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METHODS AND MODELS IN SINGULAR PROBLEMS OF MECHANICS

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Abstract. *The developed methodology allows to consider the various mechanical systems with the united positions; to come up to the idealization problem in mechanics (with the construction of idealized models by strict mathematical way; with the substantiation of legitimization of these models; with the receiving of correctness conditions in dynamics).*
This technique gives the regular manners for the decomposition of system. It allows to investigate the complex systems by analytical (and computer-analytical) methods in problems of analysis and synthesis

It is important study, that is concerned with the development of the concepts and methods of classical stability theory in reference to the problems of singularly perturbed class systems. The various aspects of complex systems dynamics are considered. Methods of the modelling and analysis on the generalized methodology base, combining the stability theory ideas and asymptotic theory manners, are elaborated. Besides non-traditional, extended approach, formed on Lyapunov's methods, Chetayev's stability postulate and the singularity postulate, is worked out. It gives a universal tool, that makes it possible to come near to the solving fundamental problems in general modelling theory. The effective algorithm of engineering level is obtained. Here different investigated objects are interpreted as singular ones, from unified positions; effectual non-traditional technology of modelling, that uses principally non-linear approach, is established; the simple schemes of decomposition of original systems (models) and of dynamic properties are worked out; the generalization of the reduction principle, well-known in stability theory, is got for general qualitative analysis. This manner allows to construct the hierarchical sequence of simplified systems (and models) as comparison ones; to

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determine the conditions of the qualitative equivalence between original and shortened systems; to find the areas of their acceptability.

In the applications to Mechanics the elaborated methods are very effective, those enable to construct the acceptable shortened models by strict mathematical way; to substantiate strongly their correctness in dynamics; to consider specific critical cases, inherent in mechanical systems; to evaluate the corresponding errors in such transition-simplifying.

The elaborated methods are illustrated on examples from Mechanics. It is considered actual problems in theory of gyroscopic systems, electromechanical systems, robotic systems, mechanical systems with the friction, Newton's model of point mass dynamics,…

New elegant outcomes are obtained, that are interesting both for theory and for applications; also this approach is very perspective from gnosiological view point, for general knowledge theory.

1. Introduction

The subjects of research are the complex systems of the singularly perturbed class, generated by the examples of concrete physical substance, those lead to the non-linear, multi-dimensional dynamical problems. The difficulties in exact solving with the analytical methods bring the necessity of the model "narrowing" and of the original problems reduction to the problems of lesser dimension [1-10].

For example, in theory of gyrosystems (GS) there is the well-known precessional system (PS) as shortened model of lesser order [11,12]. This shortened model does not take into consideration the high-frequency components of motion. But the problem of precessional theory acceptability is not solved as yet. Also in stabilization and orientation systems dynamics the different shortened systems are used [13] as simplified models for original systems with big (small) physical parameters. But the strict validity is not examined in applied works. For this there are not the effective methods, the efficient simple algorithms. Therefore main problems are: the elaboration of systematical procedures in modelling of complex systems; the construction of correct simplified models as comparison models by strong mathematical manners; the developing of regular methods for obtaining acceptability criteria.

First strict statements in these problems area were formulated by A.M.Lyapunov (1892), creator of classical stability theory, founder of new strong fundamental approaches both for general theory and for engineering applications.

From stability investigations the ideas and results in framework of comparison method led to the reduction principle (A.M. Lyapunov, K.P. Persidskiy, I.G. Malkin) and to the comparison principle (V.M. Matrosov, R. Bellman,...). There is a close connection between these problems of singular systems and the problems of stability theory (I.M. Gradstein, N.G. Chetayev); between problems of stability theory and problems of modelling in Mechanics (N.G. Chetayev, L.K. Kuzmina).

Here above formulated problems are considered, ones are solved by methods of Stability Theory. General approach, that was founded by A.M. Lyapunov, used by N.G. Chetayev to Mechanics problems, added by P.A. Kuzmin to statement for stability with parametric perturbations, is worked out for systems of singular class.

