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STABILITY PROBLEMS OF FUZZY-DESCRIPTOR SYSTEMS

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Abstract. *This paper discusses the general problem of stability and other solution properties of descriptor systems that contain fuzzy logic components, and presents a general methodology for such an analysis. The approach is based on the second method of Lyapunov and the generic concepts which describe system behaviour. Fuzzy logic components are allowed in the system model description. These and other system nonlinearities are used to enhance system analysis. Different algebraic conditions are presented for verification of specific time-evolution properties for all system descriptor variables. These conditions are based on partial algebraic constraints contained in the model.*

INTRODUCTION

Mathematical modelling of the behaviour of numerous technical and other systems has been revived in the last four decades mainly due to the introduction of some new methodologies, such as artificial neural networks and fuzzy logic (FL). This paper will discuss an approach to formulate the stability problem and to analyse it for systems that contain FL components. Instead of the ordinary state-space models, we will consider far more general descriptor models.

FL gained in popularity mainly due to the ability of representing rough models of system behaviour via linguistic description of rules governing that behaviour. It found numerous successful applications in many engineering and science fields. It is important to note that FL components can always be represented in terms of their input-output behaviour, thus avoiding an explicit indication of their internal FL operational mechanisms. Also, if the strict mathematical analysis of the dynamic behaviour of a FL system is required, then the description of the internal FL type functioning of the FL blocks, by use of membership functions, fuzzy inference rules and defuzzification methods, is not always sufficiently transparent for efficient analysis. In this paper we discuss this problem in the context of stability analysis and argue for the approach that, in

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a simple way, converts qualitative analysis of a very general class of FL dynamic systems to a 'conventional' qualitative analysis.

A number of techniques have been proposed for the qualitative analysis of FL systems [1-17]. In principle, from the input-output point of view, a FL block can be considered as a nonlinear algebraic transfer element [7-8]. Hence, without loss of generality, we can replace any such FL block with a nonlinear function and perform the analysis in a standard fashion. This provides the background for the most efficient analysis of the class of FL dynamic systems to be considered in this paper, and this approach will be followed.

MODELS OF FUZZY LOGIC COMPONENTS

Before we define the class of models to be analysed, let us consider arbitrary FL components whose internal behaviour is governed by

$$G_{fuzzy}(x^*, x_d^*, Dx^*, u^*) = Defuzz[Inference \circ Fuzz(x^*, x_d^*, Dx^*, u^*) . \tag{1}$$

Here x_d is a function that describes the desired behaviour of the descriptor vector x. Variables x^*, x_d^*, Dx^*, u^* represent the instantaneous crisp values of x, x_d, Dx and u, respectively. Operators *Fuzz*, *Inference* and *Defuzz* represent an arbitrary fuzzification mechanism, the associated inference rule base, and the arbitrary defuzzification process, respectively. G_{fuzzy} is a vector function that represents the crisp output of the FL components.

If the operators *Fuzz*, *Inference* and *Defuzz* are specified, then (1) defines a composite FL component (one that contains a number of others) that can be uniquely represented by its input-output constitutive relation as

$$R^n \times R^p \times R^n \times R^r \ni (x, x_d, Dx, u) \to G_{fuzzy}(x, x_d, Dx, u) \in R^s . \tag{2}$$

We will consider the situation where $x_d : t \to x_d(t)$ and $u : (t,x) \to \mathrm{u}(t,x)$. Thus, $G_{fuzzy}(x,x_d(t),Dx,u(t,x))$ may be replaced by $G^*(t,x,Dx)$ for the purpose of analysis. Moreover, we introduce

Assumption 1.

Vector G^* can be decomposed as $G^*(t,x,Dx) = [G^{dT}(t,x,Dx)\ G^{aT}(t,x)]^T$.

This assumption need not necessarily cause any practical problems, since many components of the descriptor vector x are frequently constructed by using the relation $x_{i+1} = Dx_i$. If, in a particular selection of descriptor variables, such decomposition is not possible, we can always expand the set of descriptor variables and reindex them, so as to allow the above-mentioned decomposition of G^*. We need to point out that the description of the FL components (2) allows a specification of different control problems for dynamic systems that contain FL components. For example, (2) can be considered as a composite FL controller in a vector form.

