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SOME ASPECTS OF THE INTERDEPENDENCE BETWEEN PRODUCTION FUNCTION AND PRODUCTIVITY FUNCTION

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Abstract. *Determination and projection of certain functional relations in the form of production functions between the product, as one of the emergent form of results and the production factors, also in a certain form, is a very important area in complex economic analysis. Also, there are some significant relations of functional dependence between the product, particular production factors and productivity, expressed by productivity functions. The paper deals with the economic and methodological aspects of production functions and productivity function, expressed as one-variable function of living labour as well as some basic aspects of interdependence between these functions.*

1. INTRODUCTION

A general aspiration in performing any economic activity both at the level of an enterprise and economy at large is to achieve a higher business success. If, in addition, we have in mind that actual business success is a quantitative expression of economic success in realizing the desired goals, then we have to determine and express it. Proceeding from the premise that economic goals in performing the economic activity are realized by actual results, on the one hand, and that is necessary to make certain investments in reproduction to realize the goals, on the other hand, it follows that the quantitative expression of business success is a relation of the results and the investments made. In that context, certain relations between the product, as one of the emergent forms of results and certain forms of the production factors have a special significance in economic analysis.

Taking into consideration the importance and role of the product for business success of economic entities, as well as for standard of living and for overall economic development, we can set certain functional relations between the product and the

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production factors, determined and projected by certain forms of *production functions.* Simultaneously, certain relationships that express functional dependence between the product, particular production factors and the productivity have the same importance. These relations are here labeled as *productivity functions.*

In general, the paper deals with some aspects of the interdependence between the productivity and the production functions, since these categories are very complex. In Chapter 1 we discuss some questions referring to the elements and determinants (characteristics) of production functions. The elements and determinants (characteristics) of productivity function is the subject of Chapter 2. Finally, in Chapter 3 we present the basic aspects of the interdependence between production functions and productivity function.

Since there are significant differences in the concept and expressions of production function of one variable (one-variable) and production function of several variables, as well as of productivity function and taking into consideration the scope of the paper, we will on this occasion analyze the determination of production function and productivity function as the function of living labour. The relationships of interdependence between the productivity function and the production function of several variables and their determination will be the subject of our another paper.

1. ELEMENTS AND CHARACTERISTICS OF PRODUCTION FUNCTIONS

1.1. Economic aspects of production functions

Proceeding from the above statement that business success is the quantitative expression of economic success in realizing the economic goals of an enterprise, it is necessary to provide an insight into the form and dimensions of that expression. In that context, it is necessary to examine the flow of function of relevant forms of results (product, revenue, gain or profit) and relevant forms of investments (consumption, cost, committed resources or capital). Since the product is the first emerged form of result, the production function (function of production) expresses the form and character of the dependence between the product and the production factors. The revenue function expresses how total revenue depends on demand and price ($\mathrm{C} = \mathrm{F}(q,r) = qr$, where C is the total revenue, q is the demand and r is the price of product). Finally, the function of gain expresses the influence of revenue and cost on gain ($\mathrm{D} = G(\mathrm{C},\mathrm{T}) = \mathrm{C} - \mathrm{T}$, where D is the gain, and T is the cost). The subject of our discussion is the production functions.

The economic essence of production functions, in accordance with the above mentioned, lies in productive capacity and efficiency of production factors. If the capacity and efficiency are related to several production factors, then we discuss a production function of several variables, or, specially, we could analyze a function that is related to only one production factor, called the one-variable production function. The analytic expression of these functions will be further discussed in the Chapter 1.2, with emphasis on only some of their forms.

Namely, since the production function is based on the assumption of infinitesimal division and fixed substitution of the product and the production factors, such an analysis is justified at the level of the economy. This type of production functions is called the type "A" production function. However, for the purpose of determining the form and character of this functional dependence at the level of different industries or even in complex organizational systems, the Walras-Leontief production function, based on previously determined technical coefficients of interdependence between the flows of product and production factors, seems to be more applicable.

