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# THE GENERALIZED CLASSICAL METHOD OF THE CONSTRUCTION OF V–FUNCTIONS FROM THE FIRST INTEGRALS

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**Abstract**. *The new heuristic method generalizing the classical construction of v–function from first integrals is described. There is shown the generalized method keeps the feature of the classical one. It means v–functions are constructed as solutions of some completely integrable partial differential equation. The form of this equation and its order are defined by nondegenerate multiparameter function* $V(\mathbf{x},\text{a})+\alpha_q$, $\mathbf{x}\in R^n$, $\alpha\in R^{q-1}$ *($\alpha$–vector of parameters) identically. Function V is the generalization of classical linear integral sheaf.*
*The representations of v–functions are described. Classical representations (method of Chetaev integral sheaf, the construction of these functions as non-linear functions of the integrals) are supplemented by geometric structures of v–functions as envelopes of some subsets of function* $V(\mathbf{x},\text{a}) + \alpha_q$.
*The most of the investigated stability problems from classical mechanics are covered and put in order by a new method. By this method, some algebraic unsolvable stability problems of ordinary differential equations are investigated.*

**1. The classical method of the construction of v–functions.** It is well known that the investigation of the stability problems of ordinary differential equations needs the construction of v–functions. The classical method of the construction of these functions from first integrals is the more effective one. The basis of it have been put by J. Lagrange [1], A.M. Lyapunov [2], E. J. Routh [3,4]. If the system with known sign definite integral is investigated, it is the simplest case. The stability of the system is followed from the first Lyapunov theorem. The Chetaev method of integral sheaf is used in other cases as a rule. It closely connects with Routh–Lyapunov theorem which is used for the investigation of the stationary motions of mechanic systems. By that classical method, ample hamiltonian systems are researched (for example, [5–16]).

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It has been shown [17,18], that the classical method is of universal importance to the investigation of stability of the hamiltonian and reversible systems because v–functions satisfying the first Lyapunov theorem must be the sign definite first integrals.

For a number of non–hamiltonian systems, v–functions belong to the space of first integrals of some comparison systems. In this case, the energy integral of comparison system is used as v–function ordinary. By integral sheaf of comparison hamiltonian system, the stability of mechanic systems with dissipation is investigated [19–24].

The formal description of classical method is given below. Let us consider the stability of trivial solution $\mathbf{x} = \mathbf{0}$ of differential equations

$$\dot{\mathbf{x}} = \mathbf{X}(\mathbf{x}), \quad \mathbf{x} \in R^n \tag{1.1}$$

with smooth right–hand sides.

For the construction of v–function, consider some comparison system

$$\dot{\mathbf{x}} = \mathbf{X}_0(\mathbf{x})\,. \tag{1.2}$$

If (1.1) is the hamiltonian or reversible system, we identify the equations (1.2) with (1.1). Let $F_1(\mathbf{x}),...,F_{m-1}(\mathbf{x})$, $m \le n$ be the known first integrals of system (1.2). According to classical method, v–functions being found belong to the set $M = \{F(\mathbf{x}){:}\ F = B(F_1,...F_{m-1}),$ $B \in C^r(R^{m-1},R^1)\}$.

There are two classical ways to construct desired functions:

I) v($\mathbf{x}$) is the linear function of integrals $F_j(\mathbf{x})$

II) v($\mathbf{x}$) is the non–linear function of integrals $F_j(\mathbf{x})$; v($\mathbf{x}$) is represented as Chetaev integral sheaf as a rule:

$$\mathrm{v} = \sum_j \lambda_j F_j + \sum_j \mu_j F_j^2\ ,\ \lambda_j,\ \mu_j = \text{const.} \tag{1.3}$$

You can use new ways for construction of these functions also. Unfortunately, they have no applications but they are needed for the complete description of classical method.

