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Review paper

STATISTICAL INTERPRETATION AND APPLICATION OF CHAOTIC MODELS IN THE ANALYSIS OF FINANCIAL MARKETS DYNAMICS

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Abstract. *In this paper we will expose various aspects of statistical investigation and application of the nonlinear dynamical models of chaotic type. Firstly, using the formal-deductive method, we will present the definitions of some of the important classes of these models. Then we will describe the practical application of chaotic models in the investigations of dynamics in the Serbian financial market.*

Key Words: *models of dynamical chaos, parameters estimation, models application.*

1. INTRODUCTION

Non-linearity in the behavior of dynamical systems is usually characterized by their aperiodic, irregular and random trajectories. Therefore, the study of such systems, especially in the analysis of real data, associated with a number of difficulties that often prevent a more precise description of certain phenomena and the construction of appropriate theoretical models. For these reasons, chaos theory is an alternative model that can describe such a complex nonlinear dynamics in a relatively simple and easy manner. In econometric theory, this idea became particularly exposed in recent decades, when it comes to the formation of a large number of models of the type of chaos that provides a new perspective to contemporary research and mathematical modeling of empirical-stochastic effects.

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Research models of dynamical chaos in economic theory and practice are usually based on analysis of the behavior of financial series, and the practical application of the results of the theory of time series in order to describe their dynamics. In almost all models that describe the evolution of a particular financial series, it is being marked as (x_n), $n = 0,1,2,\ldots$, it is assumed about its stochastic nature. In other words, these sequences are usually interpreted as sequences of random variables that give increase to the so-called modeling *statistical uncertainty of the market*. Nonlinear stochastic systems are basic models that are then used in analyzing the behavior of prices and other financial market indexes.

However, the assumption of the stochastic nature of a financial sequence can be often replaced by an alternative "chaotic" model. In fact, there are well-known types of systems that generate deterministic sequences very similarly to the behavior of stochastic sequences. Although most researchers agree with the fact that there are not always clear indications of chaotic structure of financial data, the current trends in the financial engineering consider the dynamical models of the type of chaos indispensable in almost the whole capital research of the financial markets. They have proved to be particularly useful in modeling the evolution of financial indices in various crisis situations such as inflation, devaluation, stock market crashes, and the like.

In this paper we will give a formal definition of the nonlinear dynamical system of the type of chaos, and then describe some of the most important models of this type, which in practical studies play an increasingly important role. Finally, special emphasis was put on the application of chaotic models in the analysis of the dynamics of the Serbian financial markets, above all, in describing the dynamics of financial indexes as basic categories of economic business.

2. Theoretical Concept of Chaos

In general form, non-linear deterministic model is defined by the operator $T : R^p \to R^p$, i.e. by the recurrent sequence

$$\mathbf{x}_{n+1} = T(\mathbf{x}_n;\mathbf{a}),\ n = 0,1,2,... \tag{1}$$

where $\mathbf{x}_n = (x_n, x_{n\text{-}1}, \ldots, x_{n\text{-}p+1}) \in R^p$, p is dimension of operator T, as $\mathbf{a}$ is the parameter of mapping T and in the general case is unknown variable. In this way, the structure of T defines the basic properties of the corresponding dynamic model that describes it. If the sequence $(\mathbf{x}_n)$ converges, its limit

$$\mathbf{x}^* = \lim_{n\to\infty} \mathbf{x}_n \tag{2}$$

is uniquely determined by the equation $\mathbf{x}^*=T(\mathbf{x}^*,\mathbf{a})$. In mathematical terms, the value of $\mathbf{x}^*$ which satisfies the last equation is called *a fixed point* of T and its existence is a sufficient condition for convergence (2). That's what one of the well-known consequence of *Banach's fixed point theorem* which we now mention in a form that we will often use further on.

Theorem 1. Let $\mathbf{x}^*$ be the fixed point of continuous-differentiable operator T, where $\|T'(\mathbf{x}^*)\|<1$. Then, exists an open set $U \subseteq R^p$ which contains $\mathbf{x}^*$ that for every sequence $(\mathbf{x}_n) \subseteq U$ which satisfies the relation (1) the convergence (2) is valid.

