UNIVERSITY OF NIŠ
The scientific journal **FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic Control and Robotics** Vol.2, No 10, 2000 pp. 1061 - 1069
Editor of series: Katica (Stevanovi}) Hedrih, e-mail: katica@masfak.masfak.ni.ac.yu
Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: +381 18 547-095, Fax: +381 18 547-950
http:// ni.ac.yu/Facta

LYAPUNOV FUNCTIONS FOR LIÉNARD-TYPE NONLINEAR SYSTEMS

UDC 517.5:621.316(045)

Hayao Miyagi[1], Josiah Munda[2]

[1] University of the Ryukyus, Department of Information Engineering
1 Senbaru, Nishihara, Okinawa, 903-0213, Japan, e-mail: miyagi@sys.ie.u-ryukyu.ac.jp
[2] University of the Ryukyus, Graduate School of Science & Eng.
e-mail: munda@sys.ie.u-ryukyu.ac.jp

Abstract. *Lagrange-Charpit method has been employed to construct a generalized Lyapunov functions for the Lienard-type nonlinear system, which is important as a reprenzentative system expressing the general RLC electric circuits and networks, and mechanical spring systems etc. The functionincludes particular nonlinear terms as arbitraty functions, by which quadratic term appearing in the Lure-type Liyapunov function can be extended, By changing the forms of the arbitrary functions, the result yields all the conventional Lyapunov functions as special cases.*

Key words: *Lienard-type Nonlinear Systems, Lagrange-Charpit Method, System Stability, Lyapunov Functions.*

1 Introduction

In stability analysis of systems, it is common to utilize the Lyapunov method, and the energy-type Lyapunov function is widely used. The Lyapunov method has two main uses, i.e., establishment of the stability of a null solution of the system and determination of a stability region for the system. The latter is often important to system engineers, because a lot of systems appearing in engineering have nonlinearities in which only local stability is discussed. Moreover, the Lyapunov method gives only sufficient conditions for obtaining the stability. Hence, a Lyapunov function which gives good approximation to the true stability boundary is desired.

Most of the research in the area of stability of nonlinear systems has only dealt with the simple system given by Liénard's equation [1] – [5]. Liénard-type Nonlinear System given by n second-order differential equations, a generalization of Liénard's equation, has not been thoroughly discussed. Stability of that type of system has been studied to some level in [6] – [10].

Received May 30, 2000

This paper, which is derived from our work in [11], presents a generalized Lyapunov function for the Liénard-type Nonlinear System given by n second-order differential equations, using the Lagrange-Charpit method [4] which is a well-known technique for solving partial differential equations. The function includes an arbitrary function, by which the quadratic term appearing in the Luré-type Lyapunov function is extended. All Lyapunov functions presented so far are obtainable by means of changing the form of the arbitrary function. The Lyapunov function includes extended terms which contribute to the extension of the obtained stability boundary.

2 Liénard-type nonlinear system

The Liénard-type nonlinear system considered here is of the form [7]

$$\ddot{\boldsymbol{y}} + \boldsymbol{G}(\boldsymbol{y})\dot{\boldsymbol{y}} + \gamma(\boldsymbol{y}) = \mathbf{0} \tag{1}$$

where $\boldsymbol{y}$ is an $\boldsymbol{n}$ vector, $\boldsymbol{G}(\boldsymbol{y})(> 0)$ is a nonlinear damping defined by

$$\boldsymbol{G}(\boldsymbol{y}) = \boldsymbol{D}\begin{bmatrix} g_1(\sigma_1) & & & 0 \\ & g_2(\sigma_2) & & \\ & & \ddots & \\ 0 & & & g_m(\sigma_m) \end{bmatrix}\boldsymbol{B}^{\boldsymbol{T}} = \boldsymbol{D}\left\{diag[g_i(\sigma_i)]\right\}\boldsymbol{B}^{\boldsymbol{T}}$$

$\boldsymbol{D}$ and $\boldsymbol{B}$ are $\boldsymbol{n} \times \boldsymbol{m}$ matrices, $\gamma(\boldsymbol{y}) = \boldsymbol{B}\boldsymbol{f}(\boldsymbol{\sigma}), \boldsymbol{\sigma} = \boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{y}$ and $\boldsymbol{f}^{\boldsymbol{T}}(\boldsymbol{\sigma}) = [\boldsymbol{f_1}(\boldsymbol{\sigma_1}), \boldsymbol{f_2}(\boldsymbol{\sigma_2}), \cdots, [\boldsymbol{f_m}(\boldsymbol{\sigma_m})]$.