The set M is connected with partial differential equations [25]. Let $W(\mathbf{x},\mathbf{C}) = C_1F_1 + ...\ C_{m-l}F_{m-1}$, $\mathbf{C} = (C_1,...,C_{m-1})$ be the nondegenerate integral sheaf. The set M then is coincided with smooth solution space of some completely integrable system of partial differential equations of first order:

$$H_1(\mathbf{x},\mathbf{p}) = 0, \ldots, H_{1+n-m}(\mathbf{x},\mathbf{p}) = 0\,,\quad \mathbf{p} \in R^n,\quad p_j = \frac{\partial z}{\partial x_j}\,, \tag{1.4}$$

Here, $W + C_m$ is the complete integral of (1.4). It means, the system $p_j = \partial W/\partial x_j$ ($j = 1,...,\ n$) is solvable for $(m-1)$ parameters $C_j$ identically. The elimination of these parameters from the remaining $(m - n + 1)$ equations reduce them to the form being the same as (1.4). $H_j$ are the linear functions of $\mathbf{p}$ because $W + C_m$ is linear function of $C_j$. If $m = n$, the system (1.4) is equivalent to one equation

$$\sum_{j=1}^{n} X_{0j}(\mathbf{x})\, p_j = 0\,. \tag{1.5}$$

It is known [25] that arbitrary solution z($\mathbf{x}$) of equations (1.4) can be derived from complete integral $W + C_m$ by Lagrange method of variation of parameters $C_j$ : $z(\mathbf{x}) = C_1(\mathbf{x})F_1(\mathbf{x}) + \ldots + C_{m-1}(\mathbf{x})F_{m-1}(\mathbf{x}) + C_m(\mathbf{x})$, where $(\mathbf{C}(\mathbf{x}), C_m(\mathbf{x}))$ is the solution of Pfaffian equation

$$\sum_{j=1}^{m-1} W'_{C_j}\, dC_j + dC_m = 0 \,. \tag{1.6}$$

The geometric interpretation of this method is known also [25, 26]. Let π be arbitrary regular $l$–surface from the space of parameters $C_1,...,C_m$ $(0 \le l \le m-1)$; let $(W + C_m)_\pi$ be the restriction of the family $(W + C_m)$ to this surface. The arbitrary solution $z(\mathbf{x})$ of equations (1.4) is the envelope of certain $l$–parametric family $(W + C_m)_\pi$ locally. In the case $l = 0$, the envelope is the function $(W + C_m)_\pi$ by definition. Hence it follows, the solution space of (1.4) (that is, the set M) consists of the envelopes of various families $(W + C_m)_\pi$ when index π passes by the set of regular $l$–surfaces $(0 \le l \le m-1)$ belonging to the space of parameters $C_1,...,C_m$.

Let us introduce the further notations. Let M be the subspace of envelopes of families $(W + C_m)_\pi$ when index π passes by the set of various regular $l$–surfaces belonging to the space of parameters $C_1,...,C_m$. It is obviously

$$M = \bigcup_{l=0}^{m-1} M_l \,.$$

The subspace $M_0$ has the simplest construction because it consists of the functions $W(\mathbf{x},\mathbf{C}) + C_m$, $(\mathbf{C}, C_m) = \mathbf{const}$. The other subspace $M_l$ is filled by envelopes. In the case $n$ = m, the smooth structure of M is described in paper [27].

So, the classical method may be supplement. The Lyapunov functions are the solutions of completely integrable system of partial differential equations; therefore they have the following representations:

III) $\mathrm{v}(\mathbf{x}) = C_1(\mathbf{x})F_1(\mathbf{x}) + \ldots + C_{m-1}(\mathbf{x})F_{m-1}(\mathbf{x}) + C_m(\mathbf{x})$,

where $(\tilde{N}_1(\mathbf{x}),...,\mathrm{C_m}(\mathbf{x}))$ is the solution vector of (1.6)

IV) $\mathrm{v}(\mathbf{x})$ is the envelope of certain $l$–parameter family $(W + C_m)_\pi$ , dim $\pi = l$.

Thus, the classical method consists of four ways I) – IV).

It should be noted that the representation of Lyapunov functions as envelopes of certain families of functions is typical for stability problems. Indeed, according to Chetaev's method, Lyapunov functions must be seek as the integral sheaf (1.3). If $\sum_j \mu_j^2 \ne 0$, $\mathrm{v}(\mathbf{x})$ does not belong then to $M_0$ . Hence, $\mathrm{v}(\mathbf{x})$ belongs to the subset $\bigcup_{l=1}^{m-1} M_l$ . So, $\mathrm{v}(\mathbf{x})$ is the envelope of a certain family $(W + C_m)_\pi$ . If $\mathrm{v}(\mathbf{x})$ is represented as arbitrary non–linear function of known integral, we come to this conclusion also.

