s}_j)^{\bullet},$$

$$m_i \boldsymbol{c}_i \times \sum_{j=1}^{i-1} (\boldsymbol{\omega}_j \times \boldsymbol{s}_j)^{\bullet} + \mathbf{s}_i \times \sum_{j=i+1}^{n} m_j (\boldsymbol{\omega}_j \times \boldsymbol{c}_j + \sum_{k=1}^{j-1} \boldsymbol{\omega}_k \times \boldsymbol{s}_k)^{\bullet} =$$

$$= m_i \boldsymbol{c}_i \times \sum_{j=1}^{i-1} (\boldsymbol{\omega}_j \times \boldsymbol{s}_j)^{\bullet} + \boldsymbol{s}_i \times \sum_{j=i+1}^{n} m_j [(\boldsymbol{\omega}_j \times \boldsymbol{c}_j)^{\bullet} + \sum_{k=1}^{i} (\boldsymbol{\omega}_k \times \boldsymbol{s}_k)^{\bullet} + \sum_{k=i+1}^{j-1} (\boldsymbol{\omega}_k \times \boldsymbol{s}_k)^{\bullet}] =$$

$$= \boldsymbol{a}_i \times \sum_{j=1}^{i-1} (\boldsymbol{\omega}_j \times \boldsymbol{s}_j)^{\bullet} + m_i^* \boldsymbol{s}_i \times (\boldsymbol{\omega}_i \times \boldsymbol{s}_i)^{\bullet} + \mathbf{s}_i \times \sum_{j=i+1}^{n} (\boldsymbol{\omega}_j \times \boldsymbol{a}_j)^{\bullet},$$

we can accomplish the transition to parameters of the augmented bodies in the equations (1.1). As result we derive:

$$(\boldsymbol{A}_i^* \boldsymbol{\omega}_i)^{\bullet} + \boldsymbol{a}_i \times \sum_{j=1}^{i-1} (\boldsymbol{\omega}_j \times \boldsymbol{s}_j)^{\bullet} + \mathbf{s}_i \times \sum_{j=i+1}^{n} (\boldsymbol{\omega}_j \times \boldsymbol{a}_j)^{\bullet} = \boldsymbol{M}_i + \boldsymbol{L}_i - \boldsymbol{L}_{i+1} + \boldsymbol{s}_i \times \sum_{j=i+1}^{n} \boldsymbol{F}_j. \quad (1.3)$$

We note that the bodies B_n and B_n^* are the same one (since $m_n^* = 0$). Then by virtue of (1.2) we have $\boldsymbol{A}_n^* = \boldsymbol{A}_n,\ \boldsymbol{a}_n = m_n \boldsymbol{c}_n$.

If the bodies of the system are coupled by elastic joints then the equations (1.1) take the form:

$$(\boldsymbol{A}_i^* \boldsymbol{\omega}_i)^{\bullet} + \boldsymbol{a}_i \times \sum_{j=1}^{i-1} (\boldsymbol{\omega}_j \times \boldsymbol{s}_j)^{\bullet} + \mathbf{s}_i \times \sum_{j=i+1}^{n} (\boldsymbol{\omega}_j \times \boldsymbol{a}_j)^{\bullet} =$$
$$= \boldsymbol{M}_i + \boldsymbol{L}_i^1 - \boldsymbol{L}_{i+1}^1 + \boldsymbol{L}_i^2 - \boldsymbol{L}_{i+1}^2 + \boldsymbol{s}_i \times \sum_{j=i+1}^{n} \boldsymbol{F}_j. \quad (1.4)$$

Here $\boldsymbol{L}_i^1$ and $\boldsymbol{L}_i^2$ are respectively the elastic moment and the moment of the constraint reaction in O_i characterizing the affecting of B_{i-1} on B_i.

The equations system (1.3) as well as the system (1.1) is not closed in a general case: in addition to $\boldsymbol{\omega}_i$ it has unknown moments $\boldsymbol{L}_i$ too. So for its closure an information about an interaction character between B_i and B_{i-1} and a constraint in the point O_1 is needed.