The nonlinear functions $\boldsymbol{g_i}(\boldsymbol{\sigma_i})$ and $\boldsymbol{f_i}(\boldsymbol{\sigma_i})$ are assumed to be continuous, differentiable and to satisfy the following conditions:

(i) . $\boldsymbol{g_i}(\boldsymbol{\sigma_i}) > 0$ for $\boldsymbol{\sigma_i} \neq 0$

(ii) . $\boldsymbol{\sigma_i}\boldsymbol{f_1}(\boldsymbol{\sigma_1}) > 0$ for $\boldsymbol{\sigma_i} \neq 0$

(iii) . $|\boldsymbol{\Psi_i}(\boldsymbol{\sigma_i})| \to \infty$ as $|\boldsymbol{\sigma_i}| \to \infty$
where $\boldsymbol{\Psi_i}(\boldsymbol{\sigma_i}) = \int_0^{\sigma_i} \boldsymbol{g_i}(\boldsymbol{\sigma_i})d\boldsymbol{\sigma_i}$
Making the replacements: $\boldsymbol{y} = \boldsymbol{x_1}, \dot{\boldsymbol{y}} = \boldsymbol{x_2}$, we can rewrite system (1) in the form of first-order simultaneous equations as

$$\dot{\boldsymbol{x}} = \boldsymbol{h}(\boldsymbol{x}); \quad (\boldsymbol{h}(\mathbf{0}) = \mathbf{0}) \tag{2}$$

where $\boldsymbol{x} = [\boldsymbol{x_1^T}, \boldsymbol{x_2^T}]^{\boldsymbol{T}}, \boldsymbol{h}(\boldsymbol{x}) = [\boldsymbol{h_1^T}, \boldsymbol{h_2^T}]^{\boldsymbol{T}}, \boldsymbol{x_i} = [x_{i1}, x_{i2}, \cdots, x_{in}]^T, \boldsymbol{h_i} = [h_{i1}, h_{i2}, \cdots, h_{in}]^T; (i = 1, 2)$ and $\boldsymbol{x} = \mathbf{0}$ is the equilibrium state which is asymptotically stable.

3 Lyapunov function for Liénard-type nonlinear systems

The problem concerning the stability analysis of the equilibrium state of the system (2) is formally taken up by the search for a Lyapunov function $V = V(x)$ which satisfies the partial differential equation

$$F(x, V, P) = P^T h(x) + \psi(x) = 0 \tag{3}$$

where $P = \frac{\partial V}{\partial x} = [P_1^T, P_2^T], P_i = [P_{i1}, P_{i2}, \cdots, P_{in}]^T$ and $\psi(x)$ is an arbitrary non-negative function whose opposite sign may be the time derivative of the obtained Lyapunov function. Separation of P into P_1, P_2 simplifies the application of the Lagrange-Charpit method to second-order differential equations.
In this section, we derive the generalized Lyapunov function for Liénard-type nonlinear systems.

3.1 Generalized Lyapunov function for Liénard-type nonlinear systems

Stability of Liénard-type nonlinear system (1), which is a generalization of Liénard's equation, has been studied in [7] for the case $D = B$ and $D^T B = diag[\lambda_i]$ (λ_i: positive constants). In this section, we construct a generalized Lyapunov function which includes all Lyapunov functions presented so far, using the Lagrange-Charpit method.
The characteristic equation with $\frac{\partial F}{\partial V} = 0$ becomes

$$\begin{aligned} &\frac{dx_{11}}{h_{11}(x)} = \cdots = \frac{dx_{1n}}{h_{1n}(x)} = \frac{dx_{21}}{h_{21}(x)} = \cdots = \frac{dx_{2n}}{h_{2n}(x)} \\ &= \frac{-dP_{11}}{\frac{\partial F}{\partial x_{11}}} = \cdots = \frac{-dP_{1n}}{\frac{\partial F}{\partial x_{1n}}} = \frac{-dP_{21}}{\frac{\partial F}{\partial x_{21}}} = \cdots = \frac{-dP_{2n}}{\frac{\partial F}{\partial x_{2n}}} \end{aligned} \tag{4}$$