2. Problem overview and background

We shall consider the mechanical systems, taking the Lagrange's equations (or their generalized form) as initial mathematical model

$$\frac{d}{dt}\frac{\partial T}{\partial q^{\cdot}}-\frac{\partial T}{\partial q}=Q,\quad \frac{dq}{dt}=q^{\cdot} \tag{2.1}$$

q is k-dimensional vector of generalized coordinates; k is the number of freedom degrees; T is kinetic energy of system. (2.1) is full model (FM).

In case of gyroscopic systems (GS), that are modeled by mechanical systems with the fast rotors, assuming the eigen kinetic moments of gyroscopes are big, they introduce big parameter H and go from FM to simplified precessional model (SM). This SM has ($k/2$)-degrees of freedom. Here we have the transition to model of lower order, and it gives non-regular problems.

$$\text{GS: FM}(k) \xrightarrow[H]{} \text{SM}(k/2)\ ? \tag{2.2}$$

Also for the stabilization systems, modeled as electromechanical systems (EMS) with ($n + u$) – number of freedom degrees in the analysis of the stabilization state, assuming the following systems are quick-operating, they introduce small parameter μ (corresponding to small constants of time for electrical circuits) and go to simplified model SM. One does not take into consideration the fast electrical processes and has the lesser number of freedom degrees. On the other hand, for the investigations of dynamical processes in follow systems they introduce another SM (with the inertialess mechanical subsystem). Here also we have non-regular problems.

$$\begin{gathered}\text{EMS: FM}(n+u) \xrightarrow[\mu]{} \text{SM}(n+(u/2))\ ? \\ \text{or FM}(n+u) \xrightarrow[H]{} \text{SM(n+(u/2))}\ ?\end{gathered} \tag{2.3}$$

In robotic-systems (RS) dynamics the researchers go to simplified models of lower order (for example, neglecting some real properties, connected with the non-absolute rigidity of some links or with small inertia of some elements, etc.).

$$\begin{gathered}\text{RS: FM}(k) \xrightarrow[H_1]{} \text{SM}(k_1)\ (k_1<k)\ ? \\ \text{or: FM}(k) \xrightarrow[\mu_1]{} \text{SM}(k_2)\ (k_2<k)\quad ?\end{gathered} \tag{2.4}$$

H_1, μ_1 are big, small parameters; k_1 , k_2 are the numbers of freedom degrees of SM. But the legitimacy of SM and their admissibility domains are not discussed; the conditions of acceptability are not determined; these models are not obtained in a strict mathematical way.

Note, in oscillation theory the general statement of the reduction problem of the original mechanical system to idealized model of lower order there is in early works of A.A. Andronov (1935). The strong substantiation of such reduction and the influence estimations of discarded members there are in early works of M.V. Meerov (1947), A.A. Feldbaum (1948). The general mathematical statement of this substantiation was given by A.N. Tikhonov (1948), who received the first theorem on limit process. The possibility of the singular problems solving by Lyapunov's methods was noted in early works of I.S. Gradstein (1949) and for singular problems of mechanics - by N.G. Chetayev (1953).

Returning to the main problems, we note a general peculiarities of mechanical systems, modeled as (2.1): direct introducing of small (big) parameter in initial mathematical model is non-useful (it is necessary the special transformation of variables); mechanical systems are singular systems with μ parameter in different powers; the motions in systems split up into three (or more) groups (if $\mu \to 0$); as rule, the idealized (simplified) models are not limit models; the mechanical systems are non-Tikhonov's systems (unperturbed systems are on boundary of stability domain; eigenvalues of corresponding matrices are zero or imaginary).

Therefore the direct use of known results of singular perturbations theory [3,4,5] are non-suitable. New methods and approach are necessary for mechanical singular problems.

3. Basic steps

The general approach, founded by A.M.Lyapunov, combining the methods of perturbations theory and of stability theory is developed. Such approach permits to detour the above indicated features.

