UNIVERSITY OF NIŠ
The scientific journal **FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic Control and Robotics** Vol.2, No 10, 2000 pp. 1191 - 1202
Editor of series: Katica (Stevanovi}) Hedrih, e-mail: katica@masfak.masfak.ni.ac.yu
Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: +381 18 547-095, Fax: +381 18 547-950
http:// ni.ac.yu/Facta

QUADRATIC CONSERVATION LAWS FOR ONE-DEGREE-OF-FREEDOM MASS VARIABLE OSCILLATORS

UDC 531.231.531.5:517.925.4(045)

Livija Cvetićanin

Faculty of Engineering, 21000 Novi Sad, Trg D. Obradovica 6, Yugoslavia

Abstract. *In this paper the one-degree-of-freedom mass variable oscillators are considered. The mass variation is a function of time. Due to mass variation the reactive force acts. The motion of this system is described with a second order differential equation with time variable parameters. To find the closed form solution of the equation is impossible. In this paper the conservation laws of the systems are considered. The system has a Lagrangian. To form the invariants of the system the Noether's approach is applied. It is applied for determining the conservation laws of the rheo-linear, pure-cubic oscillator and a pendulum with variable mass and length.*

1 Introduction

There are many mechanical systems with time variable mass. The mass variation is usually aperiodic.. Previously the mass variation of the systems was neglected with explanation that the influence of mass change is unessential for dynamics of the whole system. However, in the most of the machines the mass variation has a significant influence on the behavior of the system. Such systems are the rocket motors [1], where the mass of the system is varying due to decrease of the amount of fuel. In the machines in process industry (centrifuges, sieves), textile industry, cable industry, carpet industry, transportation, etc. the mass of the system is increasing or decreasing in time [2]-[7]. In dynamical analysis of the system this fact has to be taken into consideration.

The dynamics of the systems with variable mass is based on the early papers of Meshchersky [8], who introduced the reactive force which appears due to mass variation. This force depends on the relative velocity of the added or separated

Received May 18, 2000

mass. For the one-degree-of-freedom system the differential equation of motion is according to Meshcherski

$$m(t)\ \ddot{x} + \frac{\partial V(x,t)}{\partial x} = (p-1)\ \dot{m}\dot{x}, \tag{1}$$

where $(p-1)\ \dot{m}\dot{x}$ is the reactive force, p is a constant parameter, $m(t)$ is mass variation, t is time, V is the potential energy of the system, $(\dot{\cdot}) = d/dt$, $(\ddot{\cdot}) = d^2/dt^2$. The eq. (1) is a second order ordinary differential equation with time variable parameters. To find its solution in the closed form is impossible. One of the ways for analyzing of the system is to obtain the first integrals of the system [9],[10] which will enable to find the solution of (1) if their number is sufficient or to form the Lyapunov function for stability analysis [11].

The purpose of the paper is to give a systematic study of the quadratic conservation laws of the one-degree-of-freedom for various types of rheo-linear and nonlinear systems with variable mass. It is worth to say that the mass variation function and the relation between absolute velocity of the added or separated mass and the velocity of the body (coefficient p) depends on the working properties of the system. The aim is to obtain general conservation laws for various mechanical systems (rheo-linear, with quadric or/and cubic nonlinearity). For these systems the potential energy is of polynomial type. The eq.(1) is non-conservative, but has a Lagrangian of the form

$$L = m^{1-p}\frac{\dot{x}^2}{2} - m^{-p}V(x,t). \tag{2}$$

The conservation laws will be obtained applying the Noether's approach [12]. The Noether's method for determining the conservation laws is adopted for the non-conservative systems described with (1) and Lagrangian (2).