From equation (4) , $2n$ equations can be derived containing $P_1, P_2$and $\frac{\partial \psi}{\partial x_2}$ as

$$\begin{aligned} Z^1 &= \alpha D\{diag[\phi_1(\sigma_1)]\}\mathbf{1} + \beta x_2 - P_2 = 0 \\ Z^2 &= \alpha \Phi'(\sigma) x_2 - \beta \{G(x_1)x_2 + Bf(\sigma)\} + P_1 - G(x_1)P_2 + \frac{\partial \psi}{\partial x_2} = 0 \end{aligned} \tag{5}$$

where $Z^1 = [Z_1, Z_2, \cdots, Z_n]^T$, $Z^2 = [Z_{n+1}, Z_{n+2}, \cdots, Z_{2n}]^T$, $\Phi(\sigma) = D \{diag[\phi_i'(\sigma_i)]\} B^T$, $\phi_i'(\sigma_i) = \frac{d\phi_i(\sigma_i)}{\sigma_i}$. α and β are arbitrary constants, $\phi_i(\sigma_i)$ are arbitrary functions and $\mathbf{1}$ is a vector with all elements 1.

The conditions that functions $Z_1, Z_2, \ldots, Z_{2n-1}$ and F have common solution can be written in the form,

$$
\begin{aligned}
[Z_i, Z_j] &= \sum_{k=1}^{2} \left(\left[\frac{dZ_i}{dx_k} \right]^T \frac{\partial Z_j}{\partial P_k} - \left[\frac{dZ_j}{dx_k} \right]^T \frac{\partial Z_i}{\partial P_k} \right) = 0 \\
[Z_i, Z_j] &= \sum_{k=1}^{2} \left(\left[\frac{dZ_i}{dx_k} \right]^T \frac{\partial F}{\partial P_k} - \left[\frac{dF}{dx_k} \right]^T \frac{\partial Z_i}{\partial P_k} \right) = 0
\end{aligned}
\tag{6}
$$

where $i, j = 1, 2, \cdots, 2n-1, i \neq j$ and

$$
\frac{dZ_i}{dx_k} = \frac{\partial Z_i}{\partial x_k} + P_k \frac{\partial Z_i}{\partial V} \tag{7}
$$

Applying (6) to Z^1, Z^2 and F gives

$$
\begin{aligned}
[Z^1, Z^2] &= 2\left\{\beta G(x_1) - \alpha \Phi'(\sigma)\right\} - \frac{\partial^2 \psi}{\partial x_2^2} = 0 \\
[Z^2, F] &= \begin{bmatrix} \dot{x}_2^T [\alpha \frac{\partial \Phi'(\sigma)}{\partial x_{11}} - \beta \frac{\partial G(x_1)}{\partial x_{11}}] x_2 \\ \dot{x}_2^T [\alpha \frac{\partial \Phi'(\sigma)}{\partial x_{12}} - \beta \frac{\partial G(x_1)}{\partial x_{12}}] x_2 \\ \vdots \\ \dot{x}_2^T [\alpha \frac{\partial \Phi'(\sigma)}{\partial x_{1n}} - \beta \frac{\partial G(x_1)}{\partial x_{1n}}] x_2 \end{bmatrix} + \frac{\partial^2 \psi}{\partial x_1 \partial x_2} x_2 \\
&- \frac{\partial \psi}{\partial x_1} + \alpha \frac{\partial \left[\sum_{i=1}^{m} \lambda_i \phi_i(\sigma_i) f_i(\sigma_i) \right]}{\partial x_1} = 0
\end{aligned}
\tag{8}
$$

where $[Z^1, Z^2]$and $[Z^2, F]$have been defined as follows:

$$
\begin{aligned}
[Z^1, Z^2] &\equiv \begin{bmatrix} [Z_{n+1}, Z_1] & \cdots & [Z_{n+1}, Z_n] \\ \vdots & & \vdots \\ [Z_{2n}, Z_1] & \cdots & [Z_{2n}, Z_n] \end{bmatrix} \\
[Z^2, F] &\equiv \begin{bmatrix} [Z_{n+1}, F] \\ \vdots \\ [Z_{2n}, F] \end{bmatrix}
\end{aligned}
\tag{9}
$$

The condition $[Z^1, F] = 0$ becomes identical with $Z^2 = 0$.