Let the original mathematical model is (2.1) (full model), designated (3.1) here without writing;

$x = (q,q^{\cdot})$ is vector of the system state variables. The problems: how to construct the correct simplified models by a strict method; how to determine the acceptability conditions of SM in qualitative analysis; how to obtain the errors estimations.

a. First of all we must introduce the small parameter $\mu > 0$, and converting to new variables ($x \to y$, $y = L(\mu, x)$) we shall lead the system (3.1) to the form

$$M(\mu)\frac{dy}{dt} = Y(t,\mu,y) \tag{3.2}$$

Here y is $2k$-dimensional vector, $M(\mu) = ||M_{ij}(\mu)||$; $M_{ii}(\mu) = \mu^{\alpha_i} I$; $\alpha_i = 0,1,2,..., r$; $r > 0$ is constant number; I is identity matrix; $M_{ii}(\mu) = 0$ (if $i \neq j$).

Such manner allows to obtain the effective regular technique for the construction of simplified models(SM). For this we shall introduce as approximate system for (3.2) the shortened one (SS)

$$M_s(\mu)\frac{dy}{dt} = Y_s(t,\mu,y) \tag{3.3}$$

System (3.3) is obtained from (3.2) with the keeping of members, no higher than μ^s -order ($0 \le s < r$).

(3.3) is the N_s –order comparison system for (3.2) ($N_s < N$; $N=2k$). Returning to the state variables x, we shall obtain the SM, (designated (3.4) without writing): $y \to x$ and (3.3) $\to$ (3.4).

SM has k_s of freedom degrees ($k_s = N_s /2$; $k_s < k$).

We shall call (3.4) the simplified model of s–level on μ–parameter (SM$_s$). In view point of mechanics the model (3.4) is some idealized model. Moreover, we can receive the hierarchical sequence of simplified models on μ-parameter assigning s the values to $0,1,..,r-1$.

$$\mu : SS_0, SS_1, SS_2, \ldots \longrightarrow SM_0, SM_1, SM_2, \ldots$$

Remark 1. SS_0 corresponds to the degenerate system ($\mu = 0$), traditional in perturbations theory, and leads to limit model in mechanics. Taking $s = 1$, we shall have the linearized (on μ) approximate system, traditional in stability theory, that leads to simplified model of first level in mechanics.
Remark 2. Further, introducing various small and big parameters (μ, μ_1, μ_2, …), we can obtain another simplified models (and the domains of their acceptability):

$$\mu : (SS_0, SS_1, \ldots)_\mu \to (SM_0, SM_1, SM_2, \ldots)_\mu$$
$$\mu_1 : (SS_0, SS_1, \ldots)_{\mu 1} \to (SM_0, SM_1, SM_2, \ldots)_{\mu 1}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Remark 3. The research displayed: all considered mechanical systems are the singular systems of (3.2)-type (with $y = \| y_1, y_2, y_3\|^T$; $\alpha_i = 0,1,2$), the state variables are separated on three groups (low-, middle-, high-frequency) in accordance with the system structure (original model is model of Newton's Mechanics). Accordingly we can construct the idealized models of two types.

b. The following stage is the substantiation of the constructed model acceptability. We must determine the correctness conditions of shortened models, their qualitative equivalence with the full model. In conformity with the stability theory ideology we introduce the differential equations for the deviations $b = y - y^*$ [2]

$$\tilde{M}(\mu)\frac{db}{dt} = B(t,\mu,b) \tag{3.5}$$

Here $y = y(t,\mu)$ is the full system (3.2) solution with initial conditions $y_0 = y(t_0,\mu)$; $y^* = y^*(t,\mu)$ is the shortened system (3.3) solution with corresponding date $y_0^* = y^*(t_0, \mu)$. Using stability methods, analyzing system (3.5), we shall determine the necessary conditions, under which the solution of (3.5) $b = b(t,\mu)$ will have the required properties (with small enough μ, $\mu \le \mu^*$).