2 Noether's approach

The Noether's approach for obtaining conservation laws is based on the invariance of the action integral. It states [12] that for every infinitesimal transformation of the generalized coordinates and time which leaves Hamilton action integral invariant there exists a conservation law of the dynamical system. The Hamilton;s action integral for the system with Lagrangian (2) is

$$I = \int_{t_1}^{t_2} [m^{1-p}\frac{\dot{x}^2}{2} - m^{-p}V(x,t)]dt, \tag{3}$$

where $t_0 - t_1$ is the domain of integration. We are concerned with the study of the transformation properties of the integral (3) with respect to the infinitesimal transformations of coordinate, Δx, and time, Δt, given by

$$\Delta x = \bar{x}\ (\bar{t}) - x(t) = \varepsilon F(t, x, \dot{x}), \tag{4}$$

$$\Delta t = \bar{t} - t = \varepsilon f(t, x, \dot{x}), \tag{5}$$

where the space generator F and time generator f are supposed to be functions of the generalized coordinate x, time t and generalized velocity $\dot{x}$. ε is the small positive dimensionless parameter. The action integral is gauge invariant if

$$\Delta I = \Delta \int_{t_1}^{t_2} [m^{1-p} \frac{\dot{x}^2}{2} - m^{-p} V(x,t)] dt = \int_{t_1}^{t_2} \varepsilon \dot{P}(t, x, \dot{x}) dt, \tag{6}$$

where $P(t, x, \dot{x})$ is the gauge function, i.e., the condition of gauge invariance is

$$-m^{-p} \frac{\partial V(x,t)}{\partial x} F + m^{1-p} \dot{x} \dot{F} - m^{1-p} [\frac{\dot{x}^2}{2} + \frac{V(x,t)}{m}] \dot{f}$$

$$+f\{(1-p) m^{-p} \dot{m} \frac{\dot{x}^2}{2} - \frac{\partial}{\partial t}[m^{-p} V(x,t)]\} - \dot{P} = 0, \tag{7}$$

where $F \equiv F(x, \dot{x}, t), f \equiv f(x, \dot{x}, t), P \equiv P(x, \dot{x}, t)$. The relation (7) represents the basic Noether's identity for (1). After some transformations, described in [12], the following conservation law is obtained

$$m^{1-p}[\dot{x} F - (\frac{\dot{x}^2}{2} + \frac{V(x,t)}{m}) f] - P = const. \tag{8}$$

The main problem in defining the conservation law is to find the space and time generator functions (4) and (5) and the gauge function P which have to satisfy the identity (7). There are no rules for obtaining them.

In this paper only the quadratic conservation laws will be considered. The functions F, f and P will be assumed in the form which will give the conservation law with maximal order of generalized velocity $\dot{x}$ to be two.

3 Killing's equations

Let us assume that the gauge function, space generator and time generator are dependent on the general velocity $\dot{x}$ with second, first and zero order, respectively

$$P = P_0(x,t) + \dot{x} P_1(x,t) + \dot{x}^2 P_2(x,t), \tag{9}$$

$$F = F_0(x,t) + \dot{x} F_1(x,t), \tag{10}$$
$$f = f_0(x,t), \tag{11}$$

where $f_0 \equiv f_0(x,t), F_0 \equiv F_0(x,t)$, $F_1 \equiv F_1(x,t)$, $P_0 \equiv P_0(x,t)$, $P_1 \equiv P_1(x,t), P_2 \equiv P_2(x,t)$. Using the assumptions (9)-(11) the conservation law (8) is

$$\begin{aligned} const. \; = \; & \dot{x}^2 (m^{1-p} F_1 - \frac{1}{2} m^{1-p} f_0 - P_2) \\ & + \dot{x} (m^{1-p} F_0 - P_1) - (P_0 + m^{-p} V f_0), \end{aligned} \tag{12}$$

where $V \equiv V(x,t)$. Substituting (9)-(11) into Noether's identity (7) and separating the terms with the same order of the generalized velocity $\dot{x}$ a system of so called Killing's partial differential equations is obtained

$$\dot{x}^3: \quad 0 = \frac{\partial}{\partial x}(m^{1-p}F_1 - \frac{1}{2}m^{1-p}f_0 - P_2), \tag{13}$$

