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ON THE GAS LUBRICATION BY INJECTION THROUGH A PERMEABLE WALL

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Abstract. *Classical problem of flow in a gas lubricating slider bearing is considered in the paper under the assumption that the pad is made from a porous material, such as sintered metal, through which the gas is injected into the bearing under the constant ambient pressure. Both the isothermal, low Mach number gas flow in the bearing, and the same kind of flow through the porous pad, turn out to be coupled, but amenable to relatively simple analytical methods. By using Darcy's law for the flow through the porous pad, an analytic relation between the injection velocity and the pressure inside the bearing is obtained and is used for the numerical solution of the nonlinear differential equation governing the pressure. The obtained results show how the gas injection inside the bearing, even at small rates, highly increases its load, and how they can be applied in the design of externally pressurized gas bearings.*

1. INTRODUCTION

As it was shown by Montgomery and Sterry [1] in their experiments on gas-lubricated porous journal bearings in 1955, externally pressurized porous gas bearings have several advantages over conventional discrete hole admission bearings. They are simpler in construction and have better load capacities, and damping and stability properties. Nowadays they are widely used in industrial applications and further experimentally and theoretically investigated. Both incompressible oil flow and compressible gas flow in the bearing are treated. At that the flow in the bearing is usually modeled as an inertia-free flow, while the flow in the porous coating is supposed to obey classical Darcy's law. For small injection velocities both flows are coupled in the sense that neither of the two problems involved: flow in the bearing and flow the porous coating, can be solved separately, which makes the problem extremely difficult and amenable to approximate analytic methods or numerical techniques only.

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In this paper we treat a simplified problem of gas lubricating bearing: gas flow in a slider bearing with a porous pad exposed to a constant external pressure, and from this point of view the performed analysis represents an extension of the results obtained in a previous paper [2] concerned with an arbitrarily prescribed gas injection into the bearing. Under the assumption of an isothermal, low Mach number flow in the bearing, and Darcy's flow with prescribed permeability coefficient in the pad, we demonstrate how the strong interaction between these two dynamically different problems can be solved exactly. We consider also the injection of the gas into the bearing through a series of narrow slits in the pad, and show the existence of a full analogy between this case and the previous one, provided the friction factor for the flow through slits is inversely proportional to the local value of the Reynolds number, so that an equivalent value of the permeability coefficient can be found.

2. Problem statement and governing equations for the free flow

We consider the problem depicted in Fig. 1 in which a spontaneous injection/suction of the gas through a porous pad takes place under the variable pressure difference between the external, constant pressure p_a and the variable pressure inside the bearing, in order to improve the performance of the bearing. The flow in the bearing is supposed to be a steady, 2-D, isothermal, compressible flow of a perfect gas. Such a flow is governed by continuity equation, momentum equation in x and y directions (s. Fig. 1), and the equation of state. They will be written in nondimensional form by using the following scales (s. Fig. 1): δ_0 for all lengths, speed of the runner u_0 for all velocities, and pressure and density at the entrance into the bearing, p_0 and ρ_0, respectively, for pressure and density. In order to simplify this system of equations even before we write them down, we will now make the following assumption, which can be always accepted in the theory of lubrication. Let the maximum angle of inclination of the pad contour toward the x-axis, α_{max} (s. Fig. 1), be small enough, so that it can serve as a small parameter $\varepsilon : \alpha_{max} = \varepsilon$. In this case the local thickness of the gas film $\delta(x)$ will be a slowly varying function of x, and all physical quantities, like both velocity components, pressure and density will be also slowly varying function of x. To make these slow variations explicit, we will introduce the following slow coordinate $\xi = \varepsilon x$, instead of x. Also, since the inclination of the pad contour actually determines the ratio between velocity components u and v in x and y direction, respectively, v will be much less then u throughout the bearing, so that we can write: $v(x,y) = \varepsilon V(\xi,y)$, where $V(\xi,y)$ is an order one transverse velocity component. Further, we will assume that $\gamma M_0^2 / \mathrm{Re} = \lambda\varepsilon$, $\lambda = O(1)$, where γ is the