In the particular case, when all the bodies are coupled by spherical joints $(\boldsymbol{L}_i^2 = 0)$ in which the elastic regenerating moments $\boldsymbol{L}_i^1 = -\kappa^2 (\boldsymbol{e}_{i-1} \times \boldsymbol{e}_i)$ are acting for $i = 2,3,\ldots,n$ ($\boldsymbol{e}_i = \boldsymbol{s}_i / |\boldsymbol{s}_i|$), then the equations (1.4) yield

$$(\boldsymbol{A}_i^* \boldsymbol{\omega}_i)^{\bullet} + \boldsymbol{a}_i \times \sum_{j=1}^{i-1} (\boldsymbol{\omega}_j \times \boldsymbol{s}_j)^{\bullet} + \mathbf{s}_i \times \sum_{j=i+1}^{n} (\boldsymbol{\omega}_j \times \boldsymbol{a}_j)^{\bullet} =$$
$$= \boldsymbol{M}_i + \boldsymbol{s}_i \times \sum_{j=i+1}^{n} \boldsymbol{F}_j - \kappa^2 (\boldsymbol{e}_{i-1} \times \boldsymbol{e}_i - \boldsymbol{e}_i \times \boldsymbol{e}_{i+1}).$$

At last, if in the points O_i there are ideal spherical joints and the external forces affecting on the system are the supporting force in O_1 and the gravity force (i.e. $\boldsymbol{L}_i = 0$, $\boldsymbol{F}_i = m_i g \boldsymbol{v}$, $\boldsymbol{M}_i = m_i g \boldsymbol{c}_i \times \boldsymbol{v}$, where $\boldsymbol{v}$ is the unit vector of the gravity force, g is its acceleration), then the equations (1.3) are presented in the kind:

$$(\boldsymbol{A}_i^* \boldsymbol{\omega}_i)^{\bullet} + \boldsymbol{a}_i \times [\sum_{j=1}^{i-1} (\boldsymbol{\omega}_j \times \boldsymbol{s}_j)^{\bullet} - g\,\mathbf{v}] + \boldsymbol{s}_i \times \sum_{j=i+1}^{n} (\boldsymbol{\omega}_j \times \boldsymbol{a}_j)^{\bullet} = 0. \quad (1.5)$$

2. Permanent rotations of the system about the vertical vector for the problem (1.5)

We will assume below that the each body from the studied system rotates with a constant angular velocity. Such motions of the system are named to be permanent rotations. Then for the body B_i the vector $\mathbf{v}_i$ fixed in the inertial space exists:

$$\boldsymbol{\omega}_i = \omega_i\ \mathbf{v}_i,\ \ \omega_i = const. \tag{2.1}$$

Taking into account the known equality: $\dot{\mathbf{v}}_i = \mathbf{v}_i' + \boldsymbol{\omega}_i \times \mathbf{v}_i$ (the prime designates the relative derivative on time), by virtue of (2.1) we conclude that the vector $\mathbf{v}_i$ as well as the vector $\boldsymbol{\omega}_i$ will be constant in the coordinate system rigidly associated with B_i. Therefore, based on (2.1) we transfer the equations (1.5) to the moving axes:

$$\omega_i^2\ (\mathbf{v}_i \times \boldsymbol{A}_i^* \mathbf{v}_i) + \boldsymbol{a}_i \times \left[\sum_{j=1}^{i-1} \omega_j^2 [\mathbf{v}_j \times (\mathbf{v}_j \times \boldsymbol{s}_j)] - g\ \mathbf{v} \right] + \boldsymbol{s}_i \times \sum_{j=i+1}^{n} \omega_j^2\ [\mathbf{v}_j \times (\mathbf{v}_j \times \boldsymbol{a}_j)] = 0. \tag{2.2}$$

Next, if all the vectors $\mathbf{v}_i$ are collinear to an axis, then the system performs the permanent rotations about this axis. In particular, for

$$\mathbf{v}_i = \mathbf{v} \tag{2.3}$$

the system permanently rotates about the vertical vector. In the following bellow we will consider only such motions.