$$\dot{x}^2 \quad : \quad 0 = \frac{\partial}{\partial x}(m^{1-p}F_0 - P_1) + \frac{\partial}{\partial t}(m^{1-p}F_1 - \frac{1}{2}f_0 m^{1-p} - P_2)$$
$$+2(p-1)\frac{\dot{m}}{m}(m^{1-p}F_1 - \frac{1}{2}f_0 m^{1-p} - P_2), \tag{14}$$

$$\dot{x} \quad : \quad 0 = -\frac{\partial}{\partial x}(P_0 + m^{-p}Vf_0) - 2m^{-1}\frac{\partial V}{\partial x}(F_1 m^{1-p} - \frac{f_0}{2}m^{1-p} - P_2)$$
$$+\frac{\partial}{\partial t}(m^{1-p}F_0 - P_1) - (1-p)\frac{\dot{m}}{m}(m^{1-p}F_0 - P_1), \tag{15}$$

$$\dot{x}^0: \quad 0 = \frac{1}{m}\frac{\partial V}{\partial x}(m^{1-p}F_0 - P_1) + \frac{\partial}{\partial t}(P_0 + m^{-p}Vf_0). \tag{16}$$

Integrating the eq.(13) it is evident that the function depends only on time, i.e.

$$m^{1-p}F_1 - \frac{1}{2}m^{1-p}f_0 - P_2 = \theta(t). \tag{17}$$

¿From the eq.(14) it is

$$m^{1-p}F_0 - P_1 = xc(t) + \varphi(t), \tag{18}$$

where

$$c(t) = -\dot{\theta} - 2(p-1)\frac{\dot{m}}{m}\theta. \tag{19}$$

Substituting (17) and (18) into (16) and integrating it is

$$P_0 + m^{-p}Vf_0 = -\int \frac{1}{m}\frac{\partial V}{\partial x}[xc(t) + \varphi(t)]dt. \tag{20}$$

Using the relations (17), (18) and (20) and the eq.(12) it can be stated:

Theorem 1 *The system (1) has the conservation law*

$$const. = \dot{x}^2\,\theta(t) + \dot{x}\,[xc(t) + \varphi(t)] + \int \frac{1}{m}\frac{\partial V}{\partial x}[xc(t) + \varphi(t)]dt, \tag{21}$$

where the functions $\theta(t), \varphi(t)$ and $\phi(t)$ satisfy the so called potential function

$$0 = \int \frac{1}{m}\frac{\partial V}{\partial x}[xc(t) + \varphi(t)]dt - \frac{2V}{m}\theta(t) + \frac{x^2}{2}\,\dot{c}\,(t) + x\,\dot{\varphi}\,(t)$$
$$-(1-p)\frac{\dot{m}}{m}[\frac{x^2}{2}c(t) + x\varphi(t)] + \phi(t). \tag{22}$$

For the known potential energy function the eq.(22) is separated in a system of equations with the same degree of generalized coordinate x. The number of so obtained equations depends on the degree of the polynomial function which describes the potential energy.

Remark 1 *For the polynomial potential function* $V = \sum_{i=1}^{n} x^i$ *where*

$$n \leq 2,$$

we obtain three equations in (22) which enable us to denote all three unknown functions without any limitation. For $n > 2$ *the number of equations is larger than the number of unknown functions. The system of equations gives us the three unknown functions and* $(n-3)$ *limitations. Usually it is connected with the form of the mass variation or the parameter of reactive force* p. *It destroys the generality of the suggested method and gives some partial conservation laws for special cases.*

Remark 2 *The same form of conservation law (21) is retained for another assumptions for the gauge function, time and space generators.. For example:*

The gauge function and space generators are linear functions of $\dot{x}$,

$$\begin{aligned} P &= P_0(x,t) + \dot{x}\, P_1(x,t), \\ F &= F_0(x,t) + \dot{x}\, F_1(x,t), \end{aligned}$$

and

$$f = f_0(x,t), \tag{23}$$

or the gauge function, time and space generators are functions of generalized coordinates and time

$$P = P_0(x,t), \quad F = F_0(x,t), \quad f = f_0(x,t). \tag{24}$$

Let us consider some special cases of the potential energy.