Here we must solve the singular problems: stability problem (under which conditions the stability property for system (3.3) (for (SM) will entail this property for the system (3.2)) (for FM) [6]; solutions proximity problem (under which conditions the corresponding solutions of shortened system and full system will be close on time infinite interval); note, this problem is reduced to the problem of set stability; other singular problems (on speedness; of optimal parameters; etc.) [7,9].

c. The problem of correctness is connected with the of μ-value estimation problem, with permissible parameters domains; with the errors obtaining. The regular methods, based on Lyapunov's methods (first, second), give the necessary evaluations. For this we shall introduce in consideration all shortened subsystems, accordingly to all variables groups (low-, middle-, high-frequency) for our mechanical systems. Following N.G.Chetayev, imposing the conditions, that are ensuring the stability property for full system from the stability property for every subsystems, we shall get the sought expressions for μ-values estimation. For considered systems of (3.2) type (remark 3) we shall have three subsystems and the sought estimation $\mu^* = inf\{\mu_1^*, \mu_2^*, \mu_3^*\}$. Here every μ_i^* is determined from the corresponding conditions. In particular, for μ_1^*-value we can obtain the expression

$$\sup_j = |\operatorname{Re}\Delta_j(\mu)| \le \inf_j |\operatorname{Re}\lambda_{j0}(\mu)| \tag{3.6}$$

where $\lambda = \lambda(\mu)$ is the root of the characteristic equation $D(\lambda,\mu) = 0$ for full system; λ_0 is the corresponding root of degenerate equation $D_0(\lambda) = D(\lambda,0) = 0$; $\lambda(\mu) = \lambda_0 + \Delta(\mu)$; $\lambda_0 \neq 0$. Analogous expressions we shall construct for μ_2^*, μ_3^*. Also by those manners we can obtain the evaluation for the stability domain.

4. Applied problems

Elaborated technique enables to reveal a general rule for mechanical systems: the differential equations can be represented in form (3.2); the variables can be divided into three groups; the idealized models can be constructed as SM_0 and SM_1 (limit model and linearized model). Moreover the correctness conditions of these idealizations are determined by developed methods (both in non-critical and in critical specific cases, corresponding to these systems, when eigenvalues are on the imaginary axis).

As illustration consider these general propositions on the concrete mechanical system of stabilization, modeled as EMS. Let the mathematical model is the Lagrange-Maxwell's equations, written in the form [14]

$$\begin{gathered}\frac{d}{dt}a_M\dot{q}_M + b_M\dot{q}_M + g\dot{q}_M = Q'_M + Q_{ME} + Q''_M \\ \frac{d}{dt}a_E\dot{q}_E + b_E\dot{q}_E = Q'_E + Q_{EM} + Q''_E, \quad \frac{dq_M}{dt} = \dot{q}_M\end{gathered} \tag{4.1}$$

q_M, q_E are n-dimensional vectors of generalized coordinates (of Lagrange, Maxwell).

System (4.1) is the $(2n + u)$–order one (full model, FM). For the simplification of FM we shall follow the scheme of p.3. We must consider the concrete cases.

a. EMS with the fast gyroscopes.

Here we assume $g = g^* H$, $H = 1/\mu$, $\mu > 0$ is small parameter. The required transformation of variables is constructed:

$$\tau = \mu t, \quad y = \left\| y_1, y_2, y_3 \right\|^T, y_1 = a_M q'_M, \qquad y_2 = a_E\dot{q}_E,$$
$$y_3 = \left\| q_{M1}; q_{M4}; \mu^2 a_{M1} q'_M + (\mu b_{M1} + g_1^*) q_M \right\|^T, \quad ( )' = \frac{d}{d\tau}( )$$

In new variables the system (4.1) has the form (3.2) with $\alpha_1 = 2$, $\alpha_2 = 1$, $\alpha_3 = 0$. In accordance with our results the simplified models are constructed as $(SM_1)_\mu$ and $(SM_0)_\mu$:

$$\begin{gathered}b_M q_M^0 + g q_M^0 = Q'_M + Q_{ME} + \tilde{Q}''_M, \ \frac{dq_M}{dt} = \dot{q}_M \\ \frac{d}{dt}a_E q_E^0 + b_E\dot{q}_E = Q'_E + Q_{EM} + Q''_E,\end{gathered} \tag{4.3}$$

$$\begin{gathered}g\dot{q}_M = Q'_M + Q_{ME} + \overline{Q}''_M, \\ b_E\dot{q}_E = Q'_E + \overline{Q}''_E, \quad \frac{dq_M}{dt} = \dot{q}_M \, .\end{gathered} \tag{4.4}$$

(4.3) is the $(n + u)$–order system; (4.4) is $(n + (u/2))$–order system. The conditions of correctness of these idealized models and the μ–values appraisals are determined by using technique. Note, that (4.3) is a known model (precessional model); but (4.4) is a new model.

b. EMS with the quick operating followers.

