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GENERAL SIMILARITY METHOD FOR UNSTEADY MHD FREE CONVECTION PROBLEMS ON THE VERTICAL WALL

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Abstract. The paper is concerned with the nonstationary free convection near the heated vertical wall with an arbitrary temperature distribution or thermal flux. The present external magnetic field is homogenous and perpendicular to the wall. The problem has been considered in Busineski's approximation. By using the general similarity method and introducing three sets of parameters a system of two universal equations is obtained. Along with the universal equations, two integral equations of the observed problem have been obtained. The system of universal equations has been solved in a determined approximation in this paper. A part of the results obtained in the paper is given in the form of diagrams.

1. INTRODUCTION

The problems of MHD-loose convection in the boundary layer are generally nonautomodelling. However, at certain ratios of temperature change law and magnetic field upon an overflowed surface, these problems are automodelling. For instance, for constant thermal barrier conditions of the first kind the automodelling of the loose convection is secured if the exterior magnetic field is changed according to the law $Bx^{1/4} = const$ [1]. For linear change of wall temperature it is B = const, and for constant barrier conditions of the second kind it is $Bx^{1/5} = const$. In the previous papers, the integral method has been predominantely used for solving automodelling problems [2].

This paper is going to consider the nonautomodelling problem, more specifically the unstable MHD-loose convection on the vertical wall with arbitrary temperature changes or thermal flux on the wall. The problem is considered in the Busineska approximation. To solve the problem, a general similarity method will be formed, and for the formation of this method, the ideas given in the papers [3], [4], [5], [6] will be used. We have opted for the formation of general similarity method because of its advantages over other methods. With this method, the integration of the universal equation is performed only

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once and the obtained results are conveniently stored. The obtained universal results can be used for reaching general conclusions about the development of the boundary layer, but also for calculations of particular problems with the boundary layer.

2. MATHEMATICAL ANALYSIS

MHD-loose convection equations in a laminary unstable boundary layer on the vertical wall (plate) have the following form

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + \beta \cdot g(T - T_\infty) - Nu$$
$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2} + \frac{N}{c_p} u^2$$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
(1)

and the boundary and initial conditions of the first kind are:

$$u = 0, v = 0, T = T_w(x,t) \text{ for } y = 0$$

$$u \to 0, T \to T_{\infty} \text{ for } y \to \infty$$

$$u = u_1(x,y), T = T_1(x,y) \text{ for } t = t_0$$

$$u = u_0(t,y), T = T_0(t,y) \text{ for } x = x_0.$$
(2)

The denotations used in the equations (1) and the boundary and initial conditions (2) are customary for this theory: x, y - longitudinal and transversal coordinate, respectively, t - time, u, v - longitudinal and transversal velocity of the boundary layer, respectively, v - coefficient of the kinematic viscosity, β -coefficient of thermal expansion of fluid, g - gravity acceleration; T_w, T, T_∞ - temperature on the wall, boundary layer and infinity, respectively; $N = \sigma B^2/\rho$, σ - electroconductivity of fluid, ρ - density of fluid, B - magnetic induction, c_p - specific heat at constant pressure; a = v/Pr, Pr - Prantle's number; $u = u_1(x,y), T = T_1(x,y)$ - distribution of longitudinal velocity and temperature in the boundary layer at time $t = t_0$, respectively; $u = u_0(t,y), T = T_0(t,y)$ - plottings of longitudinal velocity and temperature in the cross section of the boundary layer $x = x_0$, respectively.

Boundary and initial conditions of the second kind could also be used. There would be no significant differences at the formation of the method, even the obtained universal equations would be the same.