**2. The generalized heuristic method of the construction of v–functions from first integrals.** In spite of the effectiveness of classical method, its applied region is restricted by construction of sufficient stability conditions of conservative system as a rule. Indeed, the most of v–functions satisfying the asymptotic stability theorems or instability theorems are not the integrals of the system being investigated as well as of the comparison system.

Let us consider the generalized heuristic method of the construction of v–functions free from the lacks above mentioned. The idea of this method is as follows: v–functions being found belong to certain functional space generalizing the set M. Let us describe this new space.

So, let

$$V(\mathbf{x},\mathrm{a})+\alpha_q\,,\ q\geq m,\ \ \mathrm{a}=(\alpha_1,\ldots\alpha_{q-1}) \tag{2.1}$$

be the smooth q–parameter function and

$$\operatorname{rank} V_{\mathbf{x},\mathrm{a}}=\min(n,q-1)$$

in every point of some range of values **x** and $\boldsymbol{\alpha}$. We will assume also that $W(\mathbf{x},\mathbf{C})+C_m$ is a special case of $V$, i.e.

$$W(\mathbf{x},\mathbf{C})=(V+\alpha_q)\Big|_{\lambda^{m-1}}, \tag{2.2}$$

Here $\lambda^{m-1}=(\mathrm{a}(\mathbf{C}),\alpha_q(\mathbf{C}))$ is a regular ($m$–1) surface from the space of parameters $\alpha_j$ .

Let us introduce the notations: (*) is $(\mathbf{x}_0,\boldsymbol{\alpha}_0,\alpha_{q0})$; $\pi$ is the regular $l$–surface from the space of parameters $\boldsymbol{\alpha}$, $\alpha_q$ and $(\boldsymbol{\alpha}_0,\ \alpha_{q0})\in\pi$ ; $K_l^{(*)}[V]$ is the set of envelopes of various families $(V+\alpha_q)|_\pi$ at a point $\mathbf{x}_0$ when index $\pi$ passes by the set of regular $l$–surfaces which include the fixed point $(\boldsymbol{\alpha}_0,\ \alpha_{q\,0})$.

**Definition 1** [28]. We will call the space

$$T[V]=\bigcup_{l=0}^{\min(q-1,n-1)}T_l[V],\ T_l[V]=\bigcup_{(*)}K_l^{(*)}[V]$$

a functional extensions of the solution space of equations (1.4) and also define $S=(q-n)$ to be a degree of this space.

It is obviously, $\mathrm{M}\subseteq T[V]$. It follows from the definition that $T[V]$ consists of envelopes of various subfamilies of function $V+\alpha_q$. In the classical case ($m-n\leq S\leq 0$), the definition of $T[V]$ repeats literally the geometric description of the solution space of the completely integrable system of partial differential equations provided $V(\mathbf{x},\boldsymbol{\alpha})+\alpha_q$ is the complete integral of this system. Hence it follows, $T[V]$ of the classical case is the solution space of a certain system of type (1.4). The number of such system equals to $(n-q+1)$.

Let us consider the non–classical case S > 0. It follows from condition (2.2), $M$ and $T[V]$ have the "structural co-ordination" [28], i.e. $\mathrm{M}\subseteq T[V]$, in addition to it $M_l\subset T_l\,[V]$, $l=0,\ldots,\ q-1$ . The set $T_0\ [V]$ is the simplest subspace among other subspaces $T_l\ [V]$ because it consists of functions $V(\mathbf{x},\ \boldsymbol{\alpha})+\alpha_q$ , $(\boldsymbol{\alpha}_,\ \alpha_q)=$ **const** only.

The problem of the relation between the differential equations and new functional spaces is discussed below. There are two aspects. Establishing the relation between $T[V]$ and the set of singular integral manifolds of linear Pfaffian equation with covariant property to degree S represents the first aspect. Theorems connected $T[V]$ with smooth solution space of non–covariant partial differential equation of high order are the second aspect.

Let us consider Pfaffian equation associated with $V$:

$$\sum_{j=1}^{q-1} V'_{\alpha_j}(\mathbf{x},\mathrm{a})d\alpha_j + d\alpha_q = 0 \ (q > n)\,. \tag{2.4}$$

**Definition 2.** If

$$\operatorname{rank}\frac{\partial(\mathrm{a},\alpha_q)}{\partial\mathbf{x}} < n\,, \tag{2.5}$$

we call the solution $(\boldsymbol{\alpha}(\mathbf{x}),\alpha_q(\mathbf{x}))$ of equation (2.4) a singular solution.