GENERAL MODEL OF FL SYSTEMS CONSIDERED

Let $t \in R$, $u \in R^r$ and $x \in R^n$ denote time, the input vector and the vector of descriptor variables, respectively, that are to be used in modelling system behaviour. Let also

$D \equiv d/dt$ stand for the ordinary derivative operator. We will consider a general form of models of time-continuous systems given by a set of implicit differential equations

$$f(t, x, Dx, G_{fuzzy}, u) = 0 \in R^p \tag{3}$$

where $p \neq n$ may be allowed. $G_{fuzzy} \in R^S$ represents the vector of crisp outputs of FL components contained in the system. Systems with such models are denoted as fuzzy-descriptor (FD) systems, although the term "fuzzy implicit differential systems" is equally appropriate. Let a^T be the transpose of a vector a and let us assume that the vector of descriptor variables x is decomposed as $x = [y^T z^T]^T$ In the special case the models (3) contain the canonical form of models linear in Dy, and these are of the form

$$\begin{gathered} P(t, y, z, G_{fuzzy}, u)Dy = f(t, y, z, G_{fuzzy}, u), \\ g(t, y, z, G_{fuzzy}, u) = 0, \end{gathered} \tag{4}$$

where the second equation in (4) can be considered as the "output" equation for the system. The vector z represents the so-called "redundant", i.e. "unwanted", descriptor variables. From this viewpoint, if it is possible to eliminate z from (4), then y is a candidate for the vector of the state variables. Such a process is aimed at the elimination of z from both equations in (4), and then inverting the matrix P which multiplies Dy. In most cases this elimination procedure, which results in the model reduction, is not possible. It should be pointed out that the matrix P can even be rectangular and this introduces additional problems. Some examples of physical systems where P can be rectangular are known; for example in electrical-electronic circuits. The results of this paper do not require such model reduction, i.e. the elimination of the redundant variables.

Models (3) and/or (4) appear in a natural way in modelling physical systems when the model equations are written in the sparse form. They are here altered by the explicit introduction of the input-output description (2) of FL components, since we are interested in the analysis of FD systems. The descriptor models have become more popular in the last 20 years, due to the following advantages over the models in state-space form:

- there is a great simplicity in deriving the equations (3) or (4) and in this regard there is no necessity for the elimination of the unwanted (redundant) variables, as there is no need for the formulation of the state variable models; consequently, most of the problems associated with the existence of the state variable models are surmounted;
- there is a tight relation between the system's physical variables and the variables in models (3) and (4);
- the models (3), (4) preserve the sparsity of system matrices (that is many of the entries in the system matrices are equal to zero);
- the structure of the physical system is well reflected in models (3) and (4).

However, although descriptor model formulation has these evident advantages over state-space formulation, descriptor models generally suffer the disadvantage of not being supported by developed methods of analysis of their dynamic behaviour, as is the case with the state-space models. The surveys of different results concerning both continuous and discrete descriptor systems can be found in [18-26]. A systematic introduction and presentation of the results relating to the application of Lyapunov's direct method (LDM) for the analysis of descriptor (implicit) differential systems is given in [19] and [25-27]. In [19] and [27] it is shown that the algebraic constraints imposed by the model nonlinearities can be used efficiently to enhance the qualitative analysis of descriptor systems. In this paper we will combine this approach with the concept of FL blocks as algebraic transfer elements. And we will present new results on the general methodology for the qualitative analysis of FD systems that are based on the utilisation of algebraic constraints of the system model and components. The particular problem in any descriptor system is to have an estimate of time-evolution of all, redundant and non-redundant, descriptor variables. We will present sufficient conditions that will guarantee such information for the whole descriptor vector x. The algebraic conditions presented are easily applicable in such an analysis. As will be shown, the presence of FL blocks in the system does not pose additional difficulties in our analysis, as long as we do not explicitly involve the internal functioning of the FL components in the analysis.

As far as is known to the author the results presented in this paper are the first ones that relate to the general stability problem and the qualitative analysis of FD systems. The results are based on previous contributions given in [28].