Further on, we introduce into consideration the function of flow $\Psi(x,y,t)$ with the relations

$$\frac{\partial \Psi}{\partial y} = u, \frac{\partial \Psi}{\partial x} = -v$$
 (3)

and the equations (1) and boundary and initial conditions (2) to new expressions. Then, new dimensionless variables are introduced into consideration

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$$\eta = \frac{y}{\delta}, \quad \Phi = \frac{\Psi}{u\delta}, \quad S = \frac{\beta(T - T_{\infty})}{\Theta}$$

$$\Theta = \beta(T_w - T_{\infty}), \quad U = gz\Theta, \quad z = \frac{\delta^2}{v}$$
(4)

where η - dimensionless transversal coordinate, Φ - dimensionless flow function, U-velocity ratio, S - dimensionless temperature in fluid, Θ - dimensionless temperature decrease, δ - transversal coordinate ratio. Note that with the use of boundary conditions of the second kind, Θ is represented differently.

On the basis of the idea of general similarity method in a somewhat simplified, special form, the solution of the problem can be assumed as follows

$$\Psi = U\delta\Phi[\eta; (f_{k,n}), (l_{k,n}), (g_{k,n})]$$

$$S = S[\eta; (f_{k,n}), (l_{k,n}), (g_{k,n})]$$
(5)

where the similarity parameters have the following forms

$$f_{k,n} = g^{k} z^{2k+n} \Theta^{k-1} \frac{\partial^{k+n} \Theta}{\partial x^{k} \partial t^{n}}, \quad (k, n = 0, 1, 2, ...; k \lor n \neq 0)$$

$$l_{k,n} = g^{k} z^{2(k-1)+n} \Theta^{k} \frac{\partial^{k+n} z^{2}}{\partial x^{k} \partial t^{n}}, \quad (k, n = 0, 1, 2, ...; k \lor n \neq 0)$$

$$g_{k,n} = g^{k-1} z^{2k-1+n} \Theta^{k-1} \frac{\partial^{k-1+n} N}{\partial x^{k-1} \partial t^{n}}, \quad (k, n = 0, 1, 2, ...; k \neq 0)$$
(6)

It can be noticed from the last expressions that the first parameters have the forms

$$f_{1,0} = gz^2 \frac{\partial \Theta}{\partial x}, \ f_{0,1} = \frac{z}{\Theta} \frac{\partial \Theta}{\partial t}, \ l_{1,0} = 2g\Theta z \frac{\partial z}{\partial x},$$

$$l_{0,1} = 2\frac{\partial z}{\partial t}, \ g_{1,0} = zN$$
(7)

Using now dimensionless variables (4) and the assumed solutions (5) of the equation (1), expressed by means of flow function, it is transformed into a new form.

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$$\frac{\partial^{3}\Phi}{\partial\eta^{3}} + \left(f_{1,0} + \frac{3}{4}l_{1,0}\right)\Phi\frac{\partial^{2}\Phi}{\partial\eta^{2}} - \left(f_{1,0} + \frac{1}{2}l_{1,0}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^{2} - \left(g_{1,0} + f_{0,1} + \frac{1}{2}l_{0,1}\right)\frac{\partial\Phi}{\partial\eta} + S + \frac{1}{4}\eta l_{0,1}\frac{\partial^{2}\Phi}{\partial\eta^{2}} = \sum_{\substack{k,n=0\\k\vee n\neq0}}^{\infty} \left[R_{k,n}X(\eta;f_{k,n}) + L_{k,n}X(\eta;l_{k,n})A_{k,n}\frac{\partial^{2}\Phi}{\partial\eta\partial f_{k,n}} + B_{k,n}\frac{\partial^{2}\Phi}{\partial\eta\partial l_{k,n}}\right] +$$
(8)
$$+ \sum_{\substack{k=1\\n=0}}^{\infty} \left[\Xi_{k,n}X(\eta;g_{k,n}) + C_{k,n}\frac{\partial^{2}\Phi}{\partial\eta\partial g_{k,n}}\right]$$

$$\frac{1}{\Pr} \frac{\partial^2 S}{\partial \eta^2} + \left(f_{1,0} + \frac{3}{4} l_{1,0} \right) \Phi \frac{\partial S}{\partial \eta} - \left(f_{1,0} \frac{\partial \Phi}{\partial \eta} + f_{0,1} \right) S - g_{1,0} \left(\frac{\partial \Phi}{\partial \eta} \right)^2 + \frac{1}{4} \eta l_{0,1} \frac{\partial S}{\partial \eta} =$$

