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AUTONOMOUS WEAKLY DAMPED CONSERVATIVE SYSTEMS EXHIBITING LIMIT CYCLES

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Abstract. *The present work reexamines in detail the local dynamic stability aspects of autonomous weakly damped discrete Hamiltonian systems. For such potential systems the existence of periodic attractors (limit cycles) is thoroughly discussed via the study of the effect of the algebraic form of the damping matrix on the Jacobian eigenvalues. New findings contradicting existing results are assessed. Thus, if damping is accounted for, the solution of the nondissipative symmetric systems may lose its stability through (a) a double-zero eigenvalue bifurcation, a usual Hopf bifurcation and a degenerate (isolated) Hopf bifurcation and (b) a limit cycle (dynamic) instabilty mode prior to divergence (static) mode, for which Ziegler's kinetic criterion fails. The validity of the presented theoretical findings is verified through a variety of numerical results.*

INTRODUCTION

The importance of the effect of damping on the dynamic stability of non-self adjoint (non-conservative) systems has drawn and continues to draw particular attention among researchers. This effect however was usually ignored when dealing with Hamiltonian or potential (symmetric) systems, since it was widely believed and accepted that, if these undamped systems are stable, the inclusion of damping in the analysis does not change their stability. The response of such potential dissipative systems, in the context of local (linear) analysis can be described in terms of generalized coordinates by a matrix-vector differential equation of the form:

$$[\alpha_{ij}]\ddot{q} + [c_{ij}]\dot{q} + [V_{ij}]q = 0 \tag{1}$$

where q(t) is an n-dimensional state vector and $[\alpha_{ij}]$, $[c_{ij}]$ and $[V_{ij}]$ are real $n \times n$ matrices. More specifically, matrix $[\alpha_{ij}]$ is positive definite associated with the quadratic form of the total kinetic energy, with elements being functions of the concentrated masses, i.e. $[\alpha_{ij}] = [\alpha_{ij}(m_i)]$; $[c_{ij}]$ is the positive definite, positive semi-definite or indefinite damping

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matrix, while $[V_{ij}]$ is a generalized stiffness matrix, being also a function of the external loading λ, i.e. $[V_{ij}] = [V_{ij}(\lambda)]$, which is suddenly applied with constant direction (conservative) and of infinite duration. This is a special loading case, since in the more general one λ is of varying direction (follower), a fact implying several important phenomena associated with non-potential systems, as reported in a number of pertinent studies [Inman (1983), Inman and Olsen (1988), Kounadis (1997), Kounadis and Simitses (1997) and many others not cited herein].

The present work aims to examine in detail whether similar phenomena, as the ones exhibited by non-potential systems, may also occur in the case of potential (symmetric) ones, under the effect of slight damping. Its role in conjunction with the variation of the main control parameter λ on the local dynamic stability of weakly damped Hamiltonian systems will be thoroughly discussed. In particular, it will be investigated and clarified whether symmetric weakly damped systems may under certain conditions exhibit (a) a double zero eigenvalue bifurcation and (b) a Hopf bifurcation or flutter instability prior to divergence. If either of the above is true, once again the kinetic (dynamic) stability criterion of Ziegler (associated with the vanishing of the fundamental frequency) will no longer be valid, a phenomenon evidently explored by Kounadis and co-workers for non-conservative systems.

Thus, for weakly damped symmetric systems described by eq.(1) steady-state solutions will be sought, associated with isolated periodic motions (limit cycles – periodic attractors). Rigorous definitions and references on the rapidly developing theory of dynamical systems will not be used, although the linearized dynamics and the stability of local bifurcations arising in the systems dealt with can also be studied via well established methods, such as the center manifold technique.

The theoretical findings of the present work will be comprehensively verified by a variety of numerical examples based on the 2-D.O.F. classical inverted pendulum (Ziegler's model), for which a great amount of numerical results, based on both linear and nonlinear analyses, is available.

PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

1. Statement of the problem

We seek solutions of eq. (1) in the form

$$q = re^{\rho t} \tag{2}$$

with ρ being, in general, a complex number and r a complex vector, independent of t. Combining this expression with eq. (1) one may write

$$L(\rho(\rho = ([\alpha_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}])r = 0 \tag{3}$$

where L(ρ) is a matrix-valued function. Solutions of (1) are closely related to the algebraic properties of L(ρ), and more specifically to the nature of the Jacobian eigenvalues, which are obtained by solving the characteristic equation $\det L(\rho) = 0$, which guarantees the existence of nontrivial solutions of eqs. (1) or (3) and possesses n complex conjugate pairs of solutions of the form $\rho_i = \nu_i + \mu_i j$, $j^2 = -1$, $\nu_i, \mu_i \in R$ with corresponding complex conjugate eigenvectors. Thus, the solutions of eq.(1) associated with (2) are of the form

$$A_i e^{\nu_i t} \cos\mu_i t \text{ and } B_i e^{\nu_i t} \sin\mu_i t \tag{4}$$

with A, B constants depending on the initial conditions. Clearly, solutions (4) are bounded and tend to zero as $t \to \infty$, if all eigenvalues have negative real parts.

Furthermore, if $\bar{r}^T$ is the conjugate transpose of r, it can be readily shown that

$$\rho = \frac{1}{2\bar{r}^T[\alpha_{ij}]r}\left[-\bar{r}^T[c_{ij}]r \pm \sqrt{(\bar{r}^T[c_{ij}]r)^2 - 4(\bar{r}^T[\alpha_{ij}]r)(\bar{r}^T[V_{ij}]r)}\right] \tag{5}$$

Using the latter expressions of the complex conjugate eigenvectors

$$r = x + jy,\ \bar{r} = x - jy \tag{6}$$

introducing $\rho = \nu + j\mu$, $r = x + jy$ into eq.(3) and setting real and imaginary parts equal to zero, we obtain:

$$\left.\begin{aligned} ([\alpha_{ij}](\nu^2 - \mu^2) + \nu[c_{ij}] + [V_{ij}])x &= \mu([c_{ij}] + 2\nu\nu\alpha_{ij}])y \\ ([\alpha_{ij}](\nu^2 - \mu^2) + \nu[c_{ij}] + [V_{ij}])y &= -\mu([c_{ij}] + 2\nu\nu\alpha_{ij}])x \end{aligned}\right\} \tag{7}$$

which is a homogenous system with respect to x and y, whose determinant must vanish for a nontrivial solution.