FL COMPONENTS AND BASIC REQUIREMENTS OF FD SYSTEMS

We will consider the system model (2-3) converted to the form

$$F(t,x,Dx,G^d) = 0 \in R^p \,, \tag{5}$$

$$g(t,x,G^a) = 0 \in R^k \,, \tag{6}$$

where G^a (according to Assumption 1) does not contain Dx explicitly and where x_d and u are given in advance. We do not require that (6) contains **all** algebraic constraints obtainable from (3). Let the time interval of interest be $T = \{t \in R\colon a \leq t < \sigma\}$, $a, \sigma \in \overline{R}$, where $\overline{R}$ is the extended real number system. Let (2) and (5-6) define a FD system Σ. We denote by ψ a solution of Σ. Uniqueness of solutions is not required. When x_d and u are known functions then the notation $\psi(t,t_0,x_0)$ will denote a value of the considered solution ψ at the moment t, which at the moment t_0 emanates from the point x_0. Since uniqueness is not required we may have many solutions that emanate from (t_0,x_0). The family of such solution can be denoted by $\Psi(\cdot,t_0,x_0)$. The abbreviation for the value of a solution ψ at the moment t is $\psi(t)$. The points $p^* = \psi^*(t)$ of R^n at some $t \in T$, obtained using all solutions $\psi^*$of Σ that exist on T, constitute the set $S(t)$ The following assumption regarding the solutions of Σ is necessary.

Assumption 2.

FL components (2) are such as to satisfy a set of conditions Γ which ensure that when x_d and u are given functions, then $\forall t_0 \in T$ the set $S(t_0)$ is not empty and solutions of Σ that exist on T are differentiable.

Obviously, Assumption 2 restricts the choice of possible FL components. For example, it may be convenient to have G_{fuzzy} continuous. In such cases the relay-type FL components are excluded, which suggests that we may then restrict our choice to, say, FL components of the Takagi-Sugeno type [13-14]. What actual class of FL components is allowed depends simultaneously on the properties of (5-6) and Assumption 2. In engineering problems we are normally interested in guaranteeing specific properties of solutions for a whole class Φ of the FL components (2) that satisfy Assumptions 1 and 2. This fact will be emphasised in the properties that we are to investigate. Then we introduce

Assumption 3.

Let Φ_Σ denote an arbitrary class of FL components, whose input-output constitutive relations are governed by (2) and which satisfy Assumptions 1 and 2. The class Φ_Σ is not empty.

In the qualitative analysis of FD systems we are generally interested in the following two issues:

Question 1.

FL components (2) are predefined by the specification of operators Fuzz, Inference and Defuzz. How are we to perform the analysis of the dynamic behaviour of the system Σ?

Question 2.

The input-output constitutive relations (2) of FL components are known, i.e. we know G_{fuzzy} as a function of (x,x_d,Dx,u). Do we need the explicit usage of operators Fuzz, Inference and Defuzz of the internal functioning of FL components in order to conduct a qualitative analysis of the FD system Σ?

The answer to Question 1 leads to an algebraic exercise so as to derive $G_{fuzzy}(x,x_d,Dx,u)$ from the knowledge of operators *Fuzz*, *Inference* and *Defuzz*. This is always possible, although the mathematical expressions for a description of G_{fuzzy} may be complicated.

The answer to Question 2 is that we do not need even to know the operators *Fuzz*, *Inference* and *Defuzz* in order to successfully conduct a qualitative analysis of FD system Σ; only the input-output constitutive relations (2) of FL components are required. Assumption 2, which is also within the domain of general requirements regarding the basic properties of FD systems, does not necessitate the usage of operators *Fuzz*, *Inference* and *Defuzz*. One can, in principle, deduce some properties of the solutions of FD systems by relying only on the constitutive relations of FL components and the system topology; a similar approach as used in nonlinear circuit analysis.

PROBLEM OF STABILITY

Without entering into discussion on all possible different properties that solutions of Σ may have, we will only note that the number of possible useful descriptions of solution behaviour for any system is infinite and that it can be formulated in a quite general framework through the abstract generic qualitative and quantitative concepts [26]. In what follows we will consider only the form of concepts that relate to what is known in conventional stability analysis as "absolute" concepts, since the properties involved hold $\forall G_{fuzzy} \in \Phi_\Sigma$.