$$= \sum_{\substack{k,n=0\\k \lor n \neq 0}}^{\infty} \left[R_{k,n} Y(\eta; f_{k,n}) + L_{k,n} Y(\eta; l_{k,n}) + A_{k,n} \frac{\partial S}{\partial f_{k,n}} + B_{k,n} \frac{\partial S}{\partial l_{k,n}} \right] +$$

$$+ \sum_{\substack{k=1\\n=0}}^{\infty} \left[\Xi_{k,n} Y(\eta; g_{k,n}) + C_{k,n} \frac{\partial S}{\partial g_{k,n}} \right]$$

$$(8)$$

where for the sake of shorter expression the following denotations have been introduced:

$$\begin{aligned} R_{k,n} &= [(k-1)f_{1,0} + \frac{1}{2}(2k+n)l_{1,0}]f_{k,n} + f_{k+1,n}, \\ L_{k,n} &= [kf_{1,0} + \frac{1}{2}(2k-2+n)l_{1,0}]l_{k,n} + l_{k+1,n}, \\ A_{k,n} &= [(k-1)f_{0,1} + \frac{1}{2}(2k+n)l_{0,1}]f_{k,n} + f_{k,n+1}, \\ B_{k,n} &= [kf_{0,1} + \frac{1}{2}(2k-2+n)l_{0,1}]l_{k,n} + f_{k,n+1}, \\ (k,n &= 0,1,2,...;k \lor n \neq 0) \end{aligned} \tag{9}$$

$$\begin{aligned} \Xi_{k,n} &= [(k-1)f_{1,0} + \frac{1}{2}(2k-1+n)l_{1,0}]g_{k,n} + g_{k+1,n}, \\ C_{k,n} &= [(k-1)f_{0,1} + \frac{1}{2}(2k-1+n)l_{0,1}]g_{k,n} + g_{k,n+1} \\ (k,n &= 0,1,2,...;k \neq 0) \\ X(x_{1};x_{2}) &= \frac{\partial \Phi}{\partial x_{1}} \frac{\partial^{2} \Phi}{\partial \eta \partial x_{2}} - \frac{\partial \Phi}{\partial x_{2}} \frac{\partial^{2} \Phi}{\partial x_{1} \partial \eta} \\ Y(x_{1};x_{2}) &= \frac{\partial \Phi}{\partial x_{1}} \frac{\partial S}{\partial x_{2}} - \frac{\partial \Phi}{\partial x_{2}} \frac{\partial S}{\partial x_{1}} \end{aligned}$$

The corresponding boundary conditions have the form of

$$\Phi = 0, S = 1 \qquad for \ \eta = 0$$

$$\Phi \to 0, S \to 0 \qquad for \ \eta \to \infty$$
(10)

and they have been obtained from the boundary conditions (2).

The obtained equations (8) do not explicitly depend on the distribution of temperatures or thermal flux on the vertical wall, nor on the exterior magnetic field, so that they can in that sense be regarded as universal equations of the observed problem. The boundary conditions (10) are also universal.

Equations (8) in an approximation, i.e. with a definite number of addends on the righthand sides and boundary conditions (10) are integraled once and for all. On the basis of the obtained universal results general conclusions about the development of the

boundary layer can be drawn. Thus obtained universal results should be conveniently preserved for the use in calculations of particular problems. With the integration of equations (8) with the boundary conditions (10) and storing of the obtained results, the first phase of the method has been completed.

In order to solve the problem completely in every individual case the following values have to be determined, $f_{k,n}(x,t)$, $l_{k,n}(x,t)$ and $g_{k,n}(x,t)$, and with that $\delta(x,t)$. Solving this problem is the second phase of the method. Appropriate integral equations are necessary for the realization of the second phase of the method, particularly the equation of impulse and the equation of energy. The mentioned integral equations are obtained by the integration of equations (8), element by element along η from 0 to ∞ , and by means of integration of the first, the impulse equation is obtained and by the integration of the second, we obtain the energy equation. In order to have simplified equations, for δ we select the following value

$$\delta_S^* = \int_0^\infty S \frac{\partial \Phi}{\partial \eta} dy \tag{11}$$

which is the functional of the distribution of velocity and temperature in the cross sections of the boundary layer.