4 Examples

4.1 Rheo-linear system

There are a lot of papers dealing with the problem of conservation law in the rheo-linear system [13]-[17]. Various methods are applied for determining them. Unfortunately, till nowadays a general conservation law for these systems is not obtained.

For the linear system the potential energy is

$$V = Bx^2,$$

and the differential equation of motion is

$$m(t)\ \ddot{x} + 2Bx = (p-1)\ \dot{m}\dot{x}, \tag{25}$$

where $B = const.$ the equation (22) transforms to a system of three integro-differential equations

$$0 = 2B\int \frac{c(t)}{m}dt - \frac{2B}{m}\theta(t) + \frac{\dot{c}\ (t)}{2} - (1-p)\frac{\dot{m}}{m}\frac{c(t)}{2}, \tag{26}$$

$$0 = 2B\int \frac{\varphi(t)}{m}dt + \dot{\varphi}\ (t) - (1-p)\frac{\dot{m}}{m}\varphi(t), \tag{27}$$

$$\phi(t) = 0. \tag{28}$$

Solving the eq.(26) and (27) the unknown functions are obtained.

The another form of the eq.(26) is

$$\frac{1}{2}m^{2(1-p)}\ \dddot{u} + \frac{3}{4}\ \ddot{u}\ \frac{d}{dt}(m^{2-4p}) + \dot{u}\ [4Bm^{1-2p} + \frac{1}{4}\frac{d^2}{dt^2}(m^{2-4p})] + 2Bu\frac{d}{dt}(m^{1-2p}) = 0, \tag{29}$$

where

$$u(t) \equiv u = \frac{\theta}{m^{2(1-p)}}. \tag{30}$$

One of the solutions of the equation (27) is

$$\varphi(t) = 0, \tag{31}$$

and the corresponding conservation law is

$$const. = \dot{x}^2\ \theta(t) - \dot{x}\ x[\dot{\theta} + 2(p-1)\frac{\dot{m}}{m}\theta] - 2Bx^2\int \frac{[\dot{\theta} + 2(p-1)\frac{\dot{m}}{m}\theta]}{m}dt, \tag{32}$$

i.e.

$$\begin{aligned} const. \quad = \quad & \dot{x}^2\ m^{2(1-p)}u(t) + \dot{x}\ [-xm^{2(1-p)}\frac{d}{dt}(u(t))] \\ & + \int \frac{1}{m}\frac{\partial V}{\partial x}[\varphi(t) - xm^{2(1-p)}\frac{d}{dt}(u(t))]dt. \end{aligned} \tag{33}$$

To find the solution for (29) in the closed form is impossible. Some special cases will be considered.

4.1.1 Case 1.

Let us assume that

$$u = \int m^{p-1}dt. \tag{34}$$

Substituting (34) into (29) the following mass function is obtained

$$\frac{2}{1-2p}m^p + \dot{m}\int m^{p-1}dt = 0. \tag{35}$$

The relation (35) is satisfied for

$$m = (2t + K)^2,$$

where $K = const$. Using the results (31) and (35) it can be concluded that for the differential equation

$$(2t + K)^2 \ddot{x} + 2Bx = 4(p - 1)(2t + K)\dot{x}, \tag{36}$$

there exists a conservation law

$$\frac{(2t + K)^{1-2p}}{2(2p - 1)}[(2t + K)^2 \dot{x}^2 - 2(2p - 1)(2t + K)x\dot{x} + 2Bx^2] = const. \tag{37}$$

For the case when the impact force is zero ($p = 1$) the differential equation is

$$\ddot{x} + \frac{2B}{(2t + K)^2}x = 0, \tag{38}$$

and the first integral is

$$\frac{1}{2(2t + K)}[(2t + K)^2 \dot{x}^2 - 2x\dot{x}(2t + K) + 2Bx^2] = const. \tag{39}$$