**Theorem 1** [29]. Function z($\mathbf{x}$) belongs to the space $T[V]$ if and only if the singular solution $(\boldsymbol{\alpha}(\mathbf{x}),\ \alpha_q(\mathbf{x}))$ of equation (2.4) exists provided that

$$z(\mathbf{x}) = V(\mathbf{x},\mathrm{a}(\mathbf{x})) + \alpha_q(\mathbf{x})\,. \tag{2.6}$$

In the classical case $m - n \le \mathrm{S} \le 0$, theorem 1 is known as Lagrange theorem of variation of parameters method for system of first order partial differential equations. The condition (2.5) is realised always here because differentials $d\alpha_j(\mathbf{x})$ linear depend on from each other by equation (2.4).

Let S = 1. The singular condition (2.5) has the form

$$\det\alpha_{\mathbf{x}} = 0 \tag{2.7}$$

because the differential $d\alpha_q$ depends on the differentials $d\alpha_j$ , $j$ = 1,..., $n$ by equation (2.4).

Let us differentiate the equality (2.6) with respect to $\mathbf{x}$ taking (2.4) into account, we than have

$$z_{\mathbf{x}} = V_{\mathbf{x}}(\mathbf{x},\mathrm{a}(\mathbf{x}))\,. \tag{2.8}$$

Let us differentiate the equality (2.8) with respect to $\mathbf{x}$ also:

$$(z - V)_{\mathbf{xx}} = V_{\mathrm{a}\mathbf{x}}\cdot\mathrm{a}_{\mathbf{x}},\ \det V_{\mathrm{a}\mathbf{x}} \ne 0\,.$$

Here the derivatives are calculated with respect to explicitly variables. Hence it follows, $\boldsymbol{\alpha}$(**õ**) and V)$_{\mathbf{xx}}$ are equivalent matrixes. Therefore, conditions (2.7), (2.8) have the following form

$$\det(z(\mathbf{x}) - V(\mathbf{x},\mathrm{a}))_{\mathbf{xx}} = 0,\quad z_{\mathbf{x}} = V_{\mathbf{x}}(\mathbf{x},\mathrm{a})\,. \tag{2.9}$$

The equation (2.9) is the n–dimensional analogue of Monge–Ampere equation. In the particular case n = 2, the equality (2.9) is the classical Monge–Ampere equation:

$$rt - s^2 = \alpha r + 2bs + ct + \varphi,\quad r = z_{x_1x_1},\quad s = z_{x_1x_2},\quad t = z_{x_2x_2}\,, \tag{2.10}$$

Here the coefficients $a$, $b$, $c$, $\varphi$ are the function of $x_1$, $x_2$, $z$ and partial derivatives $z_{x1}$, $z_{x2}$. In the case under consideration these coefficients are satisfied to additional condition

$$c(dx_1)^2 - 2bdx_1dx_2 + a(dx_2)^2 \equiv d^2V(\mathbf{x},\mathrm{a}(\mathbf{x},\mathbf{z}_{\mathbf{x}})),\quad \varphi = b^2 - ac\,. \tag{2.11}$$

Therefore equation (2.10) has the parabolic type.

If $\mathbf{x}$ is the fall point of rank of matrix $(z(\mathbf{x}) - V(\mathbf{x},\alpha(\mathbf{x}))_{\mathbf{xx}}$, we will exclude it from consideration for every function $z(\mathbf{x}) \in T[V]$.

**Theorem 2** [29]. The space T[V] of degree S = 1 is the space of smooth solutions $z(\mathbf{x})$ of n–dimensional Monge–Ampere equation (2.9).

Let us consider the case S = 2.

**Theorem 3.** [30]The space T[V] of degree S = 2 is the space of smooth solutions $z(\mathbf{x})$ of some third order partial differential equation

$$F(\mathbf{x}, z(\mathbf{x}), z_{\mathbf{x}}, z_{\mathbf{xx}}, z_{\mathbf{xxx}}) = 0 \tag{2.12}$$

The case S ≥ 3 isn't being investigated now.