Let us introduce the orthonormal basis fixed with B_i in the point O_i so that the vector $\boldsymbol{v}$ coincides with the ort of the third axis. In this basis we put $\boldsymbol{a}_i = (a_i^1, a_i^2, a_i^3)$, $\boldsymbol{s}_i = (s_i^1, s_i^2, s_i^3)$ and $\boldsymbol{A}_i^* \mathbf{v} = (-A_{13}^{*i}, -A_{23}^{*i}, A_{33}^{*i})$. Then, projecting the equations (2.2) on the axes of the corresponding mobile basises with respect to (2.3), we obtain such relations:

$$\alpha_i^k + \sum_{j \neq i} \omega_j^2\ [\beta_{ij}^k\ \cos(\omega_i - \omega_j)t + \gamma_{ij}^k\ \sin(\omega_i - \omega_j)t] = 0,\ k=1,2,3, \tag{2.4}$$

where:

$$\alpha_i^k = \begin{cases} A_{k3}^{*i}\ \omega_i^2 - g a_i^k, & k = 1,2, \\ 0, & k = 3 \end{cases}$$

$$\beta_{ij}^1 = -\gamma_{ij}^2 = \begin{cases} a_i^3 s_j^1, & j < i, \\ s_i^3 a_j^1, & j > i, \end{cases} \quad \beta_{ij}^2 = \gamma_{ij}^1 = \begin{cases} a_i^3 s_j^2, & j < i, \\ s_i^3 a_j^2, & j > i, \end{cases} \tag{2.5}$$

$$\beta_{ij}^3 = \begin{cases} a_i^2 s_j^1 - a_i^1 s_j^2, & j < i, \\ a_j^1 s_i^2 - a_j^2 s_i^1, & j > i, \end{cases} \quad \gamma_{ij}^3 = \begin{cases} a_i^1 s_j^1 + a_i^2 s_j^2, & j < i, \\ a_j^1 s_i^1 + a_j^2 s_i^2, & j > i. \end{cases}$$

It is obviously that the conditions of compatibility for the system (2.4) are the conditions of existence of permanent rotations of n connected rigid bodies system.

By implication of the problem we point out that

$$\left(s_i^1\right)^2 + \left(s_i^2\right)^2 + \left(s_i^3\right)^2 \neq 0, \tag{2.6}$$

because in another case $\boldsymbol{s}_i = 0$ and it means that the body B_{i+1} is coupled with B_{i-1} in the point O_i but not with B_i.

3. THE CASE OF TWO BODIES SYSTEM

In this section we will analyze the formulas (2.4) for the most simple case when $n = 2$. Then these relations take the form:

$$\begin{aligned} &\overset{*1}{A_{l3}}\ \omega_1^2 - g a_1^l \ +\ s^3\ \omega_2^2\,[\,a_2^j\ \sin(\omega_l - \omega_j)t + a_2^l\ \cos(\omega_j - \omega_l)t\,]\ = 0, \\ &\overset{*2}{A_{l3}}\ \omega_2^2 - g a_2^l + a_2^3\,\omega_1^2\,[\,s^j\ \sin(\omega_j - \omega_l)\ t\ +\ s^l\ \cos\ (\omega_j - \omega_l)t\,]\ = 0, \qquad (3.1) \\ &\omega_l^2\ [(a_2^1 s^1 + a_2^2 s^2)\sin(\omega_1 - \omega_2)t - (a_2^2 s^1 - a_2^1 s^2)\cos(\omega_1 - \omega_2)t]\ = 0. \end{aligned}$$

In (3.1) we have $s^k = s_1^k$, $k = 1,2,3,\ l, j \in \{1,2\}$, $l \neq j$.

Further, we isolate the four cases:

1. $\omega_1 \neq \omega_2$, $\omega_1 \neq 0$, $\omega_2 \neq 0$ The equalities (3.1) will be fulfilled at the every moment of time if and only if

$$\overset{*1}{A_{13}}\,\omega_1^2 - g a_1^1 = 0,\quad \overset{*1}{A_{23}}\,\omega_1^2 - g a_1^2 = 0, \qquad (3.2)$$

$$A_{13}^2\,\omega_2^2 - g a_2^1 = 0,\quad A_{23}^2\,\omega_2^2 - g a_2^2 = 0, \qquad (3.3)$$

$$a_2^1 s^1 + a_2^2 s^2 = 0,\quad a_2^2 s^1 - a_2^1 s^2 = 0, \qquad (3.4)$$

$$a_2^1 s^3 = a_2^2 s^3 = a_2^3 s^1 = a_2^3 s^2 = 0\,. \qquad (3.5)$$

Here and below we have $\mathbf{A}_2\mathbf{v} = (-A_{13}^2, -A_{23}^2, A_{33}^2)$, $\mathbf{c}_2 = (c_2^1, c_2^2, c_2^3)$, where these vectors are represented as seen from the frame fixed in the body B_2.