4.1.2 Case 2.

For the case when the mass variation is without impact force and

$$p = 1,$$

the differential equation of motion is

$$\ddot{x} + \frac{2B}{m(t)}x = 0, \tag{40}$$

the eqs.(26) and (27) transform to a system of two independent differential equations

$$\dddot{\theta} + \frac{8B}{m}\dot{\theta} - \frac{4B}{m^2}\dot{m}\,\theta = 0, \tag{41}$$

$$\ddot{\varphi} + \frac{2B}{m}\varphi = 0. \tag{42}$$

The eq.(42) is satisfied for

$$\varphi(t) = 0, \tag{43}$$

and the conservation law is

$$const. = \dot{x}^2\,\theta(t) - \dot{x}\,x\,\dot{\theta}(t) + 2Bx^2\frac{\theta(t)}{m} + \frac{1}{2}x^2\,\ddot{\theta}(t). \tag{44}$$

Let us assume a special case. The mass variation is

$$m(t) = m_0 + \mu t,$$

where $\mu = 1$. For $B = 1/8$ the differential equation of motion is

$$\ddot{x} + \omega^2(t)x = 0, \tag{45}$$

where

$$\omega^2(t) = \frac{1}{4m(t)}. \tag{46}$$

For (45), the differential equation (41) is

$$\dddot{\theta} + \frac{\dot{\theta}}{m} - \frac{\theta}{2m^2} = 0.$$

It represents a special type of Bessel's equation [18]. The solutions of the equation are

$$\theta(t) = mJ_1^2(\sqrt{m}), \quad \theta(t) = mJ_1(\sqrt{m})Y_1(\sqrt{m}), \quad \theta(t) = mY_1^2(\sqrt{m}),$$

where J_1 is the the first order Bessel's function and Y_1 is the second order Bessel's function.

4.2 The pure cubic system

For the pure cubic nonlinear system the differential equation of motion is

$$m(t)\ddot{x} + 4Dx^3 = (p-1)\dot{m}(t)\dot{x}, \tag{47}$$

where the potential energy is

$$V = Dx^4, \tag{48}$$

and $D = const.$. Substituting (48) in (22) and separating the terms with the same order of the generalized coordinate x the following system of integro-differential equations is obtained

$$x^4: \quad \int \frac{c}{m}dt - \frac{\theta}{2m} = 0, \tag{49}$$

$$x^3: \quad 4D\int \frac{\varphi}{m}dt = 0, \tag{50}$$

$$x^2: \quad \dot{c} - (1-p)\frac{\dot{m}}{m}c = 0, \tag{51}$$

$$x: \quad \dot{\varphi} - (1-p)\frac{\dot{m}}{m}\varphi = 0, \tag{52}$$

$$x^0: \quad \phi = 0. \tag{53}$$

From the eq.(50) it is

$$\varphi = 0, \tag{54}$$

and the eq.(52) is identically satisfied. The solution of the eq.(51) is

$$c = m^{1-p}, \tag{55}$$

i.e., according to (19)

$$-m^{1-p} = \dot{\theta} + 2(p-1)\frac{\dot{m}}{m}\theta. \tag{56}$$

Substituting (55) in (49) it is

$$\int m^{-p} dt = \frac{\theta}{2m}. \tag{57}$$

The system of coupled equations (56) and (57) gives us the arbitrary function

$$\theta = \frac{3m^{2-p}}{(1-2p)\,\dot{m}}, \tag{58}$$

but also the limitation for mass variation

$$\ddot{m} = \frac{p+1}{3}\frac{\dot{m}^2}{m}. \tag{59}$$

It means that the conservation law

$$\frac{3\,\dot{m}}{1-2p} m^{1-p}(m\,\dot{x}^2 + 2Dx^4) + x\,\dot{x}\,m^{1-p} = const., \tag{60}$$

is valid only for mass variation (59).