For a description of the response properties of Σ we will use two nonlinear vector functions $q_1 : T \times R^n \ni (t,x) \to q_1(t,x) \in R^{n_1}$ and $q_2 : T \times T \times R^n \ni (t,t_0,x) \to q_2(t,t_0,x) \in R^{n_2}$. The next description of the time-evolution of solutions is convenient for our analysis.

Definition 1.

The system Σ has type A bounds of response with respect to $(q_1, q_2, \Phi_\Sigma, x_d, u, T)$, where x_d and u are prespecified functions, iff

$$(\forall t_0 \in T)\ (\forall x_0 \in S(t_0))\ (\forall t \geq t_0,\ t \in T)\ Q_1(t, t_0, \varepsilon) \subseteq Q_2(t, t_0, \delta(t, t_0, \varepsilon)\ (\forall G_{fuzzy} \in \Phi_\Sigma) \\ q_1(t, \psi(t, t_0, x_0)) \leq q_2(t, t_0, x_0) \tag{7}$$

Definition 2.

The system Σ has type B bounds of response with respect to $(q_1,q_2,M,N,\Phi_\Sigma,x_d,u,T)$, where x_d and u are prespecified functions, and where M and N are two subsets of the descriptor space R^n, iff (7) holds for all solutions ψ with $x_0 \in N \cap S(t_0)$ as long as $\psi(t,t_0,x_0) \in N$, and if there is a moment $t_f(t_0,x_0) \geq t_0$, $t_f \in T$, such that $\psi(t) \in M$ for all $t \geq t_f$, $t \in T$.

By selecting specific functions q_1 and q_2, one can achieve different descriptions for solution behaviour. From these bounds, under appropriate conditions and using the additional information from the purely algebraic equation (6), it is easily possible in some cases to derive information on the behaviour of all components of x.

We can use a more convenient description tool that involves variable sets to characterise solution properties. To illustrate this we define two sets Q_1 and Q_2 as

$$Q_1(t, \tau, \varepsilon) = \{(t, \tau, x) \in T \times T \times R^n : q_1(t, \psi(t, \tau, x)) \leq \alpha(t, \tau, \varepsilon)\},\ \forall (t, \tau, \varepsilon) \in T \times T \times A_\varepsilon,$$

$$Q_2(t, \tau, \delta) = \{(t, \tau, x) \in T \times T \times R^n : q_2(t, \tau, x) \leq \beta(t, \tau, \delta)\},\ \forall (t, \tau, \delta) \in T \times T \times A_\delta,$$

where α and δ are two suitably selected functions. Now we can introduce the following two definitions which govern properties that can be deduced from the property described by Definition 1.

Definition 3.

The system Σ possesses a generalised stability type behaviour with respect to $(q_1,q_2,\Phi_\Sigma,x_d,u,\alpha,\beta,T,A_\varepsilon,A_\delta)$, where x_d, u, α and β are prespecified functions, iff

$$(\forall t_0 \in T)\ \ (\forall t \geq t_0,\ t \in T)\ \ (\forall \varepsilon \in A_\varepsilon)\ \ (\exists \delta \in A_\delta)\ \ (\forall x_0 \in S(t_0) \cap Q_1(t, t_0, \varepsilon)) \\ (\forall \psi \in \Psi(\cdot, t_0, x_0))\ \ (\forall G_{fuzzy} \in \Phi_\Sigma)\ \ \ Q_1(t, t_0, \varepsilon) \subseteq Q_2(t, t_0, \delta(t, t_0, \varepsilon)).$$

Definition 4.