As follows from [29], the increase of one of S leads to the extension of space $T[V]$ that it becomes a degenerate subset of this extension. That procedure generates the inclusion of equation connected with $T[V]$ of degree S as an intermediate integral of equation connected with the extension space of degree (S + 1). So, equation (1.5) is the intermediate integral of equation (2.9), equation (2.9) is the intermediate integral of equation (2.12) and so on.

So, by generalized heuristic method, v–functions being found are the elements of space $T[V]$. That is, when $S \geq 0$, these functions are the integrals of some partial differential equation of (S + 1) order; when $S < 0$, these functions are the integrals of some system of partial differential equations of first order.

In classical cases ($m - n \leq S \leq 0$), space $T[V]$ is a functional closed set. It means, if $\{G_1,..., G_k\} \subset T[V]$ is an arbitrary collection of functions and $B(x_1,...,x_k)$ is the arbitrary smooth function, we then have $B(G_1,...,G_k) \in T[V]$.

However, if $S > 0$, the space $T[V]$ isn't the functional closed set because $T[V]$ is described by non-linear partial differential equation. Nevertheless, the following lemma establishes the "partial functional closure" of these spaces.

**Lemma.** Let $V(\boldsymbol{x},\boldsymbol{\alpha}) + \alpha_q$, $V = \sum_{j=1}^{q-1} \alpha_j U_j(\mathbf{x})$ $(q > n)$ be the nondegenerate family of functions which linear depends on parameters $\alpha_j$ ; let $\{G_1(\mathbf{x}),...,G_k(\mathbf{x})\}$ $(k \leq n - 1)$ is the arbitrary family of the independent functions from subspace $T_0[V]$. Here, if $B(ó_1,...,ó_k)$ is the arbitrary smooth function, we then have $B(G_1(\mathbf{x}),..., G_k(\mathbf{x})) \in T[V]$.

Proof of lemma. From lemma, it follows that $G_j$ has the following presentation:

$$G_j = \alpha_1^{(j)} U_1 + \cdots + \alpha_{q-1}^{(j)} U_{q-1} + \alpha_q^{(j)}$$

Here, $\alpha_i^{(j)} \alpha_i^{(j)}$ are the fixed values of arbitrary parameters $\alpha_i$ . Let $(W^* + C_{k+1})$ be the nondegenerate family of functions such that

$$W^* = \sum_{j=1}^{k} C_j G_j = \sum_{i=1}^{q-1} U_i \left( \sum_{j=1}^{k} C_j \alpha_i^{(j)} \right) + \sum_{j=1}^{k} C_j \alpha_q^{(j)} , \quad C_j = \text{const} . \tag{2.13}$$

From (2.13), it follows that $(W^* + C_{k+1})$ is the partial case of function $V(\mathbf{x},\boldsymbol{\alpha})+\alpha_q$ that is why $T[W^*] \subset T[V]$. The space $T[W^*]$ is the functional closed set. Indeed, if $k = n - 1$, $(W^* + C_{k+1})$ is the complete integral of the linear homogeneous differential equation of type (1.5); if $k < n - 1$, it is the complete integral of the system of differential equations of type (1.4). So, we have the classical case; therefore, if $\{G_1(\mathbf{x}),...,G_k(\mathbf{x})\}$ belong to the subspace $T_0[W^*] \subset T[W^*]$, we then have $B(G_1(\mathbf{x}),...,G_k(\mathbf{x})) \in T[W^*] \subset T[V]$. This complete the proof.

In the case $k = n$, the lemma isn't correct. As follows from examples, the function $B(G_1,..., G_n)$ doesn't belong to the space $T[V]$ in general.

This lemma simplifies the search of v–functions from the space $T[V]$ essentially. For example, if we define the solutions of the partial differential equations as the integrals, the construction of sign definite v–functions by the method of Chetaev integral sheaf is possible. The Pozaritskii theorem [31] (which establishes the criterion of sign definite of $B(G_1(\mathbf{x}),...,G_k(\mathbf{x}))$ ) as well as the results of investigation of $B(G_1(\mathbf{x}),...,G_k(\mathbf{x}))$ [32] by its Hessian remains valid.