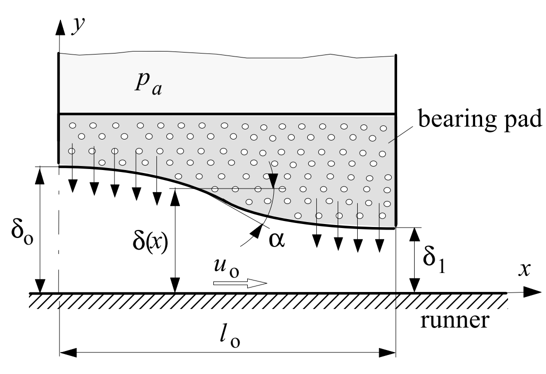

Fig. 1. Slider bearing with gas injection/suction through the bearing pad.

ratio of specific heats, M_0 is the reference Mach number defined as: $M_0 = u_0 / \sqrt{\gamma p_0 / \rho_0}$, and Re is the reference Reynolds number: $\mathrm{Re} = \rho_0 u_0 \delta_0 / \mu$ (μ is constant viscosity).

Simplified governing equations in nondimentional form will now read (some of denotations used for dimensional quantities in Fig. 1 are retained for simplicity!):

- continuity equation in which equation of state for isothermal flow in the form: $p = \rho$ is used,

$$\frac{\partial(pu)}{\partial\xi} + \frac{\partial(pV)}{\partial y} = 0 \tag{1}$$

- momentum equation in x-direction,

$$\gamma M_0^2 p \left(u \frac{\partial u}{\partial \xi} + V \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial \xi} + \lambda \frac{\partial^2 u}{\partial y^2} + O(\varepsilon^2) \tag{2}$$

- momentum equation in y-direction,

$$\frac{\partial p}{\partial y} = O(\varepsilon^2) \ . \tag{3}$$

Obviously, for high subsonic and supersonic flow inertia term in (2) is of the same order of magnitude as the dominant viscous term, and the problem is one of boundary layer type. However, for low subsonic Mach numbers inertia term can be neglected, and the flow is viscosity dominated. This case is particularly simple because equation (2), taking into account (3), can be easily integrated. Employment of no-slip boundary conditions: for $y = 0$, $u = 1$, and for $y = \delta(\xi)$, $u = 0$, then yields:

$$u = 1 - \left(1 + \frac{p'\delta^2}{2\lambda} \right) \frac{y}{\delta} + \frac{p'\delta^2}{2\lambda} \frac{y^2}{\delta^2} \ , \tag{4}$$

where $p' = \mathrm{d}p/\mathrm{d}\xi$. Strictly speaking the use of no-slip boundary condition at a permeable wall is not correct, as shown by Beavers and Joseph [3], because there exists a slip at such a wall which is proportional to the shear stress exerted by the fluid on the wall. However, for materials characterized by extremely small permeability coefficients, of which bearing coatings are usually made, such as sintered metals, coefficient of proportionality in the definition of the slip condition attains so small values that, to the degree of approximation already made here, the slip effect can be ignored.

Since we are primarily interested in the derivation of an equation for the pressure distribution inside the bearing, we will now circumvent the determination of V from (1). We will simply integrate (1) in y from 0 to $\delta(\xi)$, apply the boundary conditions: $y = 0$, $V = 0$, and for $y = \delta(\xi)$, $V = V_0(\xi)$, where $V_0(\xi)$ is unknown injection/suction velocity, to get:

$$\frac{d}{d\xi} \int_0^\delta pu \mathrm{d}y = -pV_0(\xi) \ .$$

Finally, utilizing (4) the following equation governing pressure is obtained:

$$\delta^3 p p'' + \delta^3 p'^2 + 3\delta(\delta\delta' p - 2\lambda) p' - 6\lambda(\delta' + 2V_0) p = 0 \ ,$$

with boundary conditions: for $\xi = 0$, $p = 1$, and for $\xi = L$, $p = 1$, where $L = \varepsilon l_0/\delta_0$ (s. Fig. 1). Since $v(x,y) = \varepsilon V(\xi,y)$, as stated earlier, note that the injection/suction velocity must be much smaller then the runner velocity in order for this theory to be applicable. For convenience of numerical integration of this equation we will introduce $X = \xi/L$ instead of ξ, and get:

$$\delta^3 p\frac{\mathrm{d}^2 p}{\mathrm{d}X^2} + \delta^3\frac{\mathrm{d}p}{\mathrm{d}X} + 3\delta(\delta\frac{\mathrm{d}\delta}{\mathrm{d}X}p - 2\lambda L)\frac{\mathrm{d}p}{\mathrm{d}X} - 6\lambda L\left[\frac{\mathrm{d}\delta}{\mathrm{d}X} + 2LV_0(X)\right]p = 0 \ , \tag{5}$$

with boundary conditions: for $X = 0$ and $X = 1$, $p = 1$. An interesting conclusion can now be drawn from (5). Even if $\delta = 1$ (Couette-like flow) some non-trivial pressure distribution inside the slider bearing can be induced by injection/suction of the fluid. At that, if $V_0(\mathrm{X}) \gtrless 0$ the pressure curve is concave/convex at the point of pressure extremum, indicating that $p \lessgtr 1$ inside the bearing, so that Couette-like flow with injection can still be used for lubricating purposes. Since for $\delta = 1$, $\alpha_{\max} = 0$, the definition of small parameter ε should be changed in this case. It can be redefined to be: $\varepsilon = \gamma M_0^2/\mathrm{Re}$, i.e. by choosing $\lambda = 1$. For $\lambda = 1$ $L = \varepsilon l_0/\delta_0 = \mu u_0 l_0/p_0\delta_0^2$ and plays the role of the bearing number $\Lambda = 6L$ in this problem, s. [4].

Of course, equation (5) cannot be solved before the velocity $V_0(X)$, determined by the flow through the pad, is specified.

3. Flow through the porous pad

The pad is supposed to be made of a homogenous, isotropic porous material with given permeability coefficient α, and the flow in the pad is supposed to be, lake in the bearing, steady, 2-D, isothermal, compressible flow of a perfect gas, obeying Darcy's law. Equations governing such a flow, if written in nondimensioal form by using the same scales, used already in the normalization of equations governing the flow inside the bearing, read:

$$\frac{\partial\tilde{p}}{\partial X} = -k\tilde{u} \ , \ \frac{\partial\tilde{p}}{\partial y} = -k\tilde{v} \ , \ \frac{\partial(\tilde{p}\tilde{u})}{\partial X} + \frac{\partial(\tilde{p}\tilde{v})}{\partial y} = 0 \ , \tag{6}$$

where $\tilde{p}$, $\tilde{u}$ and $\tilde{v}$ are pressure and velocity components in the bearing pad respectively, and

$$k = \frac{\delta_0^2}{\alpha}\frac{\gamma M_0^2}{\mathrm{Re}} = \frac{\delta_0^2}{\alpha}\lambda\varepsilon \ .$$

Since both transverse velocity components in the pad and in the bearing must be of the same order of magnitude, $\tilde{v}$ can be presented as $\tilde{v} = \varepsilon\tilde{V}$, $\tilde{V} = O(1)$. Then, from the second of equations (6) it follows that k must be of the order ε^{-1}, so that the order of α/δ_0^2 is ε^2. This determines the order of permeability coefficient for which the theory presented here is valid.

Further, we will introduce the slow coordinate $\xi = \varepsilon X$ instead of X for the same reason as before, and conclude that $\tilde{u} = O(\varepsilon^2)$, and is much smaller then $\tilde{v}$. Thus, the first order equations governing the flow in the pad are:

$$\frac{\partial\tilde{p}}{\partial y} = -k\tilde{v} \ , \ \frac{\partial(\tilde{p}\tilde{v})}{\partial y} = 0 \ ,$$

and can be readily solved with the boundary conditions (s. Fig. 1): for $y = b$, $\tilde{p} = p_a$, and $y = \delta(x)$, $\tilde{p} = p(\xi)$. The solutions are:

$$\tilde{p}^2 = \frac{b\,p^2 - \delta\,p_a^2}{b-\delta} + \frac{p_a^2 - p^2}{b-\delta}\,y \ , \ \tilde{V} = \beta\frac{p^2 - p_a^2}{(b-\delta)\,\tilde{p}} \ ,$$

where $\beta = \alpha / 2\lambda\delta_0^2\varepsilon^2 = O(1)$. Form here, for $y = \delta(\xi)$ we finally get the injection/suction velocity $V_0(\xi)$:

$$V_0(\xi) = \tilde{V}(\xi, \delta(\xi)) = \beta\frac{p^2 - p_a^2}{(b-\delta)p} \ , \tag{7}$$

to be used in the integration of equation (5).