The unknown functions $\frac{\partial\psi}{\partial x_2}$ and ψ are determined from (9). As the results, we have

$$\begin{aligned}\frac{\partial\psi}{\partial x_2} &= 2\{\beta G(x_1)-\alpha\Phi'(\sigma)\}x_2+B\Omega x_1)\\ \psi = x_2^T\{\beta G(x_1)-\alpha\Phi'(\sigma)\}x_2 &+ \Omega^T(x_1)B^Tx_2+\alpha\sum_{i=1}^{m}\lambda_i\phi_i(\sigma_i)f_i(\sigma_i)\\ &= \dot\sigma^T\{diag[\beta g_i(\sigma_i)-\alpha\phi_i'(\sigma_i)]\}\dot\sigma+\Omega^T(x_1)\dot\sigma+\sum_{i=1}^{m}\lambda_i\phi_i(\sigma_i)f_i(\sigma_i)\end{aligned} \tag{10}$$

where $\Omega(x_1)$ is the arbitrary vector function. In (10), inequalities

$$\beta g_i(\sigma_i)-\alpha\phi_i'(\sigma_i)>0, \qquad \alpha\phi_i(\sigma_i)f_i(\sigma_i)>0 \tag{11}$$

must be satisfied. Then we choose $\Omega(x_1)$ as

$$\Omega(x_1)=\begin{bmatrix}-2\sqrt{K_1(\sigma_1)\alpha\lambda_1\phi_1(\sigma_1)f_1(\sigma_1)}\\ -2\sqrt{K_1(\sigma_1)\alpha\lambda_1\phi_1(\sigma_1)f_1(\sigma_1)}\\ \vdots\\ -2\sqrt{K_1(\sigma_1)\alpha\lambda_1\phi_1(\sigma_1)f_1(\sigma_1)}\end{bmatrix} \tag{12}$$

in order for ψ to be the sum of perfect square forms, such that

$$\psi=\sum_{i=1}^{m}\left[\sqrt{K_i(\sigma_i)}\dot\sigma_i-\sqrt{\alpha\lambda_i\phi_i(\sigma_i)f_i(\sigma_i)}\right]^2 \tag{13}$$

where, $K_i(\sigma_i)$ $(i=1-m)$ are given as $K_i\ (\sigma_i)=\beta g_i(\sigma_i)-\alpha\phi_i(\sigma_i)$. Solving (5) for P_1 and P_2, we obtain

$$\begin{aligned}P_1 &= \alpha G(x_1)D\{diag[\phi_i(\sigma_i)]\}1+\alpha\Phi'(\sigma)x_2+\beta Bf(\sigma)-B\Omega(x_1)\\ P_2 &= \alpha D\{diag[\phi_i(\sigma_i)]\}1+\beta x_2\end{aligned} \tag{14}$$

Choosing $\alpha=1$ and $\beta=1$, the scalar function V, where

$$V(x)=\int_0^x P^T dx, \tag{15}$$

is given as

$$\begin{aligned} V &= \frac{1}{2}\{\boldsymbol{x_2} + \boldsymbol{D}\{diag[\phi_i(\sigma_i)]\}\boldsymbol{1}\}^T\{\boldsymbol{x_2} + \boldsymbol{D}\{diag[\phi_i(\sigma_i)]\}\boldsymbol{1}\} \\ &+ \sum_{i=1}^{m}\int_0^{\sigma_i}\boldsymbol{K_i(\sigma_i)}\lambda_i\phi_i(\sigma_i)d\sigma_i + \int_0^{\sigma}\boldsymbol{f^T(\sigma)d\sigma} \\ &+ 2\sum_{i=1}^{m}\int_0^{\sigma_i}\sqrt{\boldsymbol{K_i(\sigma_i)}\lambda_i\phi_i(\sigma_i)\boldsymbol{f_i(\sigma_i)}}d\sigma_i \end{aligned} \tag{16}$$

The time derivative of V is of the form

$$\dot{V} = -\sum_{i=1}^{m}\left[\sqrt{\boldsymbol{K_i(\sigma_i)}}\dot{\sigma}_i - \sqrt{\lambda_i\phi_i(\sigma_i)\boldsymbol{f_i(\sigma_i)}}\right]^2 \tag{17}$$

Now, the conditions given in (11) result in

$$\begin{gathered} \sigma_i\phi_i(\sigma_i) > 0 \qquad (\sigma_i \neq 0) \\ \boldsymbol{K_i}(\sigma_i) = g_i(\sigma_i) - \frac{d\phi_i(\sigma_i)}{d\sigma_i} \geq 0 \end{gathered} \tag{18}$$