One can easy check that by means of (2.6) the relations (3.3)–(3.5) will be satisfied by one of the ways:

$$c_2^1 = c_2^2 = s^1 = s^2 = A_{13}^2 = A_{23}^2 = 0 \qquad (3.6)$$

or

$$c_2^1 = c_2^2 = c_2^3 = A_{13}^2 = A_{23}^2 = 0. \qquad (3.7)$$

From the formulas (3.6), (3.7) we infer that under $\omega_1 \neq \omega_2$ the body B_2 performs permanent rotations only about one of its principle inertia axes. In such a case the mass center C_2 of this body should either be placed on the rotation axis ($\boldsymbol{c}_2 \parallel \mathbf{v}$ by virtue of (3.6)) or put in the suspension point O_2 ($\boldsymbol{c}_2 = 0$ according to (3.7)). For the first considered subcase the point O_2 should be located on the vertical line passing through O_1 (from (3.6) we have $\boldsymbol{s} \parallel \mathbf{v}$).

Comparing (3.2) with the relations (4.2) of the paper [9] we conclude that the body B_1 permanently rotates about an axis which contains the suspension point O_2 and belongs to the Staude's cone having been built for B_1^*. This analogue with the classical problem of rigid body mechanics will be explained as soon as we keep in mind the fact that the motion of the body B_2 does not make any influence on a motion of B_1^* when the bodies parameters are chosen with respect to the relations (3.6) or (3.7).

For this subcase the obtained conditions of existence coincide with the results of the paper [8]. From following above reasoning we deduce their uniqueness.

2. $\omega_1 \neq 0$, $\omega_2 \neq 0$. The system (3.1) can be satisfied by the relations (3.3), (3.4) and $a_1^1 = a_1^2 = \ = a_2^1 s^3 = a_2^2 s^3 = 0$. Hence

$$a_1^1 = a_1^2 = c_2^1 = c_2^2 = A_{13}^2 = A_{23}^2 = 0\,. \tag{3.8}$$

Therefore, in this case the body B_2 will permanently rotate about one of its principle barycentric axes if the barycenter of the resting body B_1^* is situated on the vertical line passing through O_1. Moreover, in general, the vector $\boldsymbol{s}$ is not collinear to $\mathbf{v}$. It is easy to show that the accomplishment of the equalities (3.8) provides the vanishing of the moment of forces which are external by respect to B_1^* relatively to the point O_1.

3. $\omega_1 \neq 0$, $\omega_2 = 0$. Now from (3.1) we get the relations (3.2), (3.4) and $a_2^1 = a_2^2 = a_2^3 s^1 = = a_2^3 s^2 = 0$. These relations can be reduced to the same conditions as for the first case. Indeed, this fact is evident because under the fulfillment of these conditions the motion of B_2 does not affect on a motion of B_1^*. This property will be valid for any values of ω_2 including $\omega_2 = 0$

4. $\omega_1 = \omega_2 = \omega$. The system (3.1) is transformed to the system of algebraic relations:

$$(A_{l3}^{*1} + s^3 a_2^l)\,\omega^2 - g a_1^l = 0, \quad (A_{l3}^{*2} + s^l a_2^3)\,\omega^2 - g a_2^l = 0,$$
$$l=1,2,\ a_2^2 s^1 - a_2^1 s^2 = 0\,,$$

which is compatible under the conditions

$$\frac{A_{l3}^{*1} + s^3 a_2^l}{A_{23}^2 + s^2 a_2^3} = \frac{a_1^l}{a_2^2},\ l = 1,2,\ \frac{A_{13}^2 + s^1 a_2^3}{A_{23}^2 + s^2 a_2^3} = \frac{a_2^1}{a_2^2},\ a_2^2 s^1 - a_2^1 s^2 = 0\,.$$

We note that the two last equalities can be presented in the kind:

$$\frac{A_{13}^2}{A_{23}^2} = \frac{c_2^1}{c_2^2} = \frac{s^1}{s^2}\,.$$

4. ABOUT COMPATIBILITY OF THE CONDITIONS SYSTEM (2.4)

In this section we will define the conditions of the compatibility of the system (2.4). For this purpose we formulate the following statement.