A special form of this production function, denoted as the type "B" production function or the Gutenberg function, expresses a form of functional dependence between the product and the production factors, taking into consideration certain limitations that are related to the integers of the quantities of product and production factors, to the technical characteristics of the aggregates (machines, equipment) that determine the quantities of factors and as well as to the influence of intensity and characteristics of production factors on the scope and character of their expanding and, so, on the flow of production function [3, p. 218]. The adequate projection of the flows and forms of interdependence at the level of technical aggregates (zones of scope) and enterprise as a whole provides an insight into particular specific characteristics and quantitative interdependence between the product and the production factors at that level. This is very important for finding reasons and factors of the current situation and for a successful management of the future economic development of an enterprise.

1.2. Methodological aspects of production functions

Let Q be a homogeneous product that in a production process represents the final results and let q denote the quantity of the product. Let, then, $U_1, U_2, \ldots, U_n$ be the factors invested in the production of a product Q and $u_1, u_2, \ldots u_n$, the investment quantities of the factors. Let us assume that the quality of the investment quantity of the factor U_i , $i = 1,2, \ldots, n$, is fixed and that there are no technical changes in the process of producing a product Q. The function

$$q = F(u_1, u_2, \ldots, u_n), \tag{1.1}$$

that is, its special form

$$q = \alpha_0 u_1^{\alpha_1} u_2^{\alpha_2} \ldots u_n^{\alpha_n} , \tag{1.2}$$

where α_i, $i = 0,1,2, \ldots, n$, are the constant parameters, is called the production function if:

a) the function is defined for all combinations of the investment quantities $u_1, u_2, \ldots u_n$, that give nonnegative values for the quantity q of a product Q,

b) $F(u_1, u_2, \ldots, u_n) = 0$ if only one investment quantity $u_i = 0$ (every invested factor U_i is necessary in the production of a homogeneous product Q),

c) $F(u_1, u_2, \ldots, u_n) > 0$ if $u_i > 0$, $i = 1,2, \ldots, n$ (the production of a product Q is impracticable without investing every production factor U_i),

d) the function F is continuous in the domain of definition and has the first and the second infinite derivatives [4, p. 26] and [7, p. 71].

The function (1.1) is, also, called *the type "A" production function* [5, p. 23].

Now, let us assume that in the production of a quantity q of a product Q, the investment quantities u_i, $i = 1,2, \ldots, m$, are variable and the quantities u_j, $j = m+1, m+2, \ldots, n$, are constant ($u_j = \lambda_j$). In that case, the production function (1.1) can be rewritten as the function of m variables, i.e.

$$q = F_1(u_1, u_2, \ldots, u_m, \lambda_{m+1}, \ldots, \lambda_n), \tag{1.3}$$

where λ_j are the fixed investment quantities of the factor U_j, which in this function serve as parameters. Hence, the production function simple can be written as

$$q = F_1(u_1,u_2, \dots ,u_m). \tag{1.3'}$$

Specially, if only the investment of labour force U is variable, i.e. the investment quantity u of labour force is variable and the investment quantities of the remaining factors are fixed (at a certain level), then the production function becomes one-variable function

$$q = f(u). \tag{1.4}$$

We have to notice that the function (1.4) is bijective.

In practice, certain discrete dependence between the investment quantities and the quantity of a product is, very often, known. Let A be the set of discrete values (the empirical set):

$$A = \{(u_i,q_i) \mid u_i,q_i \in R^+, i = 1,2, \dots ,n \}, \tag{1.5}$$

where R^+ denotes the set of positive real numbers, u_i denotes the investment quantities of labour force and q_i denotes the scope of a product Q in given production conditions.

Smoothing out the empirical curve obtained by linking the points M_i in a plane with the number pairs (u_i,q_i) , $i = 1,2, \dots ,n$ as the coordinates can be done by *linear* ($q = au+b$), *quadratic* ($q = au^2 + bu + c$), *exponential* ($q = ba^u$), *power* ($q = bu^a$) *function* [1, p. 196-198]. In that way, we can obtain the functional form of dependence (1.4).