We can easily see that $\dot{V}$ in (17) satisfies Lyapunov's criteria ;$\dot{V}(\boldsymbol{x}) \leq \boldsymbol{0}$ for $\boldsymbol{x}$ $\neq \boldsymbol{0}$, and $\dot{\boldsymbol{V}}(\boldsymbol{x})$ is not identically equal to zero along any trajectory of the system other than the origin.
Next we inspect the definiteness of $\boldsymbol{V}$ in a region around the origin. Let us rewrite the right-hand side of (**16**) except the first term, such that

$$\boldsymbol{V_0} = \sum_{i=1}^{m}\int_0^{\sigma_i}\boldsymbol{H_i(\sigma_i)d\sigma_i} \tag{19}$$

where

$$\boldsymbol{H_i(\sigma_i)} = \boldsymbol{K_i(\sigma_i)\lambda_i\phi_i(\sigma_i) + f_i(\sigma_i) + \sqrt{K_i(\sigma_i)\lambda_i\phi_i(\sigma_i)f_i(\sigma_i)}} \tag{20}$$

As $\boldsymbol{H_i(i = 1 - m)}$ in (**20**) are arranged in the forms

$$\boldsymbol{H_i(\sigma_i)} = \begin{cases} [\sqrt{\boldsymbol{K_i(\sigma_i)\lambda_i\phi_i(\sigma_i)}} + \sqrt{\boldsymbol{f_i(\sigma_i)}}]^2; & \boldsymbol{\sigma_i \geq 0} \\ -[\sqrt{-\boldsymbol{K_i(\sigma_i)\lambda_i\phi_i(\sigma_i)}} - \sqrt{-\boldsymbol{f_i(\sigma_i)}}]^2; & \boldsymbol{\sigma_i < 0} \end{cases} \tag{21}$$

we have

$$\sigma_i H_i(\sigma_i) \geq 0 \tag{22}$$

Hence, V_0 is verified to be a positive function. Thus, the scalar function V in (16) satisfies Lyapunov's criteria:-$V(x)$ is a continuous scalar function which has continuous first partial derivatives with respect to x, $V(x)=0$ for $x=0$, and $V(x)>0$ for $x \neq 0$ and becomes the Lyapunov function of the system (1).
If we choose

$$\phi_i(\sigma_i) = \alpha' \int_0^{\sigma_i} g_i(\sigma_i) d\sigma_i \qquad (0 \leq \alpha' \leq 1) \tag{23}$$

(16) is equivalent to that obtained in [7]. The Lyapunov function given in (16) is regarded as a generalized Lyapunov function for the system (1) satisfying $D = B$ and $D^T B = diag[\lambda_i]$, (λ_i: positive constants).

3.2 Example

Let us consider a Liénard-type nonlinear system [7].

$$\ddot{y}_1 + \{g_1(\sigma_1) + g_2(\sigma_2)\}\dot{y}_1 + \{g_1(\sigma_1) - g_2(\sigma_2)\}\dot{y}_2 + f_1(\sigma_1) - f_2(\sigma_2) = 0$$
$$\ddot{y}_2 + \{g_1(\sigma_1) - g_2(\sigma_2)\}\dot{y}_1 + \{g_1(\sigma_1) + g_2(\sigma_2)\}\dot{y}_2 + f_1(\sigma_1) + f_2(\sigma_2) = 0$$

where $\sigma_1 = y_1 + y_2, \quad \sigma_2 = y_2 - y_1$
We can rewrite the equation (24) in the form of (2) as

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} g_1(\sigma_1) & 0 \\ 0 & g_2(\sigma_2) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} f_1(\sigma_1) \\ f_2(\sigma_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{25}$$

where

$$D = B = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \qquad D^T B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \tag{26}$$

For the system (25), the generalized Lyapunov function (16) becomes

$$\begin{aligned}
V &= \frac{1}{2}[x_{21}+\phi_1(\sigma_1)-\phi_2(\sigma_2)]^2+\frac{1}{2}[x_{22}+\phi_1(\sigma_1)+\phi_2(\sigma_2)]^2\\
&+ 2\int_0^{\sigma_1} K_1(\sigma_1)\phi_1(\sigma_1)d\sigma_1+2\int_0^{\sigma_2} K_2(\sigma_2)\phi_2(\sigma_2)d\sigma_2\\
&+ \int_0^{\sigma_1} f_1(\sigma_1)d\sigma_1+\int_0^{\sigma_2} f_2(\sigma_2)d\sigma_2\\
&+ 2\int_0^{\sigma_1}\sqrt{2K_1(\sigma_1)\phi_1(\sigma_1)f_1(\sigma_1)}d\sigma_1\\
&+ 2\int_0^{\sigma_2}\sqrt{2K_2(\sigma_2)\phi_2(\sigma_2)f_2(\sigma_2)}d\sigma_2
\end{aligned} \tag{27}$$