So, let $V(\boldsymbol{x},\boldsymbol{\alpha})+\alpha_q$ , $V = \sum_{j=1}^{q-1} \alpha_j U_j(\mathbf{x})$ $(q > m)$ be the nondegenerate family of functions which linear depend on parameters $\alpha_j$. Let $W + C_m$ , $W = C_1F_1+...C_{m-1}F_{m-1}$ be the partial case of $V(\boldsymbol{x}, \boldsymbol{\alpha})+\alpha_q$ and $T[V]$ be the extension of $M = \{F(\mathbf{x})\colon F = B(F_1,...F_{m-1}), B \in C^r(R^{m-1},R^1)\}$.

The generalized heuristic method of the construction of v–functions from first integrals is of the following form.

I) v($\mathbf{x}$) is the linear function of integrals $U_j(\mathbf{x})$, $j = 1,...q-1$

II) v($\mathbf{x}$) is the non–linear function of the independent integrals $U_j(\mathbf{x})$, $j = 1,...,k$, $k \le n - 1$. Function v($\mathbf{x}$) may be constructed as Chetaev integral sheaf:

$$\mathrm{v} = \sum_{j=1}^{k} \lambda_j U_j + \sum_{j=1}^{k} \mu_j U_j^2 \text{ , } \lambda_j, \mu_j = \text{const}$$

III) v($\mathbf{x}$) is the function from $T[V]$ such that

$$\mathrm{v}(\mathbf{x}) = \alpha_1(\mathbf{x})U_1(\mathbf{x}) + \ldots + \alpha_{q-1}(\mathbf{x})U_{q-1}(\mathbf{x}) + \alpha_q(\mathbf{x})$$

Here, $(\alpha_1(\mathbf{x}),...,\alpha_q(\mathbf{x}))$ is the solution vector of equation (2.4) satisfying the condition of the degeneration (2.5).

IV) v($\mathbf{x}$) is the envelope of certain $l$–parameter family $(V + \alpha_q)_\pi$ where $\pi = (\mathrm{a}(C_1,...,C_l),\alpha_q(C_1,...,C_l))$ is the regular $l$–surface from the space of arbitrary parameters $\alpha$, $\alpha_q$ .

As described above, the generalized method keeps all specific characteristics of classical one and supplements it by the exit to the solution set of partial differential equations of high order.

The difference of generalized method from the classical one is as follows. The set of non–linear functions of integrals $U_j(\mathbf{x})$ doesn it cover all space $T[V]$. In applied problems, therefore, we need to use the items III), IV) of the method described. The formula (2.14) gives us the more general presentation of v($\mathbf{x}$) as a function from $T[V]$.

In keeping with [30], v–functions of most of well known stability problems belong to space $T[V]$ provided $V$ is the deformed linear integral sheaf of some comparison system.

Note, these deformations of linear sheaf belong to the restricted class of "trivial" deformations [30]. That is why, the generalized method puts in order the most of well known stability problems. Moreover, by the generalized method a algebraic unsolvable stability problems at resonances 1:1 and 1:3 were investigated [33–35]. Despite of this insolubility, some algebraic criterions of asymptotic stability were obtained.

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# GENERALISANA KLASIČNA METODA KONSTRUISANJA V-FUNKCIJA IZ PRVOG INTEGRALA

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*Opisuje se nova heuristička metoda koja generalizuje konstruisanje klasičnih v-funkcija iz prvog integrala kretanja. Generalizovana metoda prati tipičnu liniju klasične metode. To znači da su v-funkcije konstruisane od nekih potpuno integrabilnih parcijalnih diferencijalnih jednačina (ili sistema parcijalnih diferencijalnih jednačina prvog reda). Oblik ove jednačine i njen red su definisani identično pomoću nedegenerativne višeparametarske funkcije* $V(\mathbf{x},a)+\alpha_q$, $\mathbf{x}\in R^n$, $\alpha\in R^{q-1}$ *(*$\alpha$ *- vektor parametara). Funkcija V je generalizovana iz klasičnog linearnog integralnog snopa.*

*Opisane su reprezentacije v-funkcija. Klasične reprezentacije (metoda integralnog snopa Chataev-a, konstrukcija ovih funkcija u obliku nelinearnih funkcija integrala) su dopunjene geometrijskim strukturama kada se v-funkcije smatraju obvojnicom nekih podskupova funkcija* $V(\mathbf{x},a)+\alpha_q$.

*Pokazano je da je većina ispitivanih problema stabilnosti u klasičnoj mehanici pokrivena novom metodom. Ovom metodom su ipitani neki algebarski nerešivi problemi običnih diferencijanih jednačina.*