Starting from the definition of elasticity for discontinuous form of dependence [1, p. 213], we can find the coefficient of elasticity:

$$E_{q,u} = \frac{\Delta q_i}{q_i} : \frac{\Delta u_i}{u_i} = \frac{u_i(q_{i+1} - q_i)}{q_i(u_{i+1} - u_i)}\ . \tag{1.6}$$

Starting from the definition of elasticity for functional form of dependence [1, p. 214] and formula (1.4), the function of production elasticity can be expressed as

$$E_{q,i} = \frac{u}{q} q' = \frac{u}{f(u)} f'(u)\ . \tag{1.7}$$

2. ELEMENTS AND DETERMINANTS OF PRODUCTIVITY FUNCTION

2.1. Economic aspects of productivity

The problem of productivity is of primary importance in economic theory and practice. Namely, the level and dynamics of productivity influence the level and dynamics of economic development which, in its turn, affects the level and dynamics of the standard of living in a country.

There is, however, a significant difference in the concept of productivity both in theory and in practice. In some attitudes, the productivity means a production capacity of production elements, denoted as production factors. This is, primarily, relevant for human living labour, so the productivity (or productivity of labour) is, mostly, related to a capacity of producing the labour [6, p. 15]. Since the labour is one production factor, the productivity determined by such a way is treated as the productivity of one production factor (in this case: of human labour). The basic quantitative expression of the productivity is $p = q/u$, where p is the productivity, q is the physical scope of a product Q and u is the investment quantity of labour force (labour) U.

Another concept of productivity, present both in the economic theory and analysis, is based on the attitude that, besides the productivity of labour, there is a productivity of other elements, i.e. the production factors denoted as instruments and subjects of labour, as well as the land, as a another natural condition for the production process. Translating this into certain expressions of investments, we could discuss the productivity of labour ($p_u = q/u$) and the productivity of capital ($p_k = q/k$, where k is the quantity of a capital K), which is called the production coefficient. Since the production process requires the investments of both labour and capital (that is, labour, instruments and subjects of labour), we can discuss the concept of integral (total) productivity of all production factors.

Since the capability and efficiency of production are influenced by several groups of different factors, we could discuss the productivity of the factors. It, for instance, includes all organizational or disposition factors of the production process, certain social (economic-systemic and market) factors and the others. However, these factors are nonproductive ones and their material foundation is projected and based on the production process, so the factors do not result in a product or a production capacity.

The above-mentioned economic essence of productivity (the capability to produce one or all production factors) determines the basis of productivity as an economic principle of reproduction as well as a criterion (measure) of economic efficiency. Namely, from the standpoint of the reproduction principles and in accordance with an objective influence of the economic laws in a market economy, there is a principle to strive toward producing more product at lower investment quantities of living labour (a) or of all production factors (b). Since the essence and sense of basic economic principle in reproduction process are on aspiration toward achieving as higher result as lower investment, the principle of productivity concretizes the basic economic principle just on the basis and through the elements of result in the form of product and the elements of investment in the form of the investment of living labour or of all production factors [2, p. 193].

The achieved productivity is the criterion (measure) of economic quality (business success) in proportion to its determining elements. In that sense, the indicator of achieved productivity of living labour and its dynamics is one of the basic criteria of macroeconomic quality. In fact, the productivity of living labour is the most important indicator that expresses the capability and efficiency of producing living labour, as well as the level and dynamics of living standard of employees and citizens in a macroeconomy. However, from the standpoint of an enterprise, as an organizational form of mezoeconomy, the achieved productivity of living labour is a measure of its contribution to the macroeconomy (on the basis of productivity) and a measure of contribution to the quality (business success) of the enterprise, along with the economy and the profitability at that level.

In accordance with the previous statements, it is necessary to recognize and differentiate two concepts and expressions of productivity. As it was said in the introduction, in the paper we emphasize the concept of productivity of one factor (living labour). The determinants and quantification of productivity of several production factors will be the subject of our next paper.