On the other hand, the sequence $(\mathbf{x}_n)$ can have two or more points of accumulation. Limiting case is the existence of an infinite number of such points. Then, even the smallest changes of the starting value $\mathbf{x}_0$ of iterative procedure (1) cause significant changes in the values of other members of the sequence $(\mathbf{x}_n)$. Such arrays are in a class of chaotic sequences and they can only be formally defined on the basis of a number of "extra" terms which we are now describing in details, based on similar ideas as Berliner [1].

Definition 1. Let U be the open set in the Euclidian space R^p. Operator $T: U \to U$ is *sensitive to initial conditions* if exist $\delta>0$ such that for all $\mathbf{x} \in U$ exists $\mathbf{y} \in U$ and integer $n \geq 0$ for which is valid $\|T^n(\mathbf{x}) - T^n(\mathbf{y})\| > \delta$.

The previous definition implies, therefore, that the sensitive operators contain points that are "close", but that in subsequent realizations, applying the operator T, they have a significantly different trajectory. A typical situation is shown in Fig. 1, left panel, where it was taken as an example the one-dimensional operator $T(x)=1-|1-2x|$, defined in the interval $U=(0,1)$, and the initial values of points are $x=0.030$ and $y=0.031$. Suddenly, the condition of existence of such points does not mean that all values within the set U have this feature. However, most of the set of such "sensitive" points does not significantly differ from the set U, so we come to the next term.

Definition 2. The mapping $T: U \to U$ is *almost sure*[1] *expansive* if exists $\delta>0$ such that for almost all $\mathbf{x} \in U$ and almost all $\mathbf{y} \in U$ exists some integer $n \geq 0$ such that is valid $|T^n(\mathbf{x}) - T^n(\mathbf{y})| > \delta$.

In the previous definition the set U and a set of points that are "sensitive" to the initial conditions differ only up to a set of measure zero. It therefore means that relatively small changes in almost all initial values of iterative procedure (1) have a significant impact on the evolution of a sequence $(\mathbf{x}_n)$.

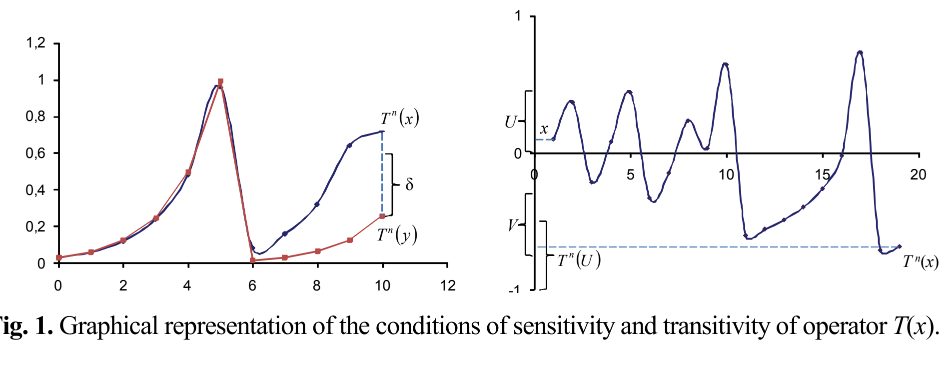

Fig. 1. Graphical representation of the conditions of sensitivity and transitivity of operator $T(x)$.

[1] Here we have the stochastic term "almost sure" that is well known from the elementary probability theory courses. The stochastic component of chaos will be discussed in further text.

Now, we will introduce another mathematical concept that further describes the specific trajectories of a chaotic model.

Definition 3. The mapping $T: R^p \to R^p$ is *topologically transitive* if for all the open sets $U, V \subseteq R^p$ exists some integer $n \geq 0$ such that is valid $T^n(U) \cap V \neq \emptyset$.

In the free interpretation, the previous definition implies a specific property of trajectories of dynamical systems that sooner or later "visit" each part of the area in which they are defined. In the Fig. 1, right panel, such a situation is presented, where the corresponding operator is $T(x)=1-2\sqrt{|x|}$. Using these concepts we can now, in the mathematical and formal sense, formulate the models of chaos.

Definition 4. The operator $T: R^p \to R^p$ generates *a model of dynamical chaos* if is valid:

(i) T is sensitive in relation to the initial conditions (in the sense of Definition 1);

(ii) T is topologically transitive (in the sense of Definition 3).