Since ν, μ, r and stiffness matrix $[V_{ij}]$ all depend directly on the loading λ, as this increases gradually from zero, the variation of $[V_{ij}]$ has a direct effect on the eigenvalues; namely $[V_{ij}]$ is positive definite for λ below the 1st buckling load (i.e. for $\lambda < \lambda_c^{(1)}$), positive semi-definite at $\lambda = \lambda_c^{(1)}$ (implying that $\det[V_{ij}] = 0$) and indefinite for $\lambda > \lambda_c^{(1)}$ (since $\det[V_{ij}] < 0$).

Assuming slight (weak) damping due to physical considerations, the response of the foregoing system will be discussed in connection with the algebraic structure of the dissipation matrix $[c_{ij}]$, while the initial equilibrium path (for small λ) is assumed to be asymptotically stable, regardless of the amount of damping accounted for.

In addition to the above, one must quote the classical Routh-Hurvitz stability criteria, which will not be further addressed, since their use is associated with cumbersome mathematical obstacles and sometimes with reduced adequacy, especially for multi D.O.F. systems.

2. Mathematical analysis

Focusing our attention on the effect of the algebraic structure of definite damping matrices on the local dynamic stability of the systems under consideration, three characteristic $[c_{ij}]$ cases will be discussed in detail, namely when the damping matrix is positive definite, positive semi-definite and finally indefinite.

At this point it must be reminded that the simplest steady-state, i.e. equilibria (fixed points) are not affected by damping, a fact not valid for all others, as for instance for periodic motions.

a. Case 1: $[c_{ij}]$ positive definite

In this first case the quantity under the radical in eq.(5) is negative for $\lambda < \lambda_{(1)}^{c}$ and hence [Kounadis (2000)] the aforementioned equation yields complex conjugate eigenvalues with negative real parts as long as the above inequality is valid. At the static (divergence) critical state identified by $\lambda = \lambda_{(1)}^{c}$, implying $\det[V_{ij}] = 0$, it is evident that there exist one zero and one negative eigenvalue and hence, if the nondissipative system is stable, the corresponding dissipative one is asymptotically stable. Thus, from eq.(8) it follows that at least one pair of complex conjugate eigenvalues with negative real parts transforms at the 1st buckling load into one zero (becoming afterwards positive) and one negative (gradually decreasing) eigenvalue, moving in opposite directions along the real axis.

b. Case 2: $[c_{ij}]$ positive semi-definite

This second case, for which $\det[c_{ij}] = 0$, will be investigated in connection with the sign of quadratic form $Q_1 = \bar{r}^T[V_{ij}(\lambda)]r$. Two subcases will be considered, related to positive definite and positive semi-definite Q_1 respectively.

Proceeding with the first subcase, this occurs when $\lambda < \lambda_{(1)}^{c}$. The eigenvalues of the corresponding characteristic equation and the related eigenvectors are associated, in general, with $Q_1 > 0$. Thus, from eq.(5) it follows that all eigenvalues have negative real parts. At a certain value of $\lambda > 0$ however, the corresponding quadratic quantity may become equal to zero, and since the damping matrix is positive semi definite, we get

$$[c_{ij}]r = 0 \tag{8}$$

yielding in the sequel that

$$([\alpha_{ij}]\rho^2 + [V_{ij}(\lambda\lambda)], \quad r = 0 \tag{9}$$

Clearly, if **r** is an eigenvector of the nondissipative system (9) satisfying eq.(8), then r is also an eigenvector of the dissipative system of eq.(3) with purely imaginary eigenvalues. Setting $\rho = \pm j\mu$ into eq.(9), one determines the corresponding eigenvector r, which is r. Furthermore, the fact that the damping matrix is positive definite implies that one of its eigenvalues is zero and hence

$$([c_{ij}] - 0I_n)\, r = 0 \tag{10}$$

Thus, the real eigenvector $r \neq 0$, corresponding to the zero eigenvalue of $[c_{ij}]$ can be readily determined, while for the foregoing subcase eq.(5) yields

$$-\rho^2 = \mu^2 = \frac{\bar{r}^T[V_{ij}(\lambda\lambda)]}{\bar{r}^T[\alpha_{ij}(m_i)]r} \tag{11}$$

from which μ can be obtained implicitly as a function of λ for a given matrix $[\alpha_{ij}(m_i)]$.

Combining eqs.(9) and (10) and eliminating ρ^2, one can determine a load $\lambda > 0$, defined as the critical load corresponding to a local dynamic bifurcation. Inserting this load into eq. (11) we get the pair of conjugate purely imaginary eigenvalues implying periodic motion. Such a load $\lambda = \lambda_H(c_{ij}, m_i)$ corresponds to a degenerate Hopf bifurcation since it can be conveniently proven [Kounadis (2000)] that the transversality condition

$dv/d\lambda|_{\lambda=\lambda_H} \neq 0$ is violated. From a practical point of view however, it is more important the case of dynamic instability occurring prior to divergence, i.e. when

$$0 < \lambda_H < \lambda_{(1)}^c \tag{12}$$

If the above inequality is satisfied, we reach to the unexpected result that a symmetric (potential) weakly damped system may exhibit a dynamic bifurcation at a load smaller than the divergence (static) buckling load. This is an isolated (local) degenerate Hopf bifurcation, since the entire trivial state, which is defined by $0 < \lambda < \lambda_{(1)}^c$, excluding $\lambda = \lambda_H$, is asymptotically stable.