4.2.1 Case 1.

If the mass variation is an exponential function

$$m = e^{at}, \quad \text{or} \quad m = e^{-at},$$

and the parameter p is

$$p = 2,$$

the differential equation of motion is

$$e^{\pm at}\,\ddot{x} + 4Dx^3 = \pm a e^{\pm at}\,\dot{x}, \tag{61}$$

and the conservation law is

$$const. = \frac{1}{a}e^{\mp at}\,\dot{x}^2 \mp x\,\dot{x}\,e^{\mp at} + 2\frac{D}{a}e^{\mp 2at}x^4. \tag{62}$$

4.2.2 Case 2.

The differential equation (59) has a partial solution

$$m = (\frac{2-p}{3}t)^{\frac{3}{2-p}},$$

for $p \neq 2$. The mass variation is a function of parameter p. The differential equation of motion is

$$(\frac{2-p}{3}t)^{\frac{3}{2-p}} \ddot{x} + 4Dx^3 = (p-1)(\frac{2-p}{3}t)^{\frac{1+p}{2-p}} \dot{x}, \tag{63}$$

and the conservation law is

$$(\frac{2-p}{3}t)^{\frac{3(1-p)}{2-p}}\{\frac{3}{1-2p}(\frac{2-p}{3}t)^{\frac{1+p}{2-p}}[(\frac{2-p}{3}t)^{\frac{3}{2-p}} \dot{x}^2 + 2Dx^4] + x\,\dot{x}\} = const. \tag{64}$$

For the Levi-Civita case, when the absolute velocity of the added mass is zero and $p = 0$, the differential equation of motion is

$$\frac{d}{dt}[(\frac{2}{3}t)^{1.5}] \ddot{x} + 4Dx^3 = 0, \tag{65}$$

the conservation law is

$$(\frac{2t}{3})^{1.5}\{3(\frac{2t}{3})^2 \dot{x}^2 + 6Dx^4(\frac{2t}{3})^{0.5} + x\,\dot{x}\} = const. \tag{66}$$

For the case when the reactive force is neglected, i.e., the relative velocity of mass variation is zero and $p = 1$, the differential equation of motion is

$$(\frac{2}{3}t)^{1.5} \ddot{x} + 4Dx^3 = 0, \tag{67}$$

and the conservation law is

$$(\frac{t}{3})^3[-3(\frac{t}{3})^{1.5} \dot{x}^2 - 6Dx^4] + \frac{t}{3}x\,\dot{x} = const. \tag{68}$$

4.3 Pendulum with variable mass and length

The motion of a pendulum with variable mass and variable length is described with a differential equation

$$m(t) \ddot{x} + 4D(t)x^3 = (p-1)\, \dot{m}(t)\, \dot{x}, \tag{69}$$

where the parameters of the equation are time dependent. Substituting

$$V = D(t)x^4,$$

in the eq.(22), it is

$$\varphi(t) = 0, \quad \phi(t) = 0, \tag{70}$$

and

$$\theta(t) = D^{-\frac{1}{3}} m^{\frac{5-4p}{3}}. \tag{71}$$

The relation between the parameter $D(t)$ and mass variation $m(t)$ is

$$D(t) = \frac{m^{2p-1}}{[\int m^{p-1} dt]^3}. \tag{72}$$

It means that the generality of the problem is limited as there exists a a dependence of the length variation function on the mass variation. The conservation law is for (72)

$$\dot{x}^2 \, m^{\frac{5-4p}{3}} D^{-\frac{1}{3}} - x \, \dot{x} \, m^{1-p} + 2x^4 D^{\frac{2}{3}} m^{\frac{2-4p}{3}} = const. \tag{73}$$

4.3.1 Case 1.

Let us consider a special case when the impact force is zero

$$p = 1.$$

The variable coefficient is

$$D(t) = \frac{m}{t^3},$$

and the differential equation of motion is

$$\ddot{x} + \frac{4}{t^3} x^3 = 0. \tag{74}$$

The corresponding conservation law is

$$\dot{x}^2 \, t - x \, \dot{x} + \frac{2}{t^2} x^4 = const. \tag{75}$$

References

[1] Cornelisse, J.W., Schoyer, H.F.R., Wakker, K.F., Rocket propulsion and space-flight dynamics, Pitman, London, 1979.

[2] Bessonov, A.P., Osnovji dinamiki mehanizmov s peremnnoj massoj zvenjev, Nauka, Moscow, 1967

[3] Cveticanin, L., Dynamics of machines with variable mass, Gordon and Breach Publishers, London, 1998.

[4] Eke, F.O., Wang, S.M., Attitude behavior of a variable mass cylinder, Journal of Applied Mechanics, Vol.62, pp.935-940, 1995.

[5] Wang, S.M., Eke, F.O., Rotational dynamics of axisymmetric variable mass systems, Trans. ASME, Journal of Applied Mechanics, Vol.62, Dec., pp.970-974, 1995.

[6] Eke, F.O., Wang, S.M., Equations of motion of two phase variable mass systems with solid base, Journal of Applied Mechanics, Vol.61, pp.855-860, 1994.

[7] Strzalko, J., Grabski, J., Dynamic analysis of a machine model with time-varying mass, Acta Mechanica, Vol.112, pp.173-186, 1995.

[8] Meshcherski, I.V., Rabotji po mehanike tel peremennoj massji, Gos.izd. teh.teoreticheskoj literaturji, Moscow, 1952.

[9] Cveticanin, L., Conservation Laws in Systems With Variable Mass, Journal of Applied Mechanics, Vol., No. pp. 1994.

[10] Cveticanin, L.J., On the stability of rheo-linear rotor systems based on some new first integrals, Mechanics, Research Communications, Basic and Applied, Vol.23, No.5, pp.519-530, 1996.

[11] Cveticanin, L., The influence of reactive force on the stability of motion for one degree of freedom mechanisms with variable mass, Machine Vibration, Vol.5, pp.224-228, 1996.

[12] Vujanovic, B.D., Jones, S.E., Variational Methods in Nonconservative Phenomena, Academic Press, New York, 1989.

[13] Lewis, H.R., Class of exact invariants for classical and quantum time-dependent harmonic oscillator, Journal of Mathematical Physics, Vol.9, pp.1976-1986, 1968.

[14] Vujanovic, B.D., Conservation laws of rheo-linear dynamical system with one and two-degrees of freedom, Int. Journal of Non-Linear Mechanics, Vol.27, pp. 309-322, 1992.

[15] Vujanovic, B.D., Application of the field-momentum method to rheonomic dynamics, Int. Journal of Non-Linear Mechanics, Vol.29, pp.515-528, 1994.

[16] Vujanovic, B., Kawaguchi T., Simic, S., A class of conservation laws of linear time-dependent dynamical systems, Tensor, Vol.58, pp.243-252, 1997.

[17] Simic, S., Cubic invariants of one-dimensional Lagrangian systems, International Journal of Non-Linear Mechanics, Vol.35, pp.333-345, 2000.

[18] Abramowitz, M., Stegun, I.A., Handbook of mathematical functions (in Russian), Nauka, Moscow, 1979.

KVADRATNI ZAKONI ODRŽANJA ZA OSCILATORE PROMENLJIVE MASE SA JEDNIM STEPENOM SLOBODE

Livija Cvetićanin

U ovom radu se razmatraju oscilatori promenljive mase sa jednim stepenom slobode. Promena mase je funkcija vremena. Usled promene mase se javlja reaktivna sila. Kretanje ovog sistema je opisano diferencijalnom jednačinom drugog reda sa vremenski promenljivim parametrima. Nemoguće je naći rešenje ove jednačine u zatvorenom obliku. U ovom radu razmatraju se zakoni održanja sistema. Za formiranje invarijanti sistema primenjen je Noether-in pristup. On je primenjen za određivanje zakona održanja reo-linearnog, čisto kubnog oscilatora i klatna promenljive mase i dužine.