By introducing the thickness of impulse loss δ^{**} and thickness δ_s , which are in this case defined with the relations

$$\frac{\delta^{**}}{\delta_S^*} = \int_0^\infty \left(\frac{\partial\Phi}{\partial\eta}\right)^2 d\eta , \quad \frac{\delta_S}{\delta_S^*} = \int_0^\infty S d\eta$$
(12)

we obtain a system of two integral equations

$$\begin{split} & \frac{\partial^{2} \Phi}{\partial \eta^{2}} [0; (f_{k,n}), (l_{k,n}), (g_{k,n})] + \left(2f_{1,0} + \frac{5}{4}l_{1,0}\right) \frac{\delta^{**}}{\delta_{S}^{*}} + \\ & + \left(g_{1,0} + f_{0,1} + \frac{3}{4}l_{0,1}\right) \Phi[\infty; (f_{k,n}), (l_{k,n}), (g_{k,n})] - \frac{\delta_{S}}{\delta_{S}^{*}} = \\ & = -\left[\sum_{\substack{k,n=0\\k\vee n\neq 0}}^{\infty} \left(R_{k,n} \frac{\partial}{\partial f_{k,n}} + L_{k,n} \frac{\partial}{\partial l_{k,n}}\right) + \sum_{\substack{k=1\\n=0}}^{\infty} \Xi_{k,n} \frac{\partial}{\partial g_{k,n}}\right] \left(\frac{\delta^{**}}{\delta_{S}^{*}}\right) - \\ & -\left[\sum_{\substack{k,n=0\\k\vee n\neq 0}}^{\infty} \left(A_{k,n} \frac{\partial}{\partial f_{k,n}} + B_{k,n} \frac{\partial}{\partial l_{k,n}}\right) + \sum_{\substack{k=1\\n=0}}^{\infty} C_{k,n} \frac{\partial}{\partial g_{k,n}}\right] \Phi[\infty; (f_{k,n}), (l_{k,n}), (g_{k,n})] \\ & \frac{1}{\Pr} \frac{\partial S}{\partial \eta} [0; (f_{k,n}), (l_{k,n}), (g_{k,n})] + \left(2f_{1,0} + \frac{3}{4}l_{1,0}\right) - g_{1,0} \frac{\delta^{**}}{\delta_{S}^{*}} + \left(f_{0,1} + \frac{1}{4}l_{0,1}\right) \frac{\delta_{S}}{\delta_{S}^{*}} = \\ & = -\left[\sum_{\substack{k,n=0\\k\vee n\neq 0}}^{\infty} \left(A_{k,n} \frac{\partial}{\partial f_{k,n}} + B_{k,n} \frac{\partial}{\partial l_{k,n}}\right) + \sum_{\substack{k=1\\n=0}}^{\infty} C_{k,n} \frac{\partial}{\partial g_{k,n}}\right] \left(\frac{\delta_{S}}{\delta_{S}^{*}}\right). \end{split}$$

The first equation of the system (13) is the impulse equation of the observed problem, and the other equation of the same system is the energy equation. In the last equations:

 $\frac{\partial^2 \Phi}{\partial \eta^2}[0; (f_{k,n}), (l_{k,n}), (g_{k,n})] \text{ is dimensionless friction and } \frac{\partial S}{\partial \eta}[0; (f_{k,n}), (l_{k,n}), (g_{k,n})]$

dimensionless thermal flux on the wall surface.