Statement. Let all the values μ_l ($l = 1, \ldots, p$) are chosen so that :

1. $\mu_l \neq 0 \quad \forall\, l$; (4.1)

2. $\mu_i^2 \neq \mu_j^2 \quad \forall\, i \neq j$. (4.2)

Then the function

$$f(t) = a_0 + \sum_{l=1}^{p} (a_l \cos \mu_l t + b_l \sin \mu_l t) \tag{4.3}$$

is identically equal to zero if and only if $a_0 = a_l = b_l = 0$ $(l = 1,\ldots, p)$.

Proof. We will prove this statement for all natural values of the parameter p by the method of mathematical induction. It is easy to make sure that for $p = 1$ the statement is correct. Then assuming that the statement is valid for $p = p_*$ we shall demonstrate it for $p = p_* + 1$. If the function $f(t)$ is identically equal to zero then all its derivatives must be identically equal to zero too, therefore from (4.3) we get

$$\ddot{f}(t) = -\sum_{l=1}^{p_*+1} \mu_l^2 (a_l \cos \mu_l t + b_l \sin \mu_l t) \equiv 0\,. \tag{4.4}$$

The identity (4.4) with respect to (4.1) is written in a kind:

$$a_{p_*+1} \cos \mu_{p_*+1} t + b_{p_*+1} \sin \mu_{p_*+1} t \equiv -\sum_{l=1}^{p_*} \frac{\mu_l^2}{\mu_{p_*+1}^2} \big(a_l \cos \mu_l t + b_l \sin \mu_l t\big)\,. \tag{4.5}$$

On the base of (4.5) the relation $f(t)\equiv 0$ takes the form:

$$a_0 + \sum_{l=1}^{p_*} (1 - \mu_l^2 / \mu_{p_*+1}^2)(a_l \cos \mu_l t + b_l \sin \mu_l t) \equiv 0. \tag{4.6}$$

As the statement is valid for $p = p_*$, then the identity (4.6) is correct only under satisfaction of the conditions:

$$a_0 = 0, \quad (1 - \mu_l^2 / \mu_{p_*+1}^2)\, a_l = 0, \quad (1 - \mu_l^2 / \mu_{p_*+1}^2)\, b_l = 0 \ \ (l = 1,\ldots,p). \tag{4.7}$$

Further, by virtue of (4.2) we have $a_0 = a_l = b_l = 0$ $(l = 1,\ldots, p)$. Then $f(t) = a_{p_*+1} \cos \mu_{p_*+1} t + b_{p_*+1} \sin \mu_{p_*+1} t$. It is easy to see that the function $f(t)$ will be identically equal to zero under the conditions $a_{p_*+1} = b_{p_*+1} = 0$ and, so, the statement is proved.

For the case, when the one of the inequalities (4.1), (4.2) becomes the equality, the function $f(t)$ after the regrouping of sum terms will be presented in the kind which meets the requirements of the statement and, therefore, can be identified on its base.

Using this statement we infer that in a general case, when

$$\omega_j \neq 0,\ \ \omega_j \neq \omega_k,\ \ \omega_j - \omega_k \neq 2\omega_l,\ \ \forall\ j,k,l \in N\,, \tag{4.8}$$

we are able to satisfy the system (2.4) by the relations

$$\alpha_i^k = \beta_{ij}^k = \gamma_{ij}^k = 0 \ \ i, j \in N,\ i \neq j,\ k=1,2,3. \tag{4.9}$$

The relations (4.9) are analyzed and under its satisfaction the conditions on the system bodies parameters are determined. Here we will not cite the detailed analysis but give a mechanical interpretation of the obtained results.

Let k is an arbitrary fixed index $(1 < k \leq n)$. Let us select two subsystems, $\Sigma_1 = \{B_2, B_3, \ldots, B_{k-1}\}$ and $\Sigma_2 = \{B_k, B_{k+1}, \ldots, B_n\}$, from the considered mechanical system. Then the studied bodies chain will permanently rotate about the vertical vector if and only if :

i) the rotation axis of the body B_j from Σ_1 or Σ_2 is a principle inertia axis for B_j^* ($A_{13}^{*j} = A_{23}^{*j} = 0$, $j = 2, \ldots, n$);

ii) the bodies of the system Σ_2 are suspended in the own barycenters, i.e. the point O_j coincides with C_j^* for $j = k, k+1, \ldots, n$ ($a_j^1 = a_j^2 = a_j^3 = 0$);

iii) for the each body B_j from Σ_1 the suspension point O_j and the barycenter C_j^* should be placed on the system rotation axis, i.e. on the vertical line passing through the fixed point O_1 ($s_j^1 = s_j^2 = 0$, $a_j^1 = a_j^2 = 0$, $j = 2,3,\ldots, k-1$);

iv) as in the case of the two bodies system the body B_1 permanently rotates about an axis of the Staude's cone having been built for the body B_1^*; if the set of indexes for Σ_1 is not empty, then this axis contains the point O_2 and by this means it is isolated.