The system Σ possesses a generalised boundedness type behaviour with respect to $(q_1,q_2,\Phi_\Sigma,x_d,u,\alpha,\beta,T,A_\varepsilon,A_\delta)$, where x_d, u, α and β are prespecified functions, iff

$$(\forall t_0 \in T)\ \ (\forall t \geq t_0,\ t \in T)\ \ (\forall \delta \in A_\delta)\ \ (\exists \varepsilon \in A_\varepsilon)\ \ (\forall x_0 \in S(t_0) \cap Q_1(t, t_0, \varepsilon)) \\ (\forall \psi \in \Psi(\cdot, t_0, x_0))\ \ (\forall G_{fuzzy} \in \Phi_\Sigma)\ \ \ \ Q_1(t, t_0, \varepsilon(t, t_0, \delta)) \subseteq Q_2(t, t_0, \delta).$$

Let ∂Q denote the boundary of the set Q. We also introduce the following two mild assumptions.

Assumption 4.

Let x_d, u, α and β be prespecified functions. Then

$$(\forall \tau \in T)\ \ (\forall t \geq \tau,\ t \in T)\ \ (\forall \varepsilon \in A_\varepsilon)\ \ (\exists \delta \in A_\delta)\ \ \ \partial Q_1(t, \tau, \varepsilon) \in Q_2(t, \tau, \delta).$$

Assumption 5.

Let x_d, u, α and β be prespecified functions. Then

$$(\forall\tau\in T)\ \ (\forall t\geq\tau,\ t\in T)\ \ (\forall\delta\in A_\delta)\ \ (\exists\varepsilon\in A_\varepsilon)\ \ \partial Q_1(t,\tau,\varepsilon)\in Q_2(t,\tau,\delta)\,.$$

If the property described by Definition 1 is verified for the FD system Σ, then Assumption 4 is needed additionally to infer properties given by Definition 3, while Assumption 5 is required to infer properties given by Definition 4.

The concepts used in Definitions 3 and 4 can be easily extended to deal with the property of the system Σ described by Definition 2.

Note that many conventional "absolute" stability types, such as absolute stability, absolute uniform stability, absolute equi-stability, etc., can be obtained as special cases of Definition 3. Similarly, from Definition 4 one can get numerous "absolute" boundedness concepts including absolute boundedness, absolute uniform boundedness, absolute equi-boundedness, etc. Properties given by Definitions 1-4 are all special cases of the generic concepts from [26]. If the property given by Definition 1 is verified for the FD system Σ, then the properties of Σ described by Definitions 3 and 4 can be verified only by checking the additional Assumptions 4 or 5, as commented above. For this reason we will present only results by which one can verify the properties given by Definitions 1 and 2. The exception is Theorem 3 which relates to the non-existence of the finite forward escape time of descriptor variables.

QUALITATIVE ANALYSIS OF FUZZY DESCRIPTOR SYSTEMS

Let φ^{I} denote the inverse function of the function φ. In what follows DV_Σ will denote any convenient norm in the appropriate space and K will denote the class of strictly increasing scalar functions φ, $\varphi: R^+\to R^+$, where $R^+:=R\cap[0,+\infty[$, and where $\varphi(0)=\min[\varphi(s): s\in R^+]$ The function $V: T\times R^n\ni(t,x)\to V(t,x)\in R^+$ is assumed to be continuous and differentiable, with DV_Σ denoting the total time derivative of V in force of the system Σ. The following theorems take into account some properties that (6) may have.

Theorem 1.

Let $u(t,x)\equiv 0$ on $T\times R^n$ and let x_d be given in advance. Let Assumptions 1-3 hold; let $x=[y^T\,z^T]^T$ and let $V: T\times R^n\ni(t,x)\to V(t,x)\in R^+$ be a continuous and differentiable function. If there exist:

(i) a function $\varphi_0\in K$, such $\varphi_0(|\,y\,|)\leq V(t,x)$ when $(t,x)\in T\times R^n$,

(ii) functions $\varphi_1\in K$ and ω so that the total time derivative of V along the solutions of Σ satisfies $DV_\Sigma\leq\omega(t,V)+\varphi_1(|\,z\,|)$ when $(t,x)\in T\times R^n$,

(iii) a function $\varphi_2\in K$, such that from (6) we can obtain the estimate $|\,z\,|\leq\varphi_2(|\,y\,|)$ when $(t,x)\in T\times R^n$,

(iv) a function m, integrable and bounded on T, such that $\omega(t,V)+\varphi(V)\leq m(t)V$ when $(t,V)\in T\times R^+$, where $\varphi(V)=\varphi_1(\varphi_2[\varphi_0^I(V)])$,

then the system Σ has response bounds of type A with respect to $(q_1,q_2,\Phi_\Sigma,x_d,0,T)$, where

$q_1 = |y|$, $q_2 = \varphi_0^I[V(t,x)\exp\int_{t_0}^t m(s)ds]$; *also* $|\psi(t)|$ *is finite for finite* $t \ge t_0$ *and arbitrary consistent initial conditions.*

Theorem 2.