For the second subcase, both $[c_{ij}]$ and Q_1 are positive semi definite (with $\det[V_{ij}(\lambda)] = 0$). If the real nonzero eigenvector corresponding to the zero eigenvalue of the singular matrix $[V_{ij}(\lambda_{(1)}^c)]$ (specifying the critical principal direction) is also an eigenvector of the damping matrix, then eq. (5) has a double zero eigenvalue ($-\rho^2 = \mu^2 = 0$) and inequality (12) becomes

$$0 < \lambda_H \equiv \lambda_{(1)}^c \tag{13}$$

Contrary to the usual (or degenerate) Hopf bifurcation case, the critical dynamic load $\lambda_H \equiv \lambda_{(1)}^c$ corresponding to a double zero eigenvalue does not depend on the matrix $[\alpha_{ij}(m_i)]$ and the relevant situation is associated with a coupled flutter-divergence instability [Kounadis (1997)]. The system exhibits a dynamic instability mode , although this bifurcation occurs at a static divergence critical state, the so called Arnold-Bogdanov bifurcation. This phenomenon was also shown via a different procedure valid, however, only for the simple case of 2-D.O.F. symmetric damped systems [Kounadis and Sophianopoulos (1999)].

In view of the above, the following important findings are established: the necessary condition for a double-zero eigenvalue is the positive-definitiveness of both the damping and stiffness matrices, while the sufficient condition is the corresponding eigenvector (corresponding to the zero eigenvalue of the singular stiffness matrix) to be common to these matrices. Moreover, all the types of dynamic instability described above can also be established in terms of eigenvalue path in the ρ-complex plane, as λ varies slowly from zero [Kounadis (2000)].

c. Case 3: $[c_{ij}]$ indefinite

The symmetric damping matrix $[c_{ij}]$ is indefinite, if all its principal minor determinants are positive, except the last one, of order nxn, which is negative, i.e. $\det[c_{ij}] < 0$. In this case, all eigenvalues of $[c_{ij}]$ are positive, except one, which is negative. Then the quadratic form $\bar{r}^T[c_{ij}]r$ may be positive or negative depending to the eigenvector $r(\lambda)$. For λ smoothly increasing from zero, the above quadratic form may also be zero at a certain value $\lambda = \lambda_H$. This vanishing however can only be realized via a suitable choice of $r(\lambda)$. If this occurs for $\lambda_H > 0$ then eq.(5) yields a pair of purely imaginary eigenvalues satisfying the transversality condition [Kounadis (2000)]. It is related with a usual Hopf bifurcation with particular practical interest if $\lambda_H < \lambda_{(1)}^c$.

3. Illustrative examples

Let us consider the 2-D.O.F. cantilever model of Ziegler (under a compressive tip load) for which a rich variety of numerical results, based on both linear and nonlinear analyses, are available. The nonlinear equations of motion for the perfect bifurcational system are given by:

$$\left.\begin{aligned}&(1+m)\ddot{\theta}_1+\cos(\theta_1-\theta_2)\ddot{\theta}_2+\sin(\theta_1-\theta_2)\dot{\theta}_2^2+c_{11}\dot{\theta}_1+c_{12}\dot{\theta}_2+2\theta_1-\theta_2-\lambda\sin\theta_1=0\\&\cos(\theta_1-\theta_2)\ddot{\theta}_1+\ddot{\theta}_2-\sin(\theta_1-\theta_2)\dot{\theta}_1^2+c_{21}\dot{\theta}_1+c_{22}\dot{\theta}_2+\theta_2-\theta_1-\lambda\sin\theta_2=0\end{aligned}\right\}\quad(14)$$

subject to initial conditions

$$\theta_i(0)=\dot{\theta}_i(0)=0\,,\quad i=1,2 \tag{15}$$

Thus, according to eq.(1), the matrices involved in the corresponding linearized problem are equal to:

$$[\alpha_{ij}]=\begin{bmatrix}1+m & 1\\ 1 & 1\end{bmatrix},\quad [c_{ij}]=\begin{bmatrix}c_{11} & c_{12}\\ c_{21} & c_{22}\end{bmatrix},\quad [V_{ij}]=\begin{bmatrix}2-\lambda & -1\\ -1 & 1-\lambda\end{bmatrix} \tag{16}$$

Evidently, at the critical (divergence) state det$[V_{ij}]$ = 0, yielding the following values for the 1st and 2nd buckling loads:

$$\lambda_{(1)}^c=0.5(3-\sqrt{5})=0.381966011 \quad\text{and}\quad \lambda_{(2)}^c=0.5(3+\sqrt{5})=2.618033989 \tag{17}$$

In addition to the above, the 1st and 2nd buckling modes of the foregoing conservative system are independent, as depicted in the static equilibrium paths λ vs. θ_i (i = 1,2) of the model (for m = 1) of Fig. 1.

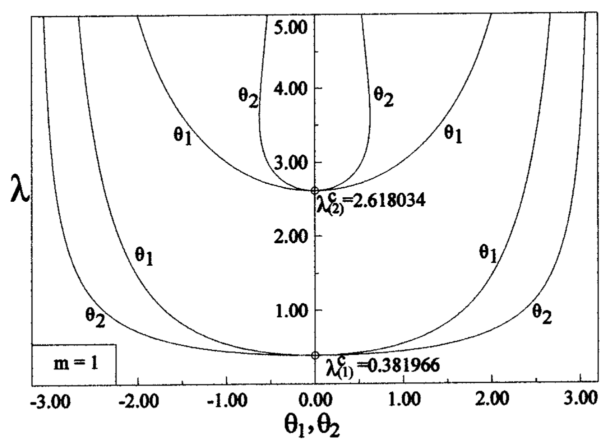

Fig. 1. Static equilibrium paths of Ziegler's symmetric system for m=1

Subsequently, we will discuss three characteristic cases of the above conservative system, associated to a usual Hopf bifurcation, a degenerate Hopf bifurcation and a double-zero eigenvalue bifurcation respectively.

(1) Usual Hopf bifurcation

The model chosen is associated with the following numerical data: $m = 1$, $c_{11} = 0.01$, $c_{12} = c_{21} = 0.0325$, $c_{22} = 0.012$ (indefinite damping matrix). It is found that for λ increasing slowly from zero, the eigenvalues cross the imaginary axis (with non-zero speed) at a loading $\lambda_H = 0.1936984$ less than the 1st buckling (divergence) load, where the Jacobian possess a pair of purely imaginary eigenvalues. This is a profound case of usual Hopf (dynamic) bifurcation occurring prior to divergence, since all the conditions required for its birth are fulfilled. Hence, the corresponding dynamic response at $\lambda = \lambda_H$ is associated with a limit cycle (isolated periodic motion independent of the initial conditions). This is shown in the phase plane portrait of Fig.2 and the relevant detail of Fig. 3.

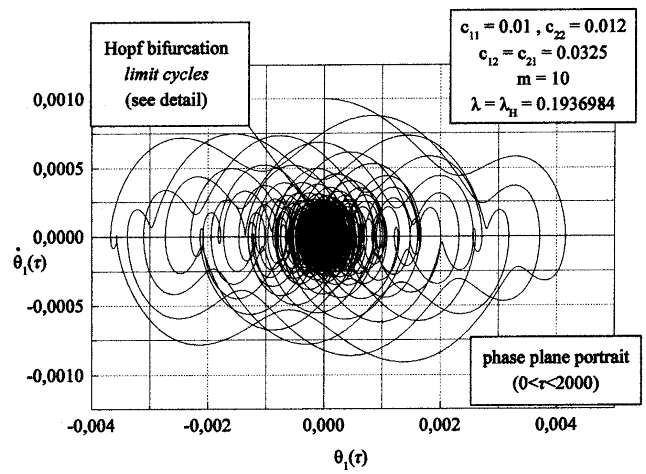

Fig. 2. Phase plane portrait of a usual Hopf bifurcation prior to divergence, for a model with m = 10, associated with a limit cycle response.

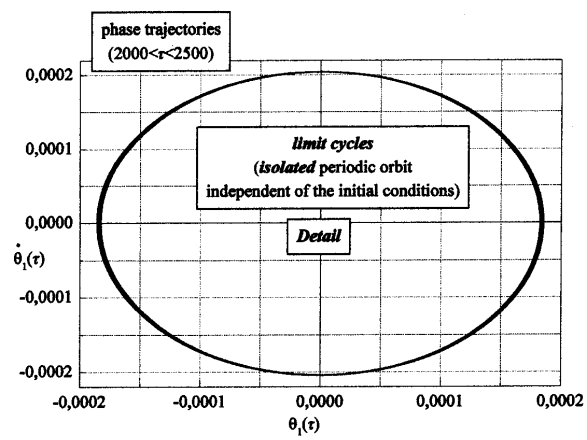

Fig. 3. Detail of Fig. 2

For $\lambda_H < \lambda < \lambda^c_{(1)}$ the system's dynamic response is similar, associated also with a periodic attractor (limit cycle) as depicted in Figs. 4 and 5, valid for $\lambda = 0.37$.

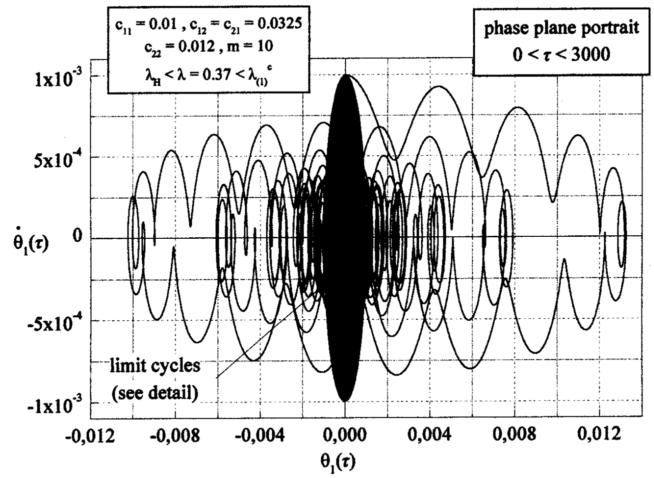

Fig. 4. Phase plane portrait of the model of Fig. 2, for $\lambda = 0.37$, associated also with a limit cycle response.

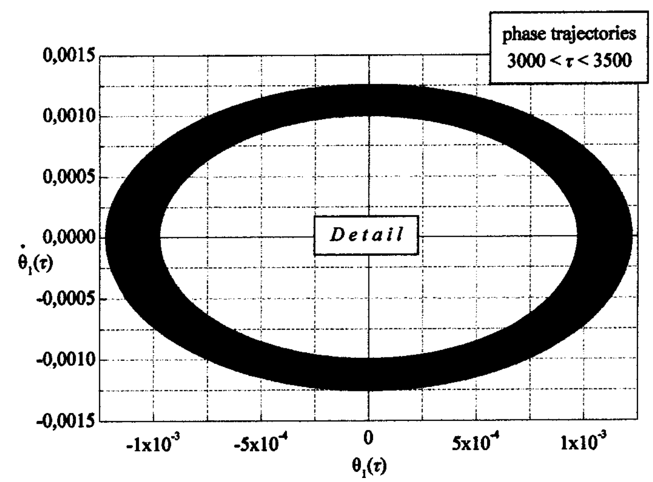

Fig. 5. Detail of Fig. 4

(2) Degenerate Hopf bifurcation

The model chosen for this case is associated with a positive semi definite damping matrix, since it is related with $c_{11} = 0.01$, $c_{12} = c_{21} = 0.002$ and $c_{22} = 0.004$ (det[cij] = 0), while $m = 10$. As shown throughout Figures 6-9, depicting the evolution of the Jacobian eigenvalues, it is evident that for a loading $\lambda = \lambda_H = 0.30769225$ (smaller than the 1st buckling load) a pair of purely imaginary eigenvalues arises; nevertheless, the transversality condition is violated and the whole phenomenon is associated with a degenerate Hopf bifurcation. Clearly, for $\lambda < \lambda_H$ the system's dynamic response is associated with a point attractor at the trivial state (origin), a fact also valid for $\lambda_H < \lambda < \lambda^c_{(1)}$, while for $\lambda = \lambda_H$ the system exhibits periodic motions with their amplitude dependent on the initial conditions. All the above phenomena are illustrated in the phase plane portraits and details of Figures 10-13. For loads higher than the 1st buckling load the linearized dynamic analysis fails to predict the actual response of the system, yielding unbounded motions (overflow), while the true response is associated with a stable point attractor, which is captured only if a nonlinear analysis is employed, as shown in Fig. 14 valid for λ=0.40.

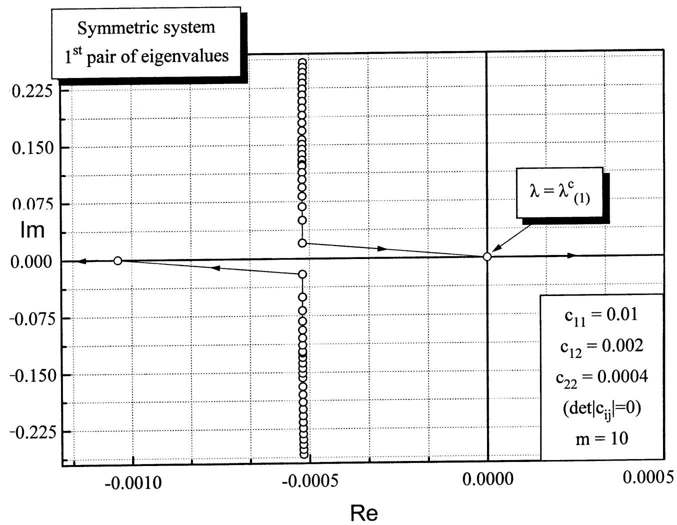

Fig. 6. Evolution of the 1st pair of eigenvalues of a symmetric system with semi definite damping matrix and m=10, in the Re – Im plane.

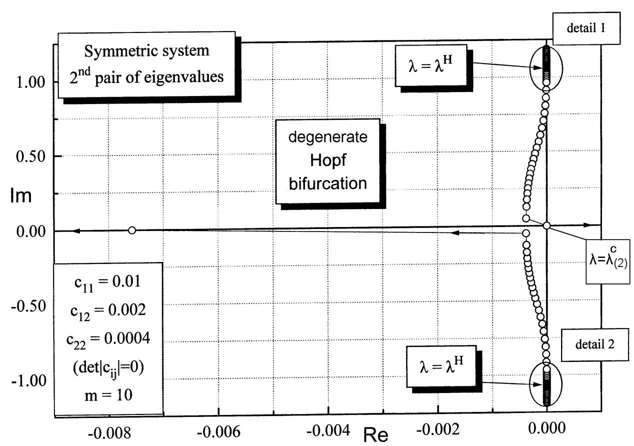

Fig. 7. Evolution of the 2nd pair of eigenvalues of a symmetric system with semi definite damping matrix and m = 10, in the Re – Im plane (degenerate Hopf bifurcation prior to divergence).

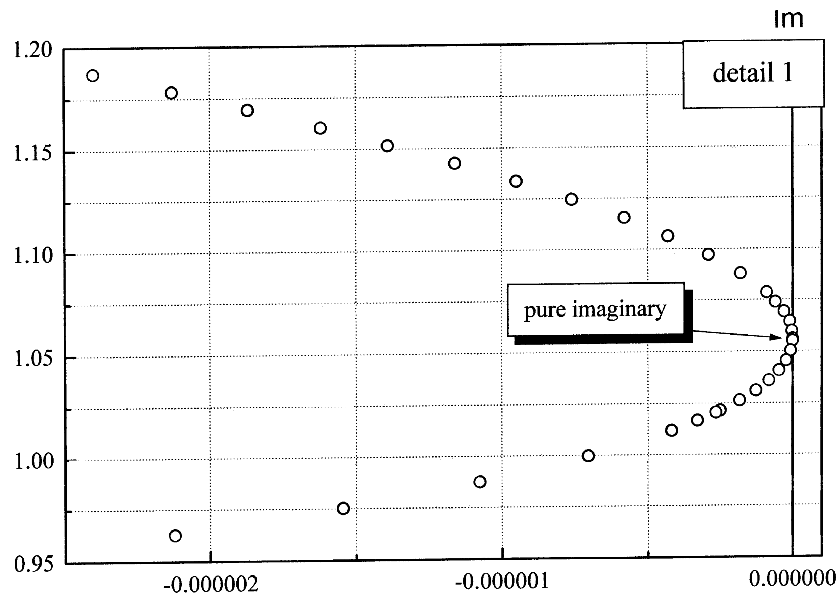

Fig. 8. Detail 1 of Fig. 7.

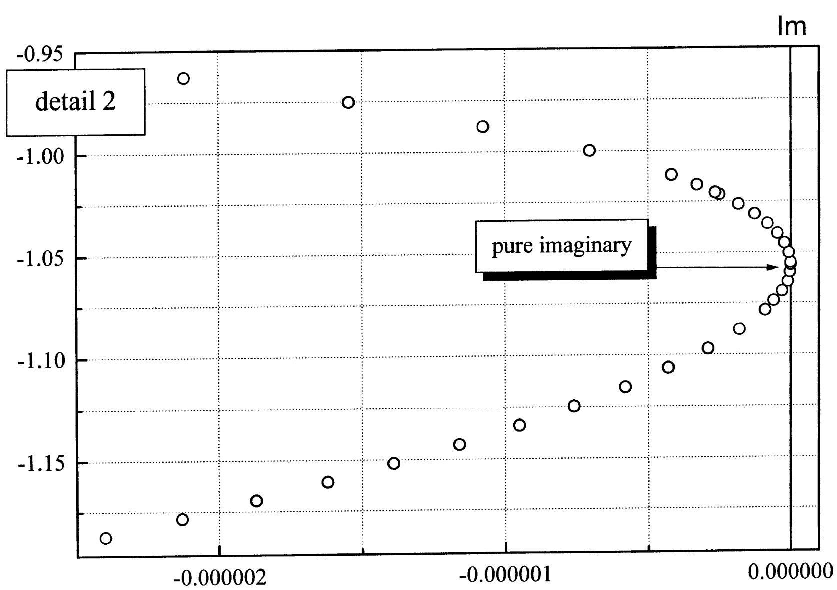

Fig. 9. Detail 2 of Fig. 7.

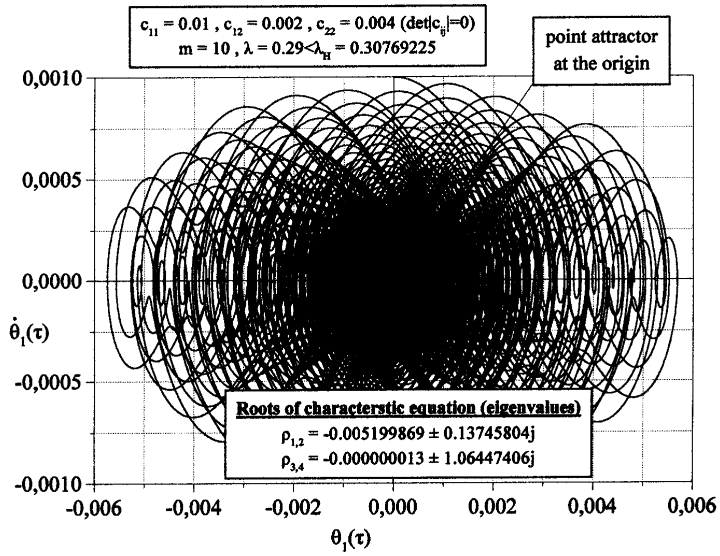

Fig. 10. Phase plane portrait of the model of Case (2) for λ=0.29<λ_H, exhibiting a point attractor at the origin.

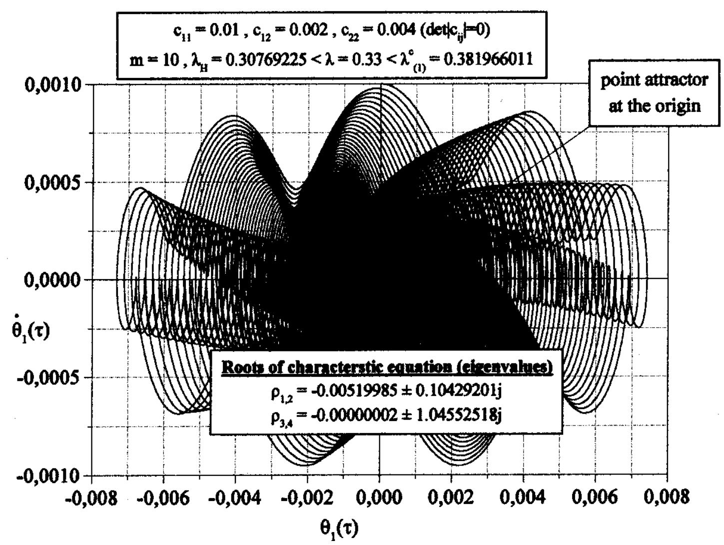

Fig. 11. Phase plane portrait of the model of Case (2) for λ_H<λ=0.33< $\lambda^c_{(1)}$, exhibiting a point attractor at the origin.

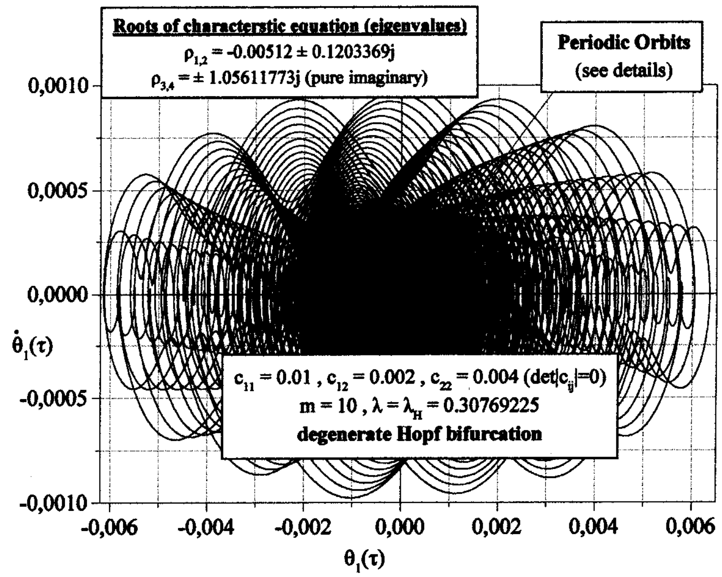

Fig. 12. Phase plane portrait of the model of Case (2) for $\lambda = \lambda_H$, associated with periodic orbits (degenerate Hopf bifurcation).

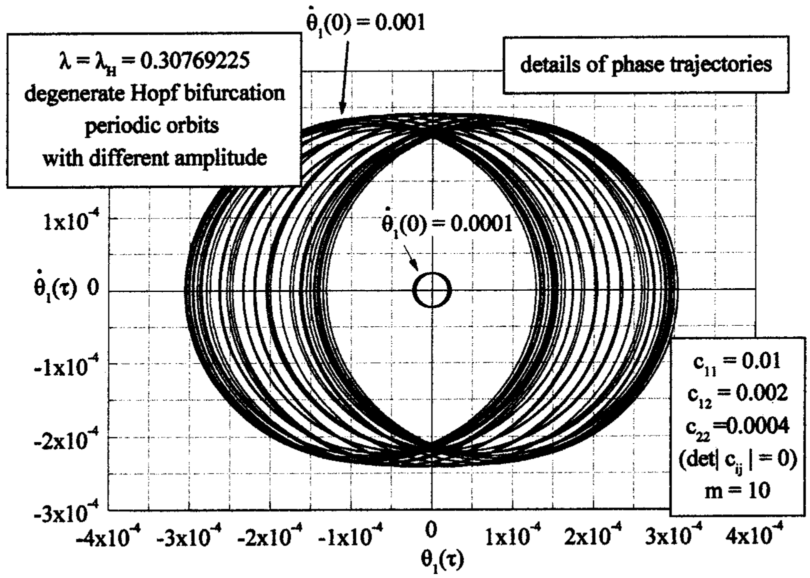

Fig. 13. Details of phase trajectories of the model of Case (2) at $\lambda = \lambda_H$ (see Fig. 12), for two different values of the initial velocity of angle θ_1, exhibiting periodic motions with different amplitudes (degenerate Hopf bifurcation).

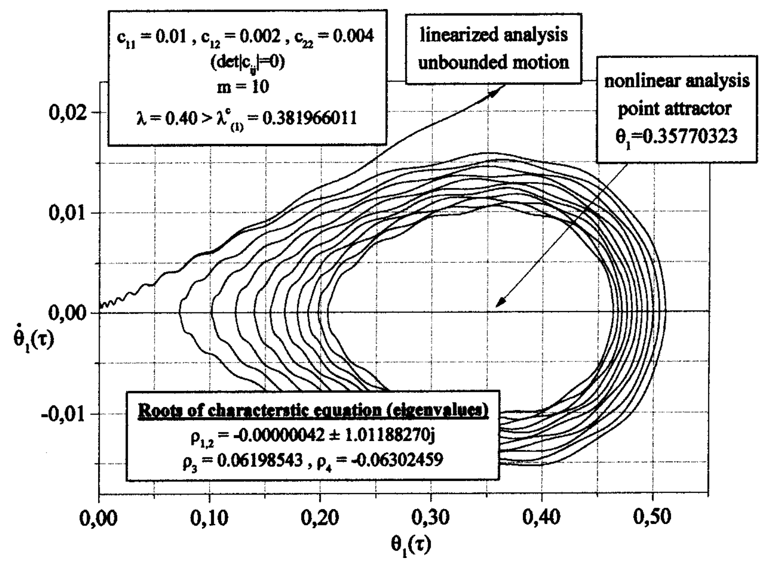

Fig. 14. Phase plane portrait of the model of Case (2) for $\lambda = 0.40$, for both linearized and nonlinear dynamic analyses.

(3) Double zero eigenvalue bifurcation (coupled flutter-divergence instability)

This last example is associated with a symmetric system with m=1 and a positive semi-definite damping matrix, since $c_{11} = 0.08$, $c_{12} = c_{21} = -0.049442719$ and $c_{22} = 0.0305572572808$ ($\det[c_{ij}] = 0$). As shown in Fig. 15 and 16, depicting the evolution of the Jacobian eigenvalues, this particular case corresponds to a coupled flutter-divergence instability, since a double zero eigenvalue bifurcation occurs for $\lambda = \lambda_H = \lambda^c_{(1)}$.

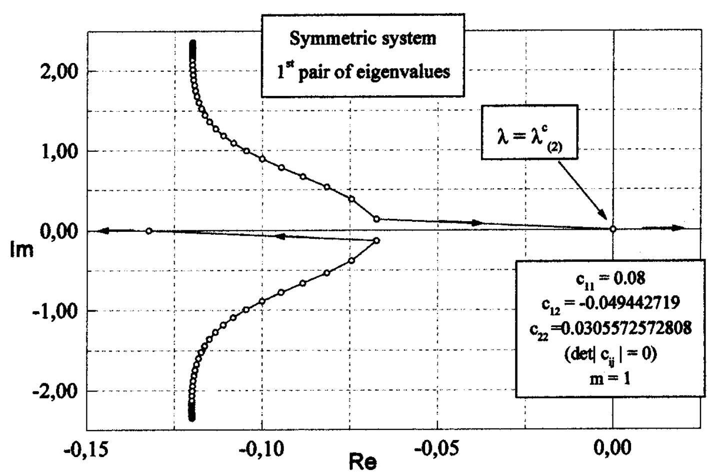

Fig. 15. Evolution of the 1[st] pair of eigenvalues for the model of Case (3) in the Re-Im plane.

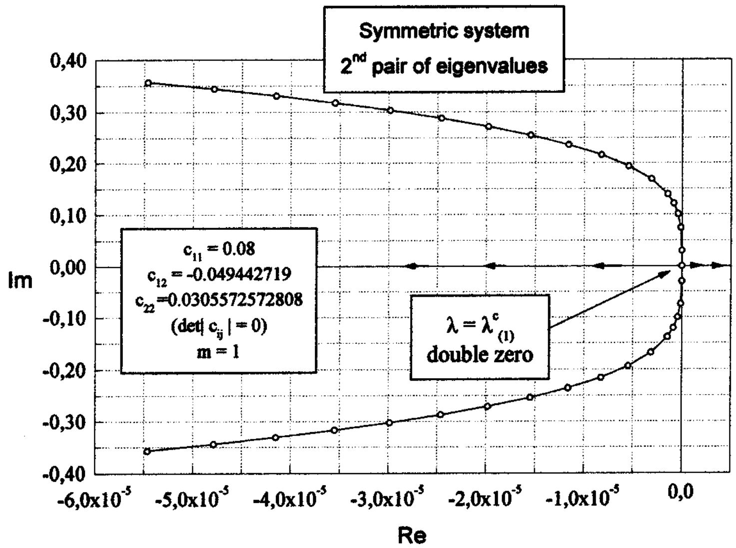

Fig. 16. Evolution of the 2nd pair of eigenvalues for the model of Case (3) in the Re-Im plane, revealing a double zero flutter-divergence instability.

Hence, the system's dynamic response is for $\lambda < \lambda_H = \lambda^c_{(1)}$ associated with a point attractor response at the trivial state, while for $\lambda = \lambda_H = \lambda^c_{(1)}$ a limit cycle (periodic attractor) response is exhibited, as shown in the phase plane portraits of Figures 17 and 18 respectively.

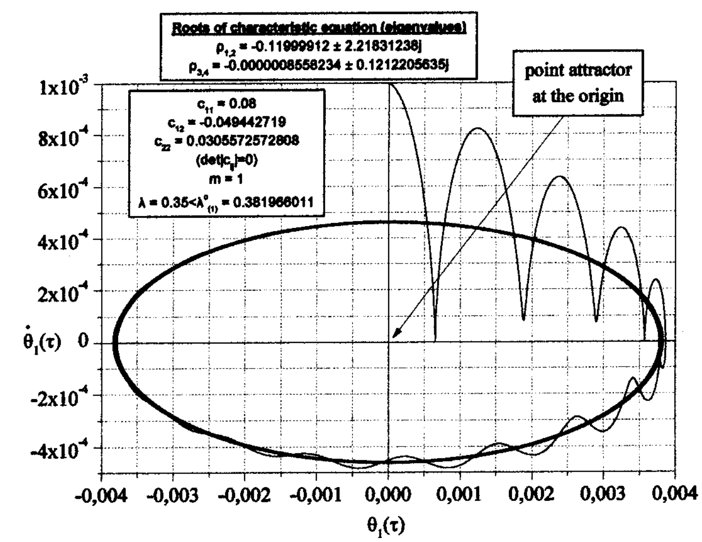

Fig. 17. Phase plane portrait of the model of Case (3) for $\lambda < \lambda_H = \lambda^c_{(1)}$, revealing a point attractor response at the origin (trivial state)

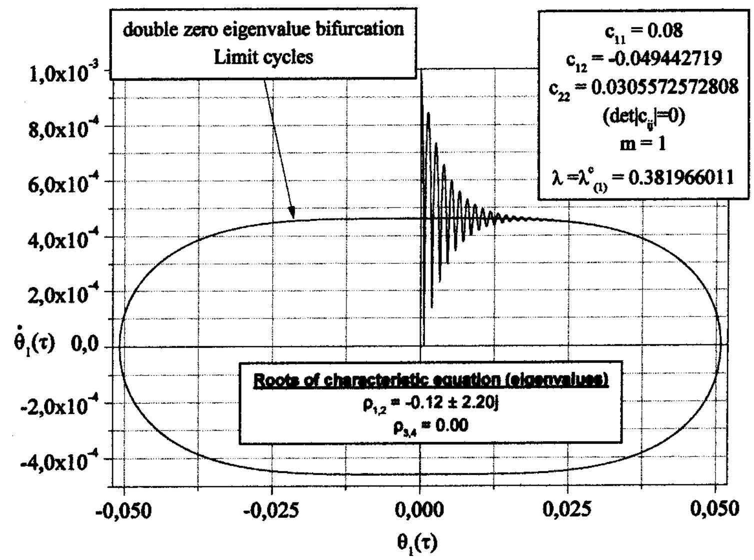

Fig. 18. Phase plane portrait of the model of Case (3) for $\lambda = \lambda_H = \lambda^c_{(1)}$, revealing a double zero eigenvalue bifurcation (limit cycle response)

Finally, for levels of the loading higher than $\lambda_H = \lambda^c_{(1)}$ the actual response of the system, being a point attractor on stable postbuckling equilibria, can only be captured if one employs a fully nonlinear dynamic analysis. Contrary, if linearized analysis is used, the corresponding response yields unbounded motions (overflow), a fact clearly perceivable in Fig. 19.

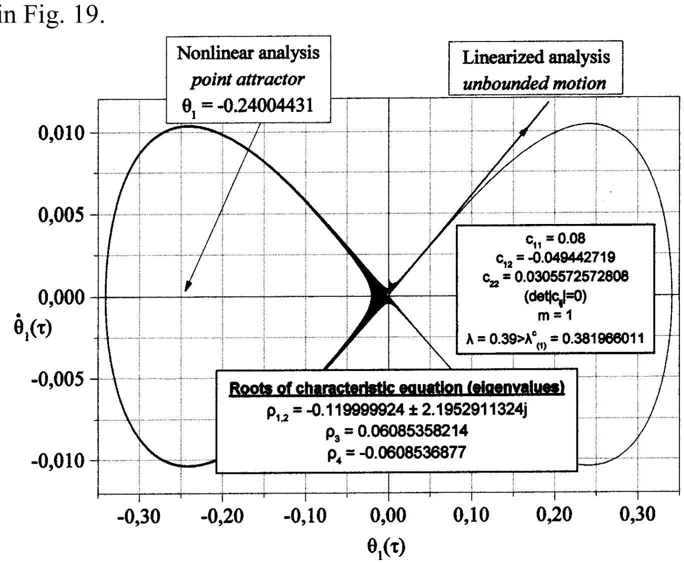

Fig. 19. Phase plane portrait of the model of Case (3), for $\lambda = 0.40 > \lambda_H = \lambda^c_{(1)}$, for both linearized and nonlinear dynamic analyses.

CONCLUSIONS

The most important conclusion of the present paper, which deals with the local dynamics and stability of autonomous weakly damped potential systems are as follows:

- Through the study of the effect of the algebraic form of the dissipation matrix on the Jacobian eigenvalues it is found that such Hamiltonian (conservative) systems may experience phenomena similar with ones widely reported for relevant non-conservative systems.
- Thus, contrary to existing results, the solution of nondissipative symmetric systems, when damping is accounted for, may lose their stability through
  a) a double zero eigenvalue bifurcation, a usual Hopf bifurcation and a degenerate (isolated Hopf bifurcation and
  b) a limit cycle (dynamic) mode of instability, occurring prior to divergence
- The theoretical findings present are validated through a variety of numerical examples.

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AUTONOMNI SLABO PRIGUŠENI KONZERVATIVNI SISTEMI SA GRANIČNIM CIKLUSOM

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Prikazani rad detaljno preispituje aspekte lokalne dinamičke stabilnosti slabo prigušenih diskretnih autonomnih Hamilton-ovih sistema. Za takve potencijalne sisteme prisustvo graničnih ciklusa (periodičnih atraktora) detaljno se razmatra kroz studiju efekta algebarskog oblika matrice prigušenja na Jacobi-jeve sopstvene vrednosti. Nova otkrića koja su u kontradikciji sa postojećim rezultatima su (a) račvanje sopstvene vrednosti dvostruke nule, uobičajeno Hopf-ovo račvanje i degenerisano (izolovano) Hopf-ovo račvanje i (b) režim nestabilnosti graničnog ciklusa (dinamički) koji prethodi režimu divergencije (statičkom), za koje Ziegler-ov kinetički kriterijum ne važi. Ispravnost teorijskih otkrića koja su ovde prikazane verifikovana je brojnim numeričkim primerima.