Let us point out that in addition to this there are a number of different ways and possibilities to present the formal definition of chaotic models. However, we believe that this formulation contributes primarily to better understanding of the essence of these models, as well as the strictly deductive way of testing their properties. Models of chaos, as well as some non-linear stochastic models, are considered good enough to describe the effects of various empirical behaviors of real financial sequences. One of the fundamental questions that often arise in financial analysis and that we are here and we deal with later, is:

Is the observed financial sequence (x_n) *stochastic or can be described above mentioned deterministic-chaotic system*?

This problem can be given a stochastic-statistical interpretation. For example, in terms of forecasting the future price trends, it is important to know how the extrapolation is made in the case of chaotic nonlinear model type. Trajectories of chaotic systems, as we shall see in the next section, can be very similar trajectories of empirical time series. However, modern trends based primarily on statistical interpretation of the dynamics of financial sequences, assume that some financial series, which are marked as $Z = (Z_n)$, defined in the form of the so-called *stochastic modifications*:

$$Z_n = X_n + \xi_n. \tag{3}$$

Here is $X = (X_n)$ a series of random variables that satisfy the recurrent relation (1), i.e. in its realization a numerical sequence (x_n) is provided. On the other hand, (ξ_n) is the so-called "white noise", i.e. the sequence of independent, same distributed random variables with pre-defined, known distribution. In this way, equality (3) represents additive decomposition of the empirical time series (Z_n) on "chaotic" component (X_n) and stochastic component (ξ_n). In practical applications, real empirical data is usually interpreted as a realization of these two sequences, thus creating the possibility of applying the appropriate mathematical and stochastic methods to study dynamics of financial sequences.

Especially, from the aspects of statisticians, it is interesting that the analysis of the recurrent relations (1) and (3), where, usually, the parameter $\mathbf{a}$, and initial the value $\mathbf{x}_0$, are considered as the unknown variables. Then, their estimation is based on the realized values of a sequence (x_n), by using well-known statistical procedures. However, even in some very simple situations, dynamical models of chaos can cause a lot of complicated and complex problems that are difficult to deal with using standard techniques.

3. SOME TYPES OF CHAOTIC MODELS

As an illustration, we will here analyze specific models of chaos that are frequently found in contemporary literature, but in practical research their importance lies primarily in the fact that they are commonly used as reference examples in the development and testing of new concepts and statistical methods in the theory of chaos.

3.1. Logistic model

First, let us consider the well-known logistic mapping, defined by a one-dimensional operator

$$T(x;a) = ax(1-x)\,. \tag{4}$$

Obviously, this operator generates a one-dimensional nonlinear dynamical system, i.e. a sequence

$$x_{n+1} = ax_n(1-x_n),\ n = 0,1,2,... \qquad 5)$$

By doing so, we will assume that $a \in [0,4]$ and $0 \le x_0 \le 1$, and in this way it is provided the condition that all values of x_n also belong to the unit interval. However, the behavior of a series (x_n) largely depends on the value of parameter a, so we will now consider some of the more typical cases that arise here:

1. For the values $0 \le a \le 1$ the sequence (x_n) converge at the zero for each starting value x_0 from interval [0,1], like we can see in Fig. 2, top left panel. We can notice that $x^* = 0$ is the fixed point of operator $T(x)$, i.e. one of the solutions of the equation $ax(1-x) = x$. On the other hand, the first derivative of the logistic mapping is $T'(x) = a(1-2x)$, from here we have $T'(0) = a \le 1$. In this way, according to the Theorem 1, we conclude that the sufficient condition is fulfilled for the above convergence.
2. Let us suppose now that the parameter value is $a \in (1,3)$. In this case, in addition to previously mentioned values x=0, the operator $T(x)$ has a fixed point $x^* = 1-a^{-1}$ for which is valid $|T'(x^*)| = |2-a| <1$. Therefore, the iterative process (5) now converges into this value for each $x \in (0,1)$. As an illustration, in Fig. 2, the top right panel, we showed the realization of this model in the case $a = 2$.
3. Finally, for the values $3 \le a \le 4$ logistics system generates sequences which, by increasing the value of the parameter a, have an increasing number of points of accumulation (Fig. 2, the bottom left panel). Such a "scenario" lasts until it reaches a famous Feigenbaum's value $a_\infty \approx 3.56994...$ Then the set of points of accumulation becomes infinite, uncountable set, and finally, for the $a \approx 4$ system goes into a state of "chaos". There are practically no points of accumulation and it is impossible to predict where they will, at some point in time n, find the values x_n (Fig. 2, bottom right panel).