Fig. 1. Values A,C and $l_{1,0}$ in function of parameters

As has been already stated, the first phase of the method is completed with the integration of equations (8) in a corresponding approximation and storing of the obtained universal results. Further on, in this paper the influence of the parameters $f_{0,1}$; $l_{0,1}$; $l_{1,0}$ and $g_{1,0}$ and the influence of other parameters will be disregarded. The influence of derivatives along parameters $f_{0,1}$; $l_{1,0}$; $l_{0,1}$ will also be disregarded, and thus, from the equation (8) the following equations are obtained

$$\Im_{1} = f_{1,0}l_{1,0}X(\eta; f_{1,0}) + \frac{1}{2}l_{1,0}g_{1,0}X(\eta; g_{1,0}) + f_{1,0}l_{0,1}\frac{\partial^{2}\Phi}{\partial\eta\partial f_{1,0}} + \frac{1}{2}l_{0,1}g_{1,0}\frac{\partial^{2}\Phi}{\partial\eta\partial g_{1,0}}$$

$$\Im_{2} = f_{1,0}l_{1,0}Y(\eta; f_{1,0}) + \frac{1}{2}l_{1,0}g_{1,0}Y(\eta; g_{1,0}) + l_{0,1}f_{1,0}\frac{\partial S}{\partial f_{1,0}} + \frac{1}{2}l_{0,1}g_{1,0}\frac{\partial S}{\partial g_{1,0}}$$
(14)

where, for the sake of brevity, \mathfrak{I}_1 and \mathfrak{I}_2 mark the lefthand sides of the first and second equation (8), respectively.

The obtained system of equations (14) represents a fiveparameter three times localized approximation of the equation system (8). Integral equations (13) in the same approximation have the form of

$$\frac{\partial^2 \Phi}{\partial \eta^2} [0; f_{1,0}; f_{0,1}; l_{1,0}; l_{0,1}; g_{1,0}] + \left(2f_{1,0} + \frac{5}{4}l_{1,0}\right) \frac{\delta^{**}}{\delta_s^*} + \left(g_{1,0} + f_{0,1} + \frac{3}{4}l_{0,1}\right) \Phi[\infty; f_{1,0}; f_{0,1}; l_{1,0}; l_{0,1}; g_{1,0}] - \frac{\delta_s}{\delta_s^*} =$$
(15)



For numerical integration of equations (14), first we substitute the derivates with relations of finite differences and thus a system of differential equations is obtained, which is resolved by the use of progonka method [7].

Solving the equations has been realised from the point $f_{1,0}$ to the right point where is $l_{1,0}$ and to left to the point where $\partial S/\partial \eta(\eta = 0) = 0$. Solving of the equations mentioned above has been carried out for the values of Prantl number Pr = 0.72 but for several values of unsteadiness parameter $f_{0,1}$. A part of the obtained results has been shown in the diagram form, in the figures 1,2,3 and 4. Because of the shorter notification in the figures 1 and 3 the following notifications are introduced $(\partial^2 \Phi / \partial \eta^2)_w = C$ and $(\partial S / \partial \eta)_w = A$, where the notifications w mean that the values refer to the values of the functions on the wall. On the figures 2 and 4 the terms $\frac{\delta^{**}}{\delta_S^*} = \int_0^\infty \left(\frac{\partial \Phi}{\partial \eta}\right)^2 d\eta$, $\frac{\delta_S}{\delta_S^*} = \int_0^\infty S d\eta$ are replaced with notifications H1

and H2, respectively.



Fig. 3. Values A, C and 11,0 in function of parametres

In figures 1 and 2 the following denotations are introduced: N for $(g_{1,0} = 0; f_{0,1} = 0; l_{0,1} = 0);$ N3 for $(g_{1,0} = 0; f_{0,1} = -0.02; l_{0,1} = 0);$ N1 for $(g_{1,0} = 0.125; f_{0,1} = 0; l_{0,1} = 0);$ N4 for $(g_{1,0} = 0; f_{0,1} = 0; l_{0,1} = 0.012);$ N2 for $(g_{1,0} = 0; f_{0,1} = 0.06; l_{0,1} = 0);$ N5 for $(g_{1,0} = 0; f_{0,1} = 0; l_{0,1} = -0.06);$

and on the figures 3 and 4 the notifications *N* to *N*3 have the next form:

$$\begin{split} N & \text{for} \left(g_{1,0} = 0.02; f_{0,1} = 0.01; l_{0,1} = 0.005 \right); \\ N2 & \text{for} \left(g_{1,0} = 0.02; f_{0,1} = -0.01; l_{0,1} = 0.005 \right); \\ N1 & \text{for} \left(g_{1,0} = 0.02; f_{0,1} = 0.01; l_{0,1} = -0.02 \right); \\ N3 & \text{for} \left(g_{1,0} = 0.02; f_{0,1} = -0.01; l_{0,1} = -0.02 \right). \end{split}$$