2.2. Methodological aspects of productivity function

If the productivity, realized in the production of a product Q, is denoted by *p,* then it is a function of the quantity q of a product Q and the investment quantity u of labour force U, invested in the production of a product Q, i.e. it is a function of two variables

$$p = G(q,u) = \frac{q}{u} . \tag{2.1}$$

If q in (2.1) is replaced from formula (1.4), then the productivity p is expressed only as a function of the investment quantity u of labour force, i.e. the productivity function can be expressed as a function of one variable

$$p = \frac{f(u)}{u} = g(u) . \tag{2.2}$$

Let the segment $[0,b]$, $b > 0$, be the domain of the definition of the production function $f(u)$ and $f(0) = f(b) = 0$. Then the productivity function (2.2) is defined in the interval $u \in (0,b]$.

If we assume that the function (1.4) is differentiable in the segment $[0,b]$, i.e. its derivative is $f'(u)$, then, on the basis of the formula (2.2), there is a derivative function of the productivity function in the interval $(0,b]$, expressed as

$$p' = \frac{f'(u)u - f(u)}{u^2} = \frac{1}{u}\left(f'(u) - \frac{f(u)}{u} \right) = g'(u) . \tag{2.3}$$

The function (2.3) is called *marginal productivity* and $f'(u)$ expresses *marginal production*, when $f(u)/u$ is *average production.*

It is supposed that the production function (1.4) in the segment $[0,b]$ has infinite second derivative $f''(u)$. From (2.3) we can obtain the second derivative of the function (2.2) in the interval $(0,b]$:

$$p'' = \frac{f''(u)}{u} - \frac{2}{u^2}\left(f'(u) - \frac{f(u)}{u} \right) = g''(u) . \tag{2.4}$$

Differentiating the relation (2.4), we can obtain the third derivative of the productivity function (2.2):

$$p''' = \frac{f'''(u)}{u} - \frac{3}{u}\left(\frac{f''(u)}{u} - \frac{2}{u^2}\left(f'(u) - \frac{f(u)}{u} \right) \right) = g'''(u) . \tag{2.5}$$

A special task here is to examine when the productivity function has a maximum, i.e. to determine the necessary and sufficient condition for a maximum of the productivity function (2.2). A necessary condition for the maximum of the function is that marginal productivity is zero or, taking the formula (2.3) into consideration, it can be expressed as

$$g'(u) = 0, \quad \text{i.e.} \quad f'(u) = \frac{f(u)}{u}. \tag{2.6}$$

Let $u_0 \in (0,b)$ be a solution of the equation (2.6). The relation (2.6) shows that for the investment quantity u_0 of labour force the marginal product $f'(u_0)$ is equal to the average product $f(u_0)/u_0$.

A sufficient condition for the maximum of the productivity function (2.2) is that its second derivative at the point u_0 is negative, or, taking the formula (2.4) into consideration, it can be expressed as

$$g''(u) < 0 \quad \text{i.e.} \quad \frac{f''(u_0)}{u_0} < 0 \quad \text{i.e.} \quad \frac{1}{u_0}(f'(u_0))' < 0. \tag{2.7}$$

The last inequality in (2.7) means, in fact, that u_0 is in the interval $(u_1,u_2) \subset (0,b)$, where the function of marginal product $f'(u)$ becomes a decreasing function. The maximum value of productivity can be obtained by substitution of u_0 in (2.2), i.e.

$$p_{\max} = \frac{f(u_0)}{u_0} = g(u_0).$$

The value of u_0 is called *the optimal investment quantity of labour force* and $p_{\max}$ is *the optimal productivity*. If the optimal investment quantity u_0 of labour force is substituted in (1.4), we obtain *the optimal quantity* $q_0 = f(u_0)$ of a product Q. The point $M(u_0,g(u_0))$ is the maximum point in the graph of the productivity function (2.2).