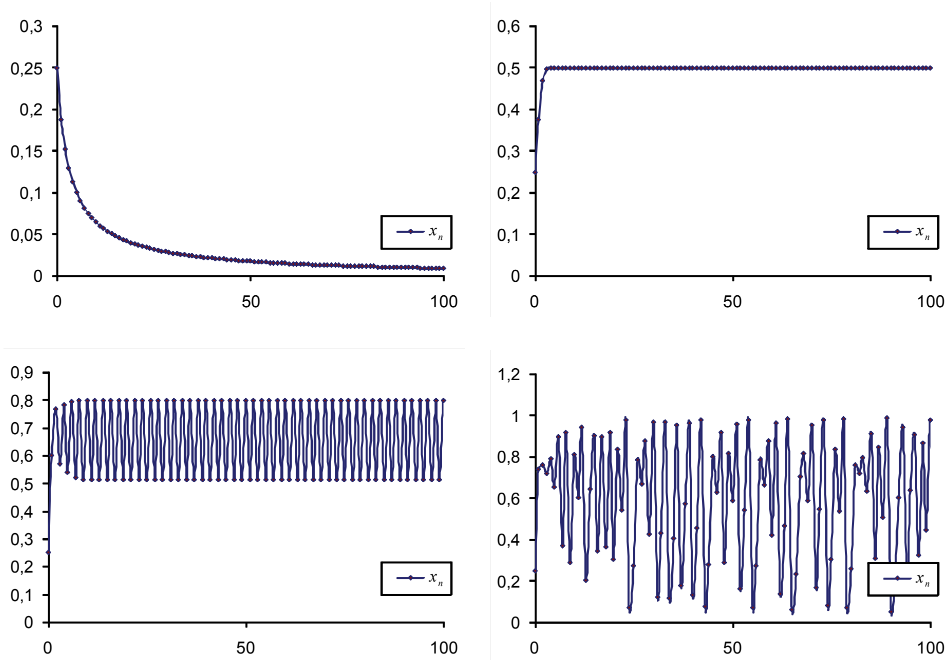

Fig. 2. Realization of the logistic model for different values of the parameter *a*.

The study of the correlation between random variables (x_n) generated with logistic mapping in the case $a \approx 4$ indicates that this series can be considered uncorrelated, i.e. interpreted as "white noise" of the chaotic type. This property is not accidental, since we have already mentioned, many chaotic systems have a probability distribution which is invariant relative to the operator T. Thus, for example, the logistics operator (4) with parameter $a = 4$ has the invariant distribution density function defined by

$$p(x) = \frac{1}{\pi\sqrt{x(1-x)}}, \quad x \in (0,1). \tag{6}$$

Therefore, if the initial value x_0 represents the realization of a random variable X_0 with the above distribution $p(x)$, then, because of invariance, the rest of the series X_n will also be random variables with the same probability distribution. The theoretical parameters of this series (mean and dispersion) are

$$E(X_n) = \frac{1}{2}, \quad D(X_n) = \frac{1}{8} \tag{7}$$

and they are very close to the obtained empirical results. Say, for example, that the initial value $x_0 = 0.1$ gives the empirical mean of 0.48887, while the empirical variance is 0.11843.