Here we assume $a_E = a_E^* \mu_1$, $A_M = A^* \mu_1$, $B_E = B^* \mu_1$; $\mu_1 > 0$ is small parameter. The necessary variables transformation are constructed:

$$\tau = \mu_1 t,\;\; y = \left\| y_1, y_2, y_3 \right\|^T, \quad y_1 = a_M q'_M,\;\; y_2 = a_E^* \dot{q}_E,$$
$$y_3 = \left\| q_{M1};\, q_{M4};\, \mu_1 a_{M1} q'_M + (b_1 + g_1) q_M \right\|^T, \quad (\,)' = \frac{d}{d\tau}(\,) \tag{4.5}$$

The system (4.1) in y-variables have the form (3.2), with $\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = 0$. Accordingly we obtain the $(SM_1)_{\mu 1}$ and $(SM_0)_{\mu 1}$ as a simplified model:

$$\frac{d}{dt} a_M \dot{q}_M + b_M \dot{q}_M + g\dot{q}_M = Q'_M + Q_{ME} + Q''_M$$
$$b_E \dot{q}_E = Q'_E + \tilde{Q}''_E, \quad \frac{dq_M}{dt} = \dot{q}_M \tag{4.6}$$

$$b_M \dot{q}_M + g\dot{q}_M = Q'_M + Q_{ME} + \overline{Q}''_M,$$
$$b_E \dot{q}_E = Q'_E + \overline{Q}''_E, \quad \frac{dq_M}{dt} = \dot{q}_M \tag{4.7}$$

(4.6) is the $(2n + (u/2))$–order system; (4.7) is the $(n + (u/2))$–order system. (4.6) is the know model (idealized model with the inertialess following systems). But (4.7) is new model (on μ_1–parameter). Also the conditions of reduction to these SM are obtained. Here as example we show the results for problem (4.1)→ (4.7).

We shall assume all functions in system (4.1) are holomorphous (in considered area) on the totality of variables, with coefficients are continuos limited functions. We must introduce the auxiliary equations:

$$d_1(\alpha) = |\, a_E \alpha + b_E + \Omega \,| = 0,$$
$$d_2(\beta) = |\, a_M \beta + b_M + g \,| = 0, \tag{4.8),}$$
$$d_3(\lambda) = \begin{vmatrix} b_{M1} + g_1 & 0 \\ (b_{M2} + g_2)\lambda & -A_M \\ \omega \;\; 0 & b_E + \Omega \end{vmatrix} = 0$$

The statement is valid:

Theorem. If equations (4.8) are satisfying the Hurwitz's conditions, then with small enough μ_1–values the stability property (asymptotic or non-asymptotic) of zero-solution of system (4.7) entails the corresponding property of stability of system (4.1) zero-solution; and for in advance given numbers $\varepsilon > 0$, $\delta > 0$, $\gamma > 0$, (ε and γ can be no matter how small) there exists such a μ_1^*–value, that in a perturbed motion, when $\mu_1 < \mu_1^*$ for $t \geq t_0 + \gamma$ there are

$$\| \dot{q}_M - \dot{q}_M^* \| < \varepsilon,\ \| q_M - q_M^* \| < \varepsilon,\ \| \dot{q}_E - \dot{q}_E^* \| < 0,$$
$$\text{if for}\quad t = t_0 \quad q_{M0} = q_{M0}^*,\ \| q_{M0} - q_{M0}^* \| < \delta,\ \| \dot{q}_{E0} - \dot{q}_{E0}^* \| < \delta$$

This theorem gives the conditions of acceptability of idealized model (4.7) (more simple than (4.6)). Also other cases and singular problems were considered and the theorems are obtained [8,14].

c. The systems with non-rigid elements.