For constructing a precise graph of the productivity function (2.2) it is necessary to examine convexity and points of inflection of the function. The graph of the productivity function (2.2) has two points of inflection $P_1(u_1,g(u_1))$ and $P_2(u_2,g(u_2))$, if $u_1, u_2 \in (0,b)$, $u_1 < u_2$ are the zeros of the second derivative, i.e. taking (2.4) into consideration that they are the solutions of the equation

$$u^2 f''(u) - 2uf'(u) + 2f(u) = 0,$$

and, also, the third derivative of this function at the points is not equal to zero, i.e. taking (2.5) into consideration, that

$$\frac{f'''(u_1)}{u_1} \neq 0 \quad \text{and} \quad \frac{f'''(u_2)}{u_2} \neq 0.$$

In the interval $u \in (u_1,u_2)$ the productivity function $g(u)$ is convex up (concave) if $g''(u) < 0$, i.e. taking (2.4) and (2.3) into consideration, if

$$\frac{f''(u)}{u} - \frac{2}{u^2}\left(f'(u) - \frac{f(u)}{u}\right) < 0 \quad \text{i.e.} \quad \frac{f''(u)}{u} - \frac{2}{u}g'(u) < 0. \tag{2.8}$$

Since, on the basis of the relation (2.6), $g'(u_0) = 0$ and, on the basis of (2.7), $f''(u_0) < 0$, where $u_0 > 0$ is the optimal investment quantity of labour force, whence u_0 is the solution of the inequality (2.8), i.e. $g''(u_0) < 0$, so $u_0 \in (u_1,u_2)$.

For $u \in (0,u_1) \cup (u_2,b)$ the productivity function $g(u)$ is convex if $g''(u) > 0$. Therefore, at the point of inflection P_1 the graph of the productivity function $g(u)$ from convexity goes to concavity, and at the point of inflection P_2 from concavity to convexity.

The productivity function $g(u)$ is not defined for $u = 0$. However, the right marginal value at that point is

$$\lim_{u \to 0^+} g(u) = \lim_{u \to 0^+} \frac{f(u)}{u} = \left[\frac{0}{0}\right],$$

then, in accordance to the L'Hospital theorem, we obtain

$$\lim_{u \to 0^+} g(u) = \lim_{u \to 0^+} f'(u) = 0,$$

since it has been supposed that marginal product $f'(u)$ for $u = 0$ is zero [4, p. 34].

Starting from certain determinants of the productivity function (2.2), resulting from the previous analysis, the graph of this function is shown in Fig. 1.

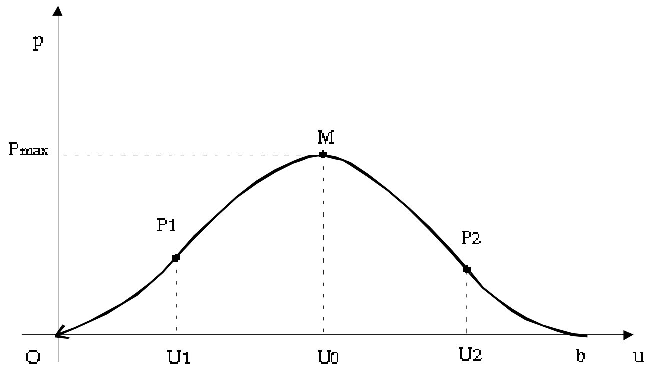

Fig. 1. The part of the graph of the function that is relevant for the economic analysis is in the interval $(0,u_0]$.

3. Interdependence between Production Function and Function of Productivity of Labour

The analysis of elements and determinants of production functions and productivity function has shown certain interdependence between them. The elements and character of the interdependence can be discussed by using certain direct relations that are valid both for a discrete set of values (*a*) and for comparing the production function and the productivity function (*b*). It is necessary, as we've mentioned above, to consider the elements and character of these functions.

(*a*) Since the productivity of labour is equal to a relation between the quantity q of a product Q and the investment quantity u of labour force U, then some aspects of the interdependence can be discussed direct by using elements of this relation. Namely, from the expression $p = q/u$ it follows $q = up$, i.e. the quantity q of a product Q can be obtained by multiplying the investment quantity u of labour force U and the level of productivity p. The expression $q = up$ is related to the elements of productivity complex in a certain period, i.e. to their real values in that period. The expression, as we can see, confirms the above mentioned attitude of the economic essence of productivity, as the expression of the investment quantity of labour force and its productive capability. Statements relating to a character of the interdependence are obvious - the volume of product is proportional to the investment quantities and the level of productivity, while the investment quantities for a certain volume of product is in inverse proportion to the level of productivity ($u = q/p$).