3.2. Baker's modified model

Chaos models described above belong to the group of models of the parabolic type, because their basic structure is described by the operator which has quadratic function. Such models have been thoroughly studied in the contemporary literature (see, for example, Tufillaro, Abbott and Reilly [9]). On the other hand, the systems in which the operator T(**x**;**a**) is not differentiable and is not a continuous function have a special treatment in research. One of them is a well-known model based on the so-called Baker's operator as a functional model. Here we consider a well-known modification of Baker's model introduced by Lopes, Lopes and Souza [5], where the one-dimensional operator $T(x;a) : [0, 1] \rightarrow [0, 1]$ is defined by the equality

$$T(x;a) = \begin{cases} \dfrac{x}{a}, & x \in [0, a) \\ \dfrac{a(1-x)}{1-a}, & x \in [a, 1) \end{cases}. \tag{8}$$

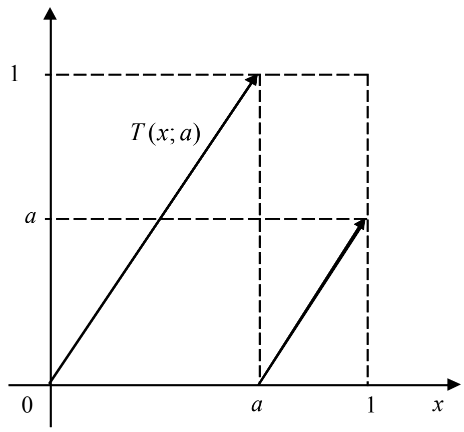

Fig. 3. The graph of Baker's operator.

We assume that $a \in (0,1)$ is an arbitrary constant, and operator $T(x;a)$, which depends on the independent variable x, is a part of linear function with a break at the point $x=a$, as shown in Fig. 3. This creates considerable possibilities of practical application of Baker's operator, or a series of (X_n) generated on the basis of this through iterative procedure (2). Specifically, parameter a, in the free interpretation, can be interpreted as a numerical indicator of intensity changes in the dynamical structure of the given financial series, for example, changes in price over a relatively short period of time. Then, a series (X_n) is an appropriate "chaotic" model of such a financial series that is often used in empirical analysis of time series. Similarly as in the case of the logistic model, the operator $T(x;a)$ also has invariant distribution which is here defined by density function

$$g(x;a)=\begin{cases}\dfrac{1}{a(2-a)}, & x\in[0,a)\\ \dfrac{1}{2-a}, & x\in[a,1).\end{cases} \tag{9}$$

Based on this, one can study the detailed structure of the correlation of stochastic modification of the model (3), which we discussed in the previous section. Firstly, by introducing assumptions about the Gaussian noise (ξ_n), which then presents a series of independent random variables with the same normal distribution, the stochastic parameters, we obtain

$$E(X_n)=0,\ D(X_n)=\sigma^2. \tag{10}$$

In this case, the correlation functions of series (X_n) is given by the expression

$$\rho(k)=\frac{C(k)-M(a)}{N(a)}, \tag{11}$$

where

$$M(a)=\left[\frac{1+a-a^2}{2(2-a)}\right]^2,\ N(a)=\left[\frac{(1-a+a^2)(5-5a+a^2)}{12(2-a)^2}\right]^2. \tag{12}$$

On the other hand, the series $C(k)$ is a solution of equation

$$C(k)=\frac{1}{2-a}\left[aB(k-1)+(1-a)^2B(k-2)+aA(k-2)\right] \tag{13}$$

with the initial conditions

$$C(0)=\frac{1+a^2-a^3}{2(2-a)},\ C(1)=\frac{a(4-a-a^2)}{6(2-a)}, \tag{14}$$

while, finally,

$$A(k)=\int_0^1 T^k(x)\,dx,\quad B(k)=\int_0^1 x\,T^k(x)\,dx. \tag{15}$$

In this way, the representation (11) of correlation functions $\rho(k)$ indicates the stationarity of time series (X_n) that, furthermore, allows the construction of consistent estimates of unknown parameters $a\in(0,1)$. Namely, the stochastic structure of series (X_n) depends on the operator $T(x;a)$. It is therefore of utmost importance to find the effective estimates of an unknown parameters, which allows the formation of appropriate mathematical and stochastic models, good enough to describe the dynamics of the observed empirical series. Using the correlation function $\rho(k)$, namely its value for $k=1$, the consistent estimate of parameter a can be obtained using the equation

$$\frac{a(4-a-a^2)}{6(2-a)}=\hat{r}, \tag{16}$$

where

$$\hat{r}=\frac{1}{N}\sum_{i=1}^{N}Z_iZ_{i-1} \tag{17}$$

is the estimated value of the first correlation of series (Z_n), as a stochastic modification of the realized series (x_n). The main difficulty that arises here is that the expression (16) is a third degree equation for the unknown parameter a, with three possible real solutions. Indeed, we find that the required estimate $\hat{a}$, as one of the solutions to equation (16), is a root of the characteristic polynomial