Let $Q_1(t,t_0,x) \subseteq Q_2(t,t_0,\delta(t,t_0,\varepsilon))$ *on* $T \times R^n$ *and let* x_d *be given in advance. Let Assumptions 1-3 hold; let* $x = [y^T\, z^T]^T$ *and let* $V: T \times R^n \ni (t,x) \to V(t,x) \in R^+$ *be a continuous and differentiable function. If there exist:*

(i) *two functions* $\varphi_0, \varphi_3 \in K$, *such that* $\varphi_0(|y|) \le V(t,x) \le \varphi_3(|y|)$ *when* $(t,x) \in T \times R^n$,

(ii) *functions* $\varphi_1 \in K$ *and* ω, *so that the total time derivative of V along the solutions of* Σ *satisfies* $DV_\Sigma \le \omega(t,V) - \varphi_1(|z|)$ *when* $(t,x) \in T \times R^n$,

(iii) *a function* $\varphi_2 \in K$, *such that from (6) we can obtain the estimate* $|z| \ge \varphi_2(|y|)$ *when* $(t,x) \in T \times R^n$

(iv) *a function m, integrable and bounded on T, such that* $\omega(t,V) - \varphi(V) \le m(t)V$ *when* $(t,V) \in T \times R^+$, *where* $\varphi(V) = \varphi_1(\varphi_2[\varphi_3^I(V)])$,

then the system Σ *has response bounds of type A with respect to* $(q_1, q_2, \Phi_\Sigma, x_d, 0, T)$, *where* $q_1 = |y|$, $q_2 = \varphi_0^I[V(t,x)\exp\int_{t_0}^t m(s)ds]$; *also* $|\psi(t)|$ *is finite for finite* $t \ge t_0$ *and arbitrary consistent initial conditions.*

Theorem 3.

Let x_d *and u be given in advance. Let Assumptions 1-3 hold; let* $x = [y^T\, z^T]^T$ *and let* $V: T \times R^n \ni (t,x) \to V(t,x) \in R^+$ *be a continuous and differentiable function. If there exist:*

(i) *a set* $\Omega \subset R^n$ *such that* Ω^c *is a bounded set (* Ω^C *being the complement in* R^n *of the set* Ω *),*

(ii) *a function* $\varphi_0 \in K$, *such that* $\varphi_0(|y|) \le V(t,x)$ *when* $(t,x) \in T \times \Omega$,

(iii) *functions* $\varphi_1 \in K$ *and* ω, *so that the total time derivative of V along the solutions of* Σ *satisfies* $DV_\Sigma \le \omega(t,V) + \varphi_1(|z|)$ *when* $(t,x) \in T \times \Omega$,

(iv) *a function* $\varphi_2 \in K$ *and a constant* $\beta \ge 0$, *such that from (6) we can obtain the estimate* $|z| \le \varphi_2(|y|) + \beta$ *when* $(t,x) \in T \times \Omega$,

(v) *a function m, integrable and bounded on T, such that* $\omega(t,V) + \varphi(V) \le m(t)V$ *when* $(t,V) \in T \times R^+$, *where* $\varphi(V) = \varphi_1(\varphi_2[\varphi_0^I(V)] + \beta)$,

then the system Σ *has* $|\psi(t)| < +\infty$ *for finite* $t \ge t_0$ *and arbitrary consistent initial conditions.*

Theorem 4.

Let x_d *and u be given in advance. Let Assumptions 1-3 hold. Let let* $x = [y^T\, z^T]^T$ *and* $V: T \times R^n \ni (t,x) \to V(t,x) \in R^+$ *be a continuous and differentiable function. If there exists*

(i) *a set* $\Omega = \{x \in R^n : |x| \ge e\}$, $e \ge 0$,

(ii) *two functions* $\varphi_0, \varphi_3 \in K$, *such that* $\varphi_0(|y|) \le V(t,x) \le \varphi_3(|y|)$ *when* $(t,x) \in T \times \Omega$,

(iii) *functions* $\varphi_1 \in K$ *and* ω, *so that the total time derivative of V along the solutions of*

Σ satisfies $DV_\Sigma \le \omega(t,V) - \varphi_1(|z|)$ for $(t,x) \in T \times \Omega$,

(iv) a function $\varphi_4 \in K$ and a constant $\beta \ge 0$, such that from (6) we can obtain the estimate $|z| \le \varphi_4(|y|) + \beta$ when $(t,x) \in T \times \Omega$,

(v) a function $\varphi_2 \in K$, such that from (6) one can also obtain $|z| \ge \varphi_2(|y|)$ when $(t,x) \in T \times N$, $N = \{x \in R^n : |y| \ge e\}$, $e \ge 0$,

(vi) a function m, integrable and bounded on T, such that $\omega(t,V) - \varphi(V) \le m(t)V$ when $(t,V) \in T \times R^+$, where $\varphi(V) = \varphi_1(\varphi_2[\varphi_3^I(V)] + \beta)$,

(vii) if $\sigma = +\infty$ and $m(t) \le -\mu < 0$ on T,

then the system Σ has response bounds of type B with respect to (q_1,q_2,M,N,Φ_Σ,x_d,u,T), where x_d, $q_2 = \varphi_0^I[V(t,x)\exp\int_{t_0}^t m(s)ds]$, and $M = \{x \in R^n : |y| \le e_1, |z| \le e_2\}$, where e_1,e_2 are constants such that $e_1 > e$, $e_2 = \varphi_4(e_1) + \beta$.

Comment.

The proofs of Theorems 1-4 follow, with obvious modifications, from the proofs of Theorems 6.1-6.4 from [19].

DISCUSSION

Theorems 1-4 generalise the corresponding theorems from [27], as well as Theorems 6.1-6.4 from [19]. The results utilise specific properties of algebraic constraints (6), as well as the input-output description of the FL components (2), and on that basis they infer information on the time-evolution of the whole descriptor vector x from the FD system **Σ**. Note that the conditions imposed on (6) do not demand solvability of (6) with respect to any of its arguments and thus the implicit character of the model of **Σ** is preserved. Having in mind our comments on the relation of generalised stability and boundedness types for the FD system **Σ**, one can use Theorems 1 and 2 to derive many specific results for verification of such concepts if, additionally, Assumptions 4 or 5 are used in connection with Definitions 3 or 4.

CONCLUSION

This paper presents some general results of the analysis of qualitative behaviour of solutions of FD systems Σ. The results use only input-output constitutive relations of FL components and avoid direct utilisation of internal FL functioning. The efficient use of the constraints implied by the algebraic equations (6) of the system model enhances the qualitative analysis that is achievable only by analysing Lyapunov functions behaviour in force of the system Σ. The results are of direct use in the analysis of singular electrical-electronic networks with FL components [29].

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PROBLEMI STABILNOSTI FUZZY-DESCRIPTOR SISTEMA

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U ovom radu razmatra se generalni problem stabilnosti i drugih svojstava rešenja descriptor systema koji sadrže komponente fuzzy logike i predstavlja opštu metodologiju za takvu analizu. Pristup se zasniva na drugom metodu Ljapunova i generičkim konceptima koji opisuju ponašanje sistema. Komponente fuzzy logike su dozvoljene u opisu modela sistema. Ove i druge nelinearnosti sistema se koriste da pojačaju analizu sistema. Za verifikaciju specifičnih svojstva koja se menjaju tokom vremena za sve promenljive deskriptora predstavljeni su različiti algebarski uslovi. Ovi uslovi se zasnivaju na parcijalnim algebarskim ograničenjima u modelu.