When we analyze the discrete set given by the relation (1.5), or, more precisely, when we determine a rate of change of productivity for two consecutive (comparative) periods, then the pointed relations of this interdependence and conditioning can be monitored considerably better.

The productivity of period i is given by the formula:

$$p_i = \frac{q_i}{u_i}, \quad i = 1,2,\dots n, \tag{3.1}$$

and the productivity of period $i + 1$ is expressed as:

$$p_{i+1} = \frac{q_{i+1}}{u_{i+1}} = \frac{q_i \pm \Delta q_i}{u_i \pm \Delta u_i} = \frac{q_i\left(1 \pm \dfrac{\Delta q_i}{q_i}\right)}{u_i\left(1 \pm \dfrac{\Delta u_i}{u_i}\right)}, \tag{3.2}$$

where $\Delta q_i = q_{i+1} - q_i$, $\Delta u_i = u_{i+1} - u_i$. If *the relative change in a product volume (investment quantity of labour force)* is labelled as K_{q_i} (K_{u_i}), i.e.

$$K_{q_i} = \frac{\Delta q_i}{q_i}\left(K_{u_i} = \frac{\Delta u_i}{u_i}\right), \tag{3.3}$$

then, from the relation (3.2) and (3.3), we obtain

$$p_{i+1} = \frac{q_i(1 \pm K_{q_i})}{u_i(1 \pm K_{u_i})}, \tag{3.4}$$

where K_{q_i} is called *the coefficient of objective change in a product volume,* and K_{u_i} – *the coefficient of objective change in an investment quantity of labour force.*

The relative change in productivity $\Delta p_i / p_i$ is called the *coefficient (rate) of change in productivity* and is denoted K_{p_i}. Then, on the basis of (3.1) and (3.4), we obtain

$$K_{p_i} = \frac{\Delta p_i}{p_i} = \frac{p_{i+1} - p_i}{p_i} = \frac{p_{i+1}}{p_i} - 1 = \frac{1 \pm K_{q_i}}{1 \pm K_{u_i}} \quad i = 1,2,\dots,n. \tag{3.5}$$

From the first part of the relation (3.3) and the relation (3.2) we obtain

$$K_{q_i} = \frac{q_{i+1} - q_i}{q_i} = p_{i+1}\frac{u_{i+1}}{q_i} - 1,$$

and, then, from (3.4), (3.5) and the second part of the relation (3.3), it follows:

$$K_{q_i} = (1 \pm K_{u_i})(1 + K_{p_i}) - 1 \quad (i = 1,2,\dots,n). \tag{3.6}$$

In an analogous way, from the second part of the relation (3.3) and the relation (3.2), we obtain

$$K_{u_i} = \frac{u_{i+1} - u_i}{u_i} = \frac{1}{p_{i+1}}\frac{q_{i+1}}{u_i} - 1,$$

and, then, from (3.4), (3.5) and the first part of the relation (3.3), it follows:

$$K_{u_i} = \frac{1 \pm K_{q_i}}{1 + K_{p_i}} - 1 \quad (i = 1,2,\dots,n). \tag{3.7}$$

The formula (3.5), (3.6) and (3,7) express the interdependence between the rate of change in productivity and the coefficients of objective change in volume of product and objective change in investment quantities of labour force.

(*b*) Let the production function (1.4) be known for a product Q, i.e. $q = f(u)$ and let its domain of definition be the segment $[0,b]$, $b > 0$. If $u_0 \in (o,b)$ denotes the quantity of labour force U invested in production of a product Q, then $f(u_0) = q_0$ is the quantity (volume) of a product Q that is realized by the investment quantity u_0 of labour force, and the relative change in volume of product (the coefficient of objective change in volume of product) is:

$$K_{q_0} = \frac{\Delta q_0}{q_0} = \frac{f(u_0 + \Delta u_0) - f(u_0)}{q_0} = \frac{f(u_0 + \Delta u_0)}{f(u_0)} - 1, \tag{3.8}$$

where $\Delta u_0 = u_1 - u_0$, $u_1 \in (0,b)$, $u_1 > u_0$ is the increment (change) in the investment quantity of labour force u.