$$P(a) = a^3 + a^2 - (4 + 6\hat{r})a + 12\hat{r}. \tag{18}$$

We will only consider the solution $a=\hat{a}$ which is located in the unit interval (0,1), which finds use for the well-known Newton's iterative procedure for the approximate determination of isolated roots of nonlinear equations. For this purpose, we form a recurrent sequence

$$a_{k+1} = a_k - \frac{P(a)}{P'(a)},\ k = 1, 2, ... \tag{19}$$

for which, under certain conditions, can be shown that it converges to the required estimate of the parameter a, depending on the choice of initial values a_0. Of course, the existence of root $a \in (0,1)$ depends on the polynomial $P(a)$, and the first estimated correlation value $\hat{r}$. As

$$P(0) = 12\hat{r}, \quad P(1) = 2(3\hat{r} - 1) \tag{20}$$

it will, for example, in the case of weaker positive correlation between series (X_n), when $0 < \hat{r} < 1/3$, the polynomial $P(a)$ has at least one of the roots which lies within the unit interval. Then, by using the iterative method (19) we can obtain a numerical series (a_k) that converges to the requested estimate. In this way, the appropriate model of the dynamics of observed financial series is formed, and it will be discussed in more details, but in the next section of this paper.

4. Application of Chaotic Models

As already noted in the introduction, nonlinear dynamical systems have proved to be very successful in describing the dynamic structure of financial series, and today they provide a fundamental theoretical basis for most empirical analysis of the behavior of different market segments. Chaos theory itself gives a full contribution to modern studies of financial time series and their mathematical and stochastic modeling. A more detailed view of these concepts and applications of nonlinear "chaotic" models in financial engineering and economic theory, in general, we can find, for instance, in the works of Bollt [2], Schittenkopf, Dorffner and Dockner [6], and so on. Here we will describe some of the possible applications of nonlinear dynamic models of the type of chaos in the research and analysis of the dynamics of some segments of the Serbian financial market.

4.1 Model of growth of national income

As one of the most interesting examples that can be found in the national literature, we will present, but from the stochastic point of view and partly generalized the work of

Jablanović [4]. Using the logistic operator, which we described in the third section, it describes the so-called *chaotic model of growth of national income*. The main parameter of this model is

$$\alpha = \frac{\beta}{\beta + \gamma - 1}, \tag{21}$$

where β is the ratio of the level of investment and growth in national income over a period of time t, and γ relationship between government spending and the national income. Thus, if Y_t denotes the current value of national income at time t, the growth model can be represented by equation

$$Y_t = \alpha Y_{t-1} - \alpha Y_{t-1}^2, \; t = 1, 2, ..., T. \tag{22}$$

In this form it is clearly equivalent to the iterative process of the logistic model (5), and this model belongs to a group of dynamic models of deterministic chaos, because of the factor of randomness completely subordinate to its mathematical formulation. Otherwise, this relatively simple model of growth of national income has the ability to generate a stable equilibrium, cycles or chaos, in accordance with the known, previously described features of the theoretical model. However, if you look at the place of the first series its stochastic modification of the form (3), which in this case follows

$$Z_t = Y_t + \xi_t, \tag{23}$$

where (ξ_t) is the previously defined white noise, we are able to estimate the unknown parameter α using standard statistical procedures. Using, for example, a well-known method of least squares, i.e. regression procedure to minimize the function

$$Q(Y_1, ..., Y_T; \alpha) = \sum_{t=1}^{T} \xi_t^2 = \sum_{t=1}^{T} (Y_t - \alpha Y_{t-1} + \alpha Y_{t-1}^2)^2 \tag{24}$$

with respect to the parameter α, we can get its estimated value

$$\hat{\alpha} = \frac{\sum_{t=1}^{T} y_t (y_{t-1} - y_{t-1}^2)}{\sum_{t=1}^{T} (y_{t-1} - y_{t-1}^2)^2}, \tag{25}$$

where (y_