Initial mathematical model is the Lagrange's equation; the small parameter is $\nu = 1/H_1$, $c = c^* H_1^2$, c is the matrix of the potential energy of elasticity forces [7].

The variables transformation is constructed and the simplified model is received: $(SM_0)_\nu$, $(SM_1)_\nu$. Here: $(SM_0)_\nu$ is a known model (the idealized model of absolutely rigid system); $(SM_1)_\nu$ is a new model.

d. The robotic systems with the small-inertia links; the gyrosystems with the small gyroscopes; the systems with big friction, etc.

5. Conclusion

Using the methodology of stability theory in combination with asymptotic approach is very perspective, both for applied problems and in general gnosiological aspect. It permits to ascertain the essence of idealized model, its harmony of the full model; to understand its mechanical-mathematical sense.

For example, our results on the substantiation of the precessional model show that this model is asymptotic model of first level on μ-parameter.

Also we can show the analogous result for mechanical systems with the big friction. Again we shall be able to note the interesting conception in regard to model of Newton's mechanics (NM) (the traditional model in engineering practice). Elaborated method allows to establish the conformity between of Newton's mechanics model (that gives as mathematical model the differential equations of second degree for each (individual) generalized coordinate) and another models: model of Aristotel's mechanics (AM), giving the differential equations of first degree; hypothetical model of mechanics, giving the differential equations of third degree; etc.).

With this approach we obtained that AM in dynamics of point mass with big friction is analogue of precessional model (PM) in dynamics of fast-rotating body:

PM is $(SM_1)_\mu$, $\mu = 1/H$; and AM is $(SM_1)_\kappa$; $\kappa = 1/H$.

Let the initial model in dynamics of point mass is NM, that gives as mathematical model

$$m\frac{dV}{dt} = F(r,V), \quad \frac{dr}{dt} = V \tag{5.1}$$

$x = (r,V)$ are the state variables; F is vector-function of force; $F(r,V) = -bV + \ldots$; b is the coefficient of friction force; let $b = b^*/\kappa$, $\kappa > 0$ is small parameter (big friction). Let other members in (5.1) have 0–power on κ.

We can construct the variables necessary transformation and change $x \to y = (y_1, y_2)$. In new variables y the system (5.1) will be reduced to the form (3.2). In accordance with our results (p.3) we receive here SS_0 and SS_1 on κ-parameter. In state variables we shall have SM_0 and SM_1, as idealized models. Here SM_1 is the linearized model on κ (simplified model of first level on κ-parameter) that corresponds to Aristotel's mechanics model.

If $F(r,V) = F_0 - bV$, F is constant force; we get (as SM_1):

$$F = bV, \quad \frac{dr}{dt} = V \tag{5.2}$$

(5.2) is the Aristotel's idealization. So, AM is the asymptotic model of 1-st level on κ-parameter (that is analogue to PM). Also we obtained the conditions of acceptability for this model (did not write here).

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METODE I MODELI U SINGULARNIM PROBLEMIMA MEHANIKE

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Značajno je izučavati, radi razvoja koncepta i metoda klasične teorije stabilnosti, referentne probleme klase singularno pobudjenih sistema. Različiti aspekti kompleksnih dinamičkih sistema su proučeni. Metode modeliranja i analize na bazi uopštene metodologije u kombinaciji sa idejama teorije stbilnosti i metodama asismptotske teorije su razradjene. Osim netradicionalnih, uopštena približenja, formirana na bazi Lyapunovljevih metoda, postulata Chetayev-ljeve stabilnosti i postulata singularnosti su razradjena. Dobijen je univerzalni alat, koji pruža mogućnost dolaženja do približnog rešenja fundamentalnih problema u opštoj teoriji modeliranja. Efektivni algoritam na inženjerskom nivou je dobijen. Razvijene metode su ilustrovane na primerima iz mehanike. Razmotreni su aktuelni problemi teorije giroskopskih sistema, elektromehaničkih sistema, mehaničkih sistema sa trenjem i drugi. Novi elegantni pristup je dobijen od interesa je za teoriju i primenu. Taj pristup je veoma perspektivan i sa gnoseološke tačke gledišta.