An inverse function of the production function (1.4) has the form

$$u = f^{-1}(q), \tag{3.9}$$

and it express how the investment quantity u of labour force U depends on the volume (quantity) q of a product Q. Let the domain of definition of the function (3.9) is the segment $[c,d] \subset R^+$ and let $q_0 \in (c,d)$ be certain quantity of a product Q, then the relative change in the investment quantity of labour force (the coefficient of objective change in the investment quantity of labour force) is obtained

$$K_{u_0} = \frac{\Delta u_0}{u_0} = \frac{f^{-1}(q_0 + \Delta q_0) - f^{-1}(q_0)}{u_0} = \frac{f^{-1}(q_0 + \Delta q_0)}{f(q_0)} - 1, \tag{3.10}$$

where $\Delta q_0 = q_1 - q_0$, $q_1 \in (c,d)$, $q_1 > q_0$, is the increment (change) in the quantity q of a product Q.

Starting from the productivity function (2.2) and the formula (3.8) and (3.10), the relative change in productivity (the rate of change in productivity) is obtained

$$K_{p_0} = \frac{\Delta p_0}{p_0} = \frac{1+K_{q_0}}{1+K_{u_0}} - 1\,, \tag{3.11}$$

where $p_o = f(u_0)/u_0$, is the productivity realized in the production of a product Q with the investment quantity $u_0 \in (0,b)$ of labour force U.

Starting from the definition of elasticity for the functional form of dependence, we obtain the elasticity of the productivity function (2.2)

$$E_{p,u} = \frac{u}{p}p' = \frac{u}{g(u)}g'(u). \tag{3.12}$$

Theorem 1: *If $E_{p,u}$ is the elasticity of the productivity function, and $E_{q,u}$ is the elasticity of the production function, then*

$$E_{p,u} = E_{q,u} - 1\,. \tag{3.13}$$

Proof: If we replace the relation (2.2) and (2.3) in the formula (3.12), we obtain

$$E_{p,u} = \frac{u}{\frac{f(u)}{u}}\frac{f^{'}(u)u - f(u)}{u^2} = \frac{f^{'}(u)u - f(u)}{f(u)} = \frac{u}{f(u)}f^{'}(u) - 1,$$

and, then, from the formula (1.7), we obtain the formula (3.13).

On the basis of the formula (3.13) we can conclude:

1. If the production function is elastic, i.e. $E_{q,u} > 1$, then the productivity elasticity is positive, i.e. $E_{p,u} > 0$.
2. If the production function is indifferent elastic, i.e. $E_{q,u} = 1$, then the productivity function is perfect nonelastic $E_{p,u} = 0$.
3. If the production function is nonelastic, i.e. $E_{q,u} < 1$, then the productivity elasticity is negative, i.e. $E_{p,u} < 0$.

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NEKI ASPEKTI MEĐUZAVISNOSTI PROIZVODNE FUNKCIJE I FUNKCIJE PRODUKTIVNOSTI

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U složenim ekonomskim analizama utvrđuju se i projektuju određene funkcionalne relacije u obliku proizvodnih funkcija između proizvoda, kao jednog od pojavnih oblika rezultata i proizvodnih faktora, takođe u određenom pojavnom obliku. Istovremeno, od ne manje važnosti su i relacije funkcionalne zavisnosti između proizvoda, pojedinih proizvodnih faktora i produktivnosti, izražene funkcijama produktivnosti. U radu se analiziraju ekonomski i metodološki aspekti proizvodnih funkcija i funkcije produktivnosti, kao jednofaktorske funkcije živog rada, a, potom, osnovni aspekti međuzavisnosti ovih funkcija.