Fig. 4. Values H1 and H2 in function of parameters

3. NUMERICAL RESULTS

From the figures 1,3 can be noticed that for $f_{0,1} < 0$ i.e., for the case when the dimensionless fluid temperature Θ decreases with the time, also the dimensionless friction on the wall surface C decreases, but the dimensionless heat flux on the wall surface A increases with respect to the absolute values. For positive values of unsteadiness parameter $f_{0,1}$ i.e., for the case of the increase of dimensionless temperature difference Θ , from the same figure, can be noticed that the increase of parameter $f_{0,1}$ increases also the dimensionless friction on the wall surface, but the dimensionless heat flux on the wall surface decreases at the absolute value.

From the figures 2,4 it can be seen that for the negative values of unsteadiness parameter $f_{0,1}$ the values of H1 and H2 decrease, but for the positive values of this parameter the values of H1 and H2 increase. If the value of $f_{0,1}$ is greater at the absolute value, so the values of H1 and H2 are smaller, but the greater positive values of unsteadiness parameter correspond to the greater values of H1 and H2.

With the increase of magnetic parameter $g_{1,0}$ value C decreases, and term A increases by the absolute value.

REFERENCES

- Cramer K.R. Several magnetohydrodynamic free convection solutions, Trans. ASME, Ser. C, 1963, vol.85, N1, p.35-40.
- 2. Gupta A.S. Steady and transient free convection of an electrically conducting fluid from a vertical plate in the in the presence of a magnetic field. Appl.Sci.Res., 1960, Vol. A9, N5, p319-333.
- Лойцянский Л.Г. Универсальные уравнения и параметрические приближения в теории ламинарного пограничного слоя. АН СССР, Прикладная математика и механика, 1965, т.29. Но 1, с.70-87.
- Saljnikov V.N. A contribution to universal solutions of the boundary layer theory. Teorijska i primenjena mehanika, 1978, br.4, p.139-163.
- Золотов Н.Л. Метод обобщеного подобия в задачах слободной конвекции с произвольным распределением температуры или теплового потока на вертикальной стенке. Изв. АН СССР, механика жидкости и газа, 1980, Но3, с.167-170.
- Saljnikov V.N., Boricic Z.B. Nikodijevic D.D. Natüliche Konvektionsströmung an einer senkrecht sthenden geheizeten porösen Platte. ZAMM 69 (1989)6, T648-T651.
- Симуни М.Л., Терентьев М.Н., Численное решение уравнений "однопараметической" теории пограничного слоя. Тр. Ленингр. Политехн. Ин-та., 1965, Ho248.

METODA UOPŠTENE SLIČNOSTI ZA NESTACIONARNE MHD PROBLEME SLOBODNE KONVENCIJE NA VERTIKALNOM ZIDU

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Ovaj rad se bavi nestacionarnim MHD problemima slobodne konvekcije u blizini zagrejanog zida sa proizvoljnim rasporedom temperature, odnosno toplotnog fluksa. Spoljašnje magnetno polje je homogeno i upravno na zid. Problem je razmatran u Busineskovoj aproksimaciji. Korišćenjem metode uopštene sličnosti i uvođenjem tri skupa parametara dobijen je sistem univerzalnih jednačina. Pored univerzalnih jednačina u radu su izvedene i dve integralne jednačine razmatranog problema. Sistem univerzalnih jednačina rešavan je metodom konačnih razlika. Deo dobijenih rezultata predstavljen je u obliku dijagrama.