with

$$\begin{aligned}
\dot{V} &= -[\sqrt{K_1(\sigma_1)}\dot{\sigma_1}-\sqrt{2\phi_1(\sigma_1)f_1(\sigma_1)}]^2\\
&+ [\sqrt{K_2(\sigma_2)}\dot{\sigma_2}-\sqrt{2\phi_2(\sigma_2)f_2(\sigma_2)}]^2
\end{aligned} \tag{28}$$

4 Conclusions

This paper has given the generalized Lyapunov function for the Liénard-type nonlinear system which is important as the general system expressing LRC electric circuits and spring systems etc.. The Lagrange-Charpit method which is a well known technique for solving partial differential equations was applied to construct the Lyapunov function. The proposed function includes arbitrary functions, by which the Luré-type Lyapunov function is extended. The result yields all conventional Lyapunov functions as special cases, selecting the arbitrary functions appropriately.

References

[1] Y. N. Yu and K. Vongsuria: "Nonlinear Power System Stability Study by Lyapunov Function and Zubov's Method", IEEE Trans. Power Apparatus and Systems, Vol. PAS-86, No.12, 1967, pp.1480–1484.

[2] A. K. DeSarkar and N. D. Rao: "Zubov's Method and Transient Stability Problem of Power Systems", Proceedings IEE, Vol.118, No.8, 1971, pp.1035–1040.

[3] F. S. Prabhakara, A. H. El-Abiad and A.J. Koivo: "Application of Generalized Zubov's Method to Power System Stability", International Journal of Control, Vol.20, No.2, 1974,pp.203–212.

[4] H. Miyagi and T. Taniguchi: "Application of the Lagrange-Charpit Method to Analyse the Power System's Stability", International Journal of Control, Vol.32, No.2, 1980, pp.371–379.

[5] S. Kawamoto, A. Ishigame, T. Imoto and T. Taniguchi: "Construction of Improved Lyapunov Function for Power System", Trans. IEE of Japan, Vol.109, No.3/4, 1989, pp.38–39.

[6] H. Miyagi, N. Miyagi and T. Ohshiro: "A Study of Constructing Lyapunov Function for Liénard-Type Non-Linear Systems", Trans. Society of Instrument and Control Engineers, Vol.24, No.4, 1981, pp.344–349.

[7] N. Miyagi and H. Miyagi: "A Study on the Stability of Liénard-Type Non-Linear Systems", Trans. JSME, Vol.56-C, No.529, 1990, pp.2427–2431.

[8] H. Miyagi and K. Yamashita: "Robust Stability of Luré Systems with Multiple Nonlinearities", IEEE Trans. on Automatic Control, Vol. 37, No.6, 1992, pp.883–886.

[9] H. Miyagi, H. Sakihara and K. Yamashita: "Stability Analysis of Fuzzy-Type Nonlinear Systems Using the Method of Lyapunov", Proc. of ITC–CSCC '97, Okinawa, Japan, pp.449–502.

[10] J. L. Munda and H. Miyagi: "Lyapunov Function for an Induction Generator - Infinite Bus Power System with Transmission Losses", Proc. of 3rd Mathmod Symposium, Vienna, Feb. 2000, pp.169–173.

[11] H. Miyagi, J. L. Munda and N. Miyagi: "Study on Lyapunov Functions for Liénard-Type Non-Linear Systems", Submitted to the Trans. of IEEJ, April 2000.

LJAPUNOVLJEVE FUNKCIJE ZA NELINEARNE SISTEME LIENARD-OVOG TIPA

Hayao Miyagi, Josiah Munda

Lagrange-Charpit-ov metod je primenjen za konstruisanje generalisanih Ljapunovljevih funkcija za nelinearne listeme Lienard-ovog tipa što je značajno kao reprezentativni sistem koji izražava opšta RLC električna kola i mreže i sisteme mehaničkih opruga itd. Funkcija uključuje partikularne nonlinearne članove kao proizvoljne funkcije pomoću kojih se može proširiti kvadratni član u Ljapunovljevoj funkciji Lure-ovog tipa. Menjanjem oblika proizvoljnih funkcija rezultat daje sve konvencionalne Ljapunovljeve funkcije kao specijalne slučajeve.