Next, we shall give all the values between 2 and n to the index k then under the fulfillment of the conditions (4.8) we will obtain all possible cases of existence of permanent rotations about the vertical vector of the n rigid bodies system.

REFERENCES

1. Staude, O. *Über permanente Rotationsaxen bei der Bewegung eines schweren Körpers um einen festen Punkt*. Journ. für die reine und angew. Math., **113** (1894), 318-334.
2. Kharlamov, P.V. *Motion equations of a rigid bodies system*, Mekhanika tv. tela, **4** (1972), 52-73.
3. Kharlamov, P.V., Kovalev, A.M. *Invariant relations method in multibody dynamics*, Nonlinear Analysis, **30 (**1997), 3817-3828.
4. Raucent, B., Samin, J.C. *Minimal parameterization of robot dynamic models*, Mech. Structures Mach.,**22** (1994), 371–396.
5. Oliviers, M., Campion,G., Samin, J.C. *Nonlinear dynamic model of a systems of flexible bodies using augmented bodies*, Multibody System Dynamics, **2** (1998), 25-48.
6. Wittenburg, J. *Dynamics of Systems of Rigid Bodies*, B.G.Tebner, Stuttgart, 1997.
7. Savchenko, A.Ya, Bolgrabskaya, I.A., Kononykhin, G.A. *Stability of motion of coupled bodies system*, Kiev: Naukova dumka, 1991.
8. Lesina, M.Eph. *On the problem of two connected bodies*, Mekhanika tv. tela, **17** (1985), 46-49.
9. Kharlamov, P.V. *On permanent rotations of a rigid body with a fixed point*, Appl. Math. and Mech., **29** (1965), 52-73.

O USLOVIMA POSTOJANJA NEPRESTANE ROTACIJE SISTEMA NEPOKRETNIH KRUTIH TELA OKO VERTIKALNOG VEKTORA

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U klasičnom problemu kretanja teškog krutog tela oko nepokretne talke, neprestane rotacije su dobro poznate i potpuno istražene kao najprostiji i vizuelno dobro predstavljeni tip kretanja. Mnogobrojne osobine ovih kretanja su utvrđene i njihov teoretski i primenjeni značaj je uopšteno poznat (ovde je lista naučnih referenci tako iscrpna da rad Staude-a mora biti istaknut pre svih). U mehanici sistema tela gde pri porastu broja tela sistema raste i broj mehaničkih parametara kao i red diferencijalnih jednačina kretanja, proučavanje uslova postojanja takvih kretanja je jedan komplikovan problem.

Uspeh analitičkog istraživanja u različitim mehaničkim problemima, naročito u dinamici sistema mnogostrukih tela je često prouzrokovan dobrim izborom oblika jednačina kretanja za proučavani objekat. U prvom delu ovog rada razmatra se novi oblik jednačina kretanja razmatra-

nog mehaničkog sistema. On je izveden iz jednačina P.V. Kharlamova korišćenjem mehaničkih parametara uvećanih tela [4,6] u ovim jednačinama. Dobijene jednačine imaju mnogo kompaktniji oblik koji je pogodan za njihovo proučavanje.

U drugom delu rada su određeni uslovi za postojanje kretanja za sistem od n teških krutih tela koji su jedan za drugim u nizu povezani u lanac idealnim sfernim zglobovima, te su ti uslovi određeni kada svako od ovih tela neprekidno rotira oko vertikalnog vektora. Odeljak broj 4 sadrži analizu ovih uslova u opštem slučaju kada su ugaone brzine ovih tela različite. U istraživanje a priori uslova o raspodeli mase i načinu njihovog spajanja se nije ulazilo. Najprostiji slučaj dva tela je proučen u trećem odeljku do detalja.