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NUMBERS OR UNDERSTANDING: ANALYTICAL AND NUMERICAL METHODS IN THE THEORY OF PLATES AND SHELLS

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Abstract. *Different connections of plates and shells theory with pure and numerical mathematics are considered. Some new ideas in the field of analytical methods of plates and shells theory, as well as trends in applied science development in the epoch of computerisation are analysed.*

1. Introduction

Computer (information) revolution is one of the most significant phenomena of our times. The opinions totally differ while evaluating it: from absolute acceptance to full negation. There is no doubt the methods of computer analysis of information bring about quality changes in all life domains and in the science as well. In the case of fundamental sciences, although computers make a lot of changes in them, they do not alter them. The more complicated problem occurs in the case of the so-called "applied sciences", many disciplines of which appeared only as a result of backwardness of suitable mathematical means, and at present should wither away while changing into chapters of calculation mathematics or engineering methods. In order to continue its existence the applied science on the one hand should intermingle actively with fundamental sciences (it should become a fundamental science itself to some extent), on the other hand it should be open to new ideas being born in fundamental sciences.

We are going to consider a specific applied science: shells and plates theory and we are going to discuss its fate. From the very beginning theory of shells and plates was

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closely connected with physics and mathematics, imposing tasks on these sciences and deriving from them ideas and methods of construction and analysis of acquired relations. At first this close relationship was conditioned by the fact that the very same scientists worked in many disciplines of the science, and thus personally realised its entity. In the later period the theory of shells and plates similar to other formal theories began its development not only according to external needs (direct demands of technology, implementation of physical and mathematical achievements), but also in connection with certain internal logic. Below we are going to dwell on the problem of theory of shells and plates intermingling with contemporary mathematics, and we are going to analyse the role of analytical and numerical methods while reaching the solutions of actual problems.

2. SOMETHING ABOUT THE THEORY ITSELF

A not complicated experiment with the thin sheet of paper will remind us that it is much easier to bend it or crash it than to stretch it. The problem is that the resistance of such a sheet of paper to stretching or squeezing B is much bigger than to bending D**.** In shells and plates theory it has been proved that B is proportional to Young's module E, and to the thickness of the sheet h, and D - Eh, so this is why when h is a small value, the quoted resistance to bending is considerably smaller than stiffness to stretching or squeezing. Thus it is understandable that the constructor's task while using the thin-walled elements essentially relies on making them stretch or squeeze and not bend. Unfortunately, one cannot fully exclude the state of bending stresses (originating from the bending moment). A very important feature of the shell comes here to help – the ability to locate the state of momentary stresses (originating from the moment) – see Figure 1.

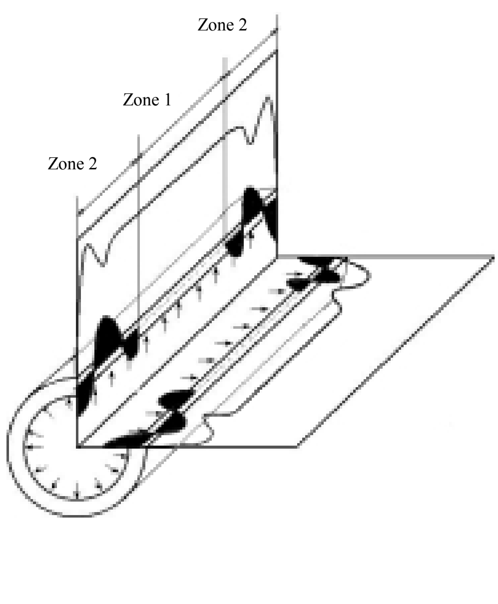

Fig. 1. High endurance of shells is described as the ability to accept the edge and surface loads at the cost of stretch deformation with regular thickness. This is the area of non-momentary state which is described in the initial equation within the boundary $h \to 0$ (zone 1 in the figure). In a well-designed shell, the top layer being properly fastened, the zones of strong bending appear on a small area. The strain states called boundary effects in the shell theory originate from here (zone 2 in the figure). Their definition is considerably facilitated due to the localisation and quick changeability. Lowering of the bending strains is usually achieved by local hardening of the construction.

In the case of plates and shells, one of the dimensions (thickness) is considerably smaller than two others so it must be replaced by the surface with some rigidity features while creating the dependencies describing the deformation or motion of such a system. G. Kirchhoff created the first satisfying theory of flat plates bending within the phenomenological approach, based on the following hypotheses:

- Rectilinear fibres perpendicular to the middle surface of the plate before the deformation become rectilinear and perpendicular to the bent surface after the deformation while keeping their length at the same time.
- There is a lack of mutual interaction between the plate layers parallel to the middle surface in the direction normal for the layers.

Later A. Love generalised the hypotheses about the bent surfaces, and he created the basic interdependencies of shells theory [1].

3. The analysis of the initial equations

Not dwelling on the history of origin of contemporary shells theory [1-3] we are going to mark that the initial equations are very complicated: they are differential equations with partial derivatives [4] of high order, the coefficients of which can be calculated in different ways. Moreover it is not always possible to remain within the linear theory, and non-linear calculus (i.e. dis-proportions of the answers to operating disturbances) extremely complicates the process of solution [5]. The methods of solutions to applied problems can be conventionally divided into "A. Lyapunov's and A. Poincaré's approaches". A. M. Lyapunov thought that having put the physical initial problem it should be solved mathematically and it would be the best if the precise solution was reached. A. Poincaré rejected this point of view that the accuracy and justification of solutions should not exceed the accuracy and justification of initial correlation. Unfortunately the choice of problems allowing for the exact solution turned out to be very narrow in theory, became very quickly exhausted and could not satisfy the practical demand. One can hope for the approximate approaches only (as in an optional physical theory, anyway). To some extent they can be conventionally divided into analytical and numerical. Conventionally, because, first of all analytical methods create the possibility of getting the numerical values, and then, very often, if one is to be limited to small numerical approximations, this method allows to get the formula, and getting the bigger precision is possible only in numerical terms. Such delimitation is generally accepted and we are going to observe it. The most important position is occupied by asymptotic approach in the theory of plates and shells. The asymptotic methods (the methods of disturbance) represent the collection of mathematical solutions to solve the problems having "small" (in a way) constituents. The negligence of these constituents enlarges the symmetry of the problems and allows for their solving, and in the extreme case for simplifying considerably the process of finding the solutions, on the other hand the introduction of corrections – allows for showing the real picture of the process or of the phenomenon. Besides, the asymptotic methods – they are much more than the set of technical approaches, they present the whole system of views and even ideologies of natural sciences [3, 6]. One would like to introduce here a short digression about the so-called engineering methods of calculation a thin walled shell uniting big stability, small weight, simplicity and technology of the product, has become one of the most popular constructions in numerous branches of modern engineering, above all in the rocket industry, aircraft industry, shipbuilding industry, civil and industrial engineering, in the construction of chemical devices. This practice requires quick results in the first place, and moreover – it demands them quickly. The engineer cannot expect mercy from the mathematicians, this is why different approximated approaches have become extremely popular. Sometimes pure mathematicians display a kind of snobbery at those methods – and unnecessarily. Our experience assured us of one thing: as a matter of fact any approximated methodology suggested and approved by a good engineer is asymptotic, and can be treated as the first approximation of the asymptotic process. In connection with this the memories of A.N. Krylov, a well-known ship constructor and mechanic, are recalled. He writes with admiration about a Russian self-taught engineer P.A.Titov who designed the first armoured Russian ship. He remembers his conversation and works with him. "Your formulas, midshipman, are exact; here, I roughly defined the size – it's correct".

Lets consider two examples of how contemporary asymptotic methods allow solving the problems in shells and plates theory, and this theory in turn makes demands on them**.**

4. THE BOUNDARY LAYER

The theory of plates and shells can be treated as the chapter in the theory of elasticity for those bodies the two dimensions of which considerably surpass the third dimension. As a result an essential small parameter appears in the equations – e.g. the ratio of the shell thickness to its radius. This is why in the theory of plates and shells the asymptotic methods in reality turn out to be the most adequate from physical and mathematical point of view. Besides, in the theory of plates and shells, as in the science of precisely defined applied character, the problem of creating the approximated methods of calculations is the most important, particularly in initial correlation the problem of eliminating of those sizes which cannot noticeably influence the final results, but they introduce difficulties not essential for the heart of the matter.

And so the initial equations of the shell theory comprise a small parameter ε in consequence of which the so-called singular disturbance takes place from the point of view of asymptotic methods. It means that the real solution deviations from the boundary ($\varepsilon = 0$) are focused in a narrow located interval – the interval of the boundary effect (Figure 1). An analogous phenomenon plays an important role in hydromechanics, where it is called the boundary layer, the viscous liquid flowing round the body everywhere apart from the narrow interval near the body surface, the liquid viscosity may be not taken into account, and the examination of the boundary layer is simplified because of its narrowness. Let's notice, by the way, that the boundary effect was created and understood in the shells theory much earlier than in the hydromechanics.

5. THE METHOD OF AVERAGING

The danger of the resistance loss appearing in the shell when the bending stresses occur is the basic "cost" of thin walls. The suitable critical loads (i.e. the loads which cause the warping of the shell) as a rule represent by themselves the boundary of construction work capability, as strong contraflexure threatens with destruction or irreversible deformation, when surpassed. In these cases, as a rule, in order to improve the situation, it is useful not to enlarge the thickness of the shell, but to strengthen it with longitudinal or lateral force elements. It is interesting to observe how wide the following strategy is used by the Nature. In order to increase their resistance to bending the leaves make use of the majority of known construction solutions. Almost all leaves have developed the system of stiff ribs, whereas the pellicles between represent the cell structure enlarging their stiffness; besides in some cases they are corrugated. The empty bamboo and grass stems that tend to become flat when bent are smartly strengthened by "nodes" or septum placed along the stems at different intervals [1]. The introduction of stiffening ribs can lead to strong stress-deformation heterogeneity of the construction state, and worsen the working conditions. It all requires sufficiently particular analysis. In practice one often passes to the scheme of homogeneous orthotropic shell, i.e. the shell which is stiff in various directions. The stiffness and inertial characteristics of strengthening elements become fuzzy on the shell surface, which is now considered to be homogeneous but endowed with a few new properties, because of the constructive properties of the object (constructive orthotropy). The introduction of constructive orthotropy creates the possibility of putting aside the peculiarity of force interaction between the ribs and sheathing, and of radical simplifying of the task. Meanwhile the construction orthotropic scheme allows for accurate enough estimation of global characteristics of the system (the frequency of deviations, displacements), it does not allow for estimation of local characteristics (stress). This difficulty can be eliminated quickly by successive introducing of the averaging method based on the separation of quick and free components of solutions. The solution of constructive orthotropy theory equations is present as the averaging, and the exact solutions allowing for the estimation of all the components of stress-deformation state are not more complicated than those obtained as a result of orthotropic scheme construction.

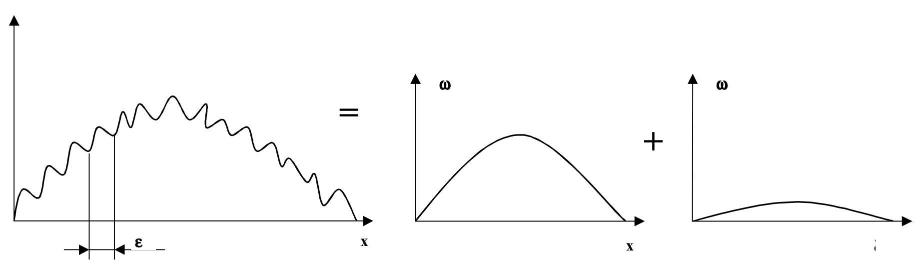

Fig. 2. The contemporary averaging method on the idea level can be described in this way: First – periodically repeated heterogeneity is separated (Fig. 1); the solution is presented as the sum of slow and quick components. The last one is considered in new "stretched" co-ordinates ('the rule of the microscope'). As a result we get two 'slow-change' systems instead of an initial 'quick-change' system.

An interesting peculiarity of asymptotic methods occurs here. The ribbed shell is obviously more complicated for calculation than the flat one. But the presence of new parameters and new structures (the layout of ribs) leads to new, broader than in the isotropic case, possibilities of asymptotic integration. The second example is connected with the perforation of plates and shells. The working out of the similar system (Figure 3a) is complicated because of multiconnection domains.

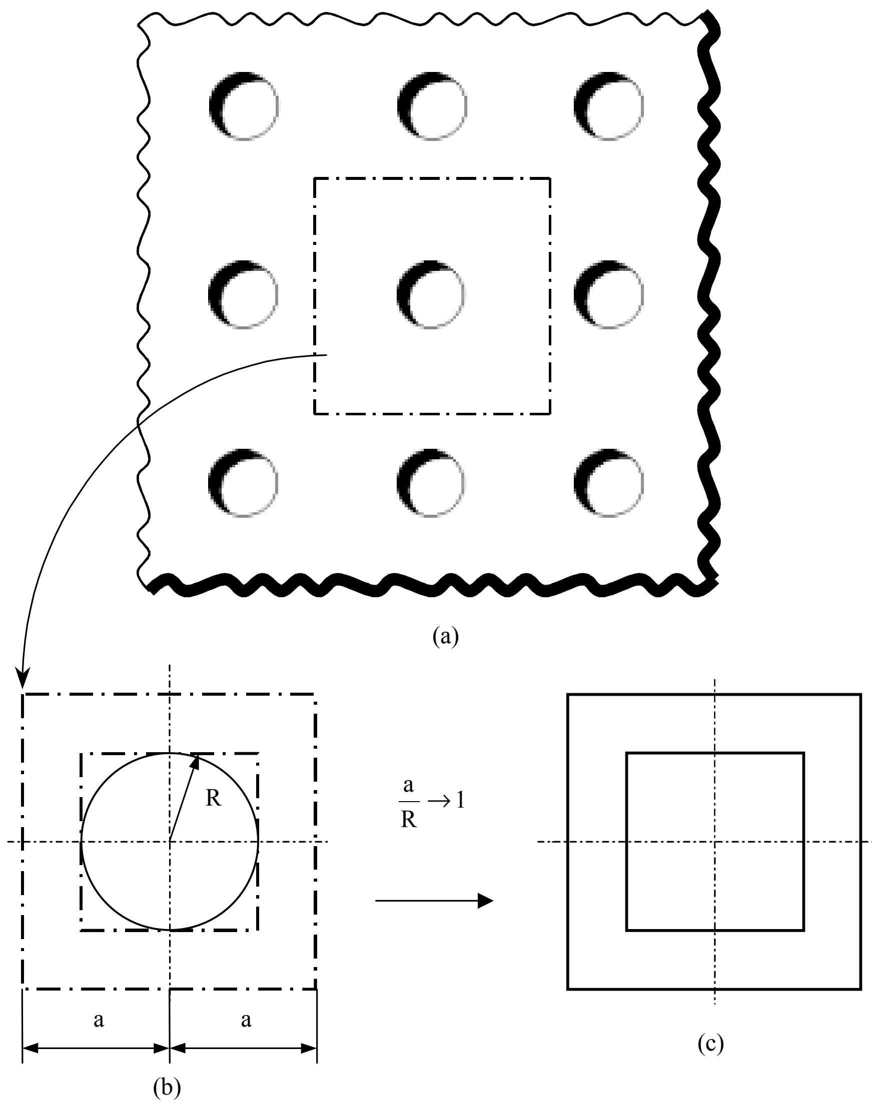

Fig. 3. Periodically corrugated shell can be replaced by a homogeneous shell with certain accepted parameters.

Obviously, the averaging method can be applied here as well, but what about the solution to the problem of partition (i.e. with separate periodically continuous sector with one opening, Fig. 3b.)? Two cases can be considered. If the opening is small the influence of the external boundary may not be taken into consideration, and the plate with the opening will be considered. But if the opening is big, then the round opening is replaced by the square one, and the initial plate (shell) – by the framework (Fig. 3b), then these boundary solutions can be bound together and we get the solution for the opening of any radius. The analysis of the quoted example and of many other engineering methods of working out shows that all clever simplifications are of asymptotic nature. What would be, then, the role of pure asymptotic approaches (**would be**, because, unfortunately, the cases of indirect application of asymptotic methods in practice are not too numerous.) First of all it is a severe evaluation of the application domain of this or another simplification (otherwise this domain would be introduced by way of very expensive, often even open-air experiments or long term settlement of accounts). In the second place, asymptotics reveals subtle effects exerting an influence on the construction work, e.g. numerous stress concentrations conditioned by the boundary layers not taken into consideration in simple schemes. Last of all we are going to consider the most important for the technique problem of optimal design. This is the task contrary to the settlement of accounts of initial system requiring frequent returns to the solutions of simple problems.

If we were to use exact numerical algorithms, then in every stage of optimisation it comes to the loss of big amount of machine time, and there can be thousands of these stages. One can take a simple engineering scheme, but there is a big fear that we shall get into the area where it cannot be applied or we 'miss' important effects. It is here that the most important are asymptotic approaches combining the simplicity with the final exactitude and precise evaluation of application area.

6. NUMERICAL METHODS IN THE PLATES AND SHELLS THEORY

Many fundamental notions and mathematical methods have their origins in the problems of plates and shells theory. Suffice to mention the notion of durability and linearization method introduced in Euler's works. Mathematical physics was generated and formed by problems of plates and shells theory to a big extent. For example the equations of elliptic type appeared in connection with membranes deformation description, and the membranes vibrations equations lead to the equations of hyperbolic type and in consequence to the theory of vibrations and waves in general [4]. Many numerical methods also originated from the plates and shells theory, e.g. Rayleigh-Ritz's method applied by the authors to use the vibrations of the plate. According to this approach the solution is presented in the form of base functions set, fulfilling the given boundary conditions, and dependent on some not specified parameters. The last ones are chosen in such a way so that the solution may ensure the minimum of the potential energy of the system. As a result I.G. Bubnov and B.G. Galerkin generalise this method in such a way that it is not necessary to integrate the energy while using it, and then the area of application of the suggested approach widens considerably. V.Z. Vlasov and L.V. Kantorovitch elaborated the method according to which a part of variables are set as the approximating functions, and the differential equations must be solved in order to explain the dependencies on the other variables. Only smooth elementary functions (trigonometric, multinominals) have been chosen as approximating for a long time. Then when the number of approximating functions is small the presented methods can be considered as analytical. At first the use of computers resolved itself into the increase of basic functions, but it turned out that it is not the better way for the computer by any means. The real revolution took place when the fact was understood that it was much comfortable to break the unknown area into some small elements and to approximate the solution by the multinomial in this small area. In other words the method of finite elements makes advantage of finite functions as basic, i.e. clearing everywhere apart from the fixed number of subdomain elements. So the possibilities of integration of differential equations increased extremely thanks to modern computers, and what is clear opens new and not known earlier horizons. At the same time some fear resides here, which was pointed clearly in the introduction to the book [7] in the domain of shells theory. "The authors are firm opponents of exchanging the fundamental domain - the theory of shells – for one of the chapters of applied sciences". This regrettable tendency is a side effect of implementation of numerical methods. In journals (and monographs) there were flowing avalanches of works with the opinions of numerical experts, realised sometimes with the use of standard packets of applied programmes. Unfortunately (or perhaps fortunately) there cannot be a pattern set for all life cases. At the same time the most important thing is the understanding of the penetrated problem, and not the number. As for the numerical methods, then when the complicated problems are set, the introductory analytical solutions may turn out to be very useful, and sometimes they are even necessary to quicken the realisation of numerical algorithm. In the domain of mechanics of deformed solid body it is primary to accept initial hypothesis and assumptions based on deep understanding of work of the material in construction.

The estimation of errors in the accepted hypothesis and assumptions – the formation of the set of equations adequately describing the construction working.

The position of the authors: sensible combination of analytical and numerical methods with the understanding of the mechanical side of the considered problem [8-12]. Let's notice that littering the literature with worthless, purely numerical papers creates specific ecological problem. Sometimes one finds it much easier to get the solution oneself than to search for it the pile of publications, and then still try to reveal real regularities in the choice of tables and graphs. It is understandable that many useless papers also appear in the domain of pure mathematics – e.g. the existence and exclusiveness theorems are carefully proved (not constructively), but it is clearly seen that their results are useless. Besides, analytical papers allow for simpler and more effective improvement than the numerical papers.

7. Conclusion

Is that so that there is an insurmountable precipice between the "analysts" and the "followers of the theory of numbers"? We are sure that this is not the case. We know from our own experience that it is much easier to reach an agreement with a clever and extensively educated "follower of the theory of numbers" than with a limited "analyst". Moreover it can be assumed that the boundary between the analysts and the "followers of the theory of numbers" will violently obliterate together with the implementation of the programme packets of the "Mathematics" type, which allow to form symbolic explanation, and also the packets giving the possibility of analytical presentation of the bulk of numerical data.

It seems that in connection with the above statements there increases the value of the qualified mechanic – 'sheller' who is able to analyse both the obtained results and the initial setting of the problem from physical and mathematical point of view. It is quite possible that the alike specialist, to a considerable degree saved by computers from the necessity of performing time-consuming formal calculations – also analytical, and having additional time and possibilities for the analyses, one should call a mechanic – analyst or knowledge engineer.

And if shells and plates specialists exist and are appreciated it means that the theory itself will not perish.

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BROJEVI ILI RAZUMEVANJE: ANALITIČKE I NUMERIČKE METODE U TEORIJI PLOČA I LJUSKI

I. V. Andrianov, J. Awrejcewicz

Razmatraju se različite veze teorije ploča i ljuski sa čistom i numeričkom matematikom. Analizirane su neke nove ideje u oblasti analitičkih metoda teorije ploča i ljuski, kao i trendovi u razvoju primenjene nauke.