Gas injection/suction through the pad can be maintained by a number of narrow slits, perpendicular to x - axis. If the flow in each of them is steady, 1-D, compressible, low Mach number flow, it is well known that the momentum equation for such a flow is (in dimensional form):

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}y} = -\frac{4\,\tau_w}{d} \ , \tag{8}$$

where d is diameter of the slit, τ_w is the local value of the wall shear stress: $\tau_w = f\,\tilde{\rho}\,\tilde{v}^2/2$, and f is the friction factor. In laminar, low Mach number, flows: $f = C/\mathrm{Re}$, where C is a constant (C=16 for pipes), and $\mathrm{Re} = \tilde{\rho}\,\tilde{v}\,d/\mu$ is the local Reynosld number. If written in nondimensional form, equation (8) attains now the form of the second of equation (6), provided $\alpha = d^2/32$, which at the same time yields the estimate $d/\delta_0 = O(\varepsilon)$, as a necessary condition for the validity of theory. The same holds for continuity equation for 1-D, isothermal flow, which is as well known: $\tilde{p}\,\tilde{v} = const$. Thus, the two problems are fully equivalent from the point of view of fluid mechanics, so that (dis)advantages of one method of injecting the gas into the bearing over the other should be sought in those characteristics of gas bearings, mentioned in the Introduction, which are not directly related to the flow problem.

4. RESULTS AND DISCUSSION

Equation (5) with $V_0(\xi)$determined by (7) is a second order nonlinear ordinary differential equation, with boundary conditions defined at two points. It can be solved by standard numerical techniques. Its numerical integration is performed in this paper for the simplest pad geometry of the form: $\delta = 1 - (1 - \delta_e)X$, where $\delta_e = \delta_1/\delta_0$ (s. Fig. 1). In this particular case $\varepsilon = \delta_0(1 - \delta_e)/l_0$, so that $L = 1 - \delta_e$ and cannot be chosen arbitrarily.

In Fig. 2 and Fig. 3 we present the results of the numerical integration of equation (5) for $\delta_e = 0{,}7$ and $\delta_e = 0{,}5$, respectively, for various p_a and fixed values of other parameters. It is seen that even for $p_a = 1$ (no difference between the pressure above the pad and outer pressure) there exists a small suction velocity produced by the pressure growth inside the bearing, pressure maximum being considerably smaller than in the classical case without any gas injection/suction into the bearing [4], or in the case of the gas suction with relatively small, constant velocity [2]. In both cases considered, pressure distribution inside the bearing is more uniform in comparison with the cases elaborated in [2] and [4].

Pressure growth with p_a is very pronounced, thus highly improving the performance of the bearing. This effect is particularly apparent for relatively small exit cross sections of the bearing, for which the pressure maximum is shifted to the right.

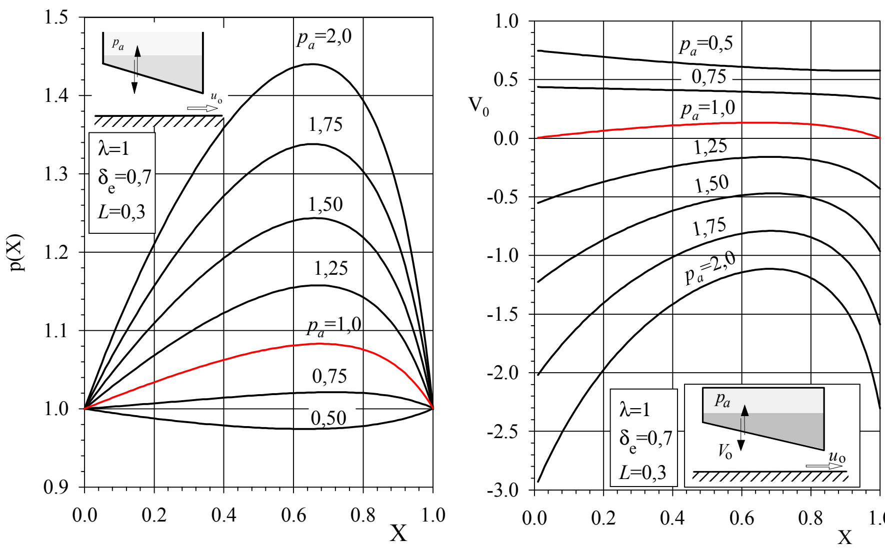

Fig. 2. Pressure and injection/suction velocity distributions inside a slider bearing for $\lambda = \beta = 1$, $b = 2$, $\delta_e = 0{,}7$, $L = 1 - \delta_e = 0{,}3$ and different p_a.

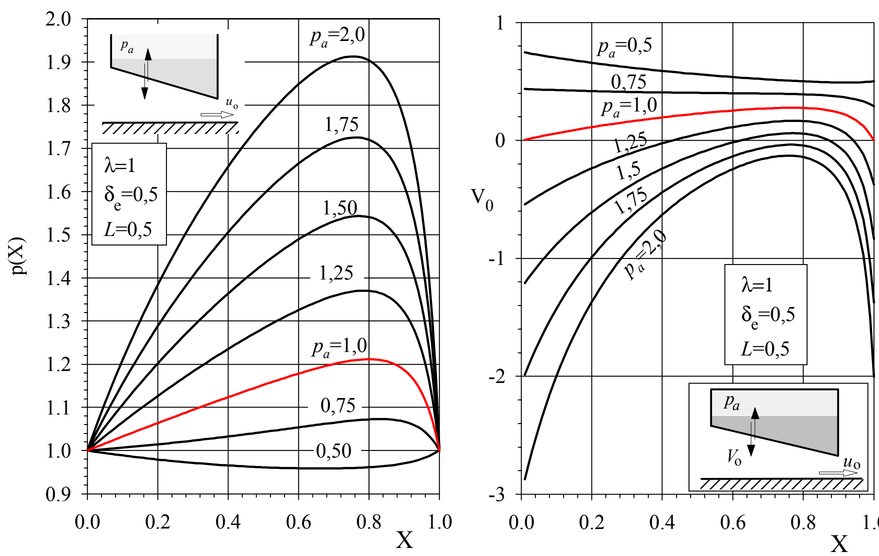

Fig. 3. Pressure and injection/suction velocity distributions inside a slider bearing for $\lambda = \beta = 1$, $b = 2$, $\delta_e = 0{,}5$, $L = 1 - \delta_e = 0{,}5$ and different p_a.

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O PODMAZIVANJU GASOM UBRIZGAVANJEM KROZ PROPUSTLJIVI ZID

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U radu se tretira klasični problem strujanja gasa u kliznom ležaju pod pretpostavkom da je nepokretni deo ležaja sačinjen od poroznog materijala, kao što je to na pr. sinterovani metal, kroz koji se gas ubrizgava u ležaj pod konstantnim spoljašnjim pritiskom. Ispostavlja se da su oba strujanja koja pri tome nastaju - izotermsko strujanje pri malom Mahovom broju u ležaju, i isto tako strujanje u poroznom nepokretnom delu ležaja, međusobno spregnuta, ali da se mogu tretirati pomoću relativno jednostavnih analitičkim metoda. Korišćenjem Darsijevog zakona za strujanje kroz poroznu sredinu dobija se analitička relacija koja povezuje brzinu ubrizgavanja sa pritiskom u ležaju, koja je zatim iskorišćena za numeričko rešavanje nelinearne diferencijalne jednačine kojom se opisuje raspored pritiska. Dobijeni rezultati pokazuju da ubrizgavanje gasa u ležaj, čak i pri relativno malim brzinama, značajno povećava nosivost ležaja, tako da se oni mogu korisno upotrebiti u konstruisanju ovakvih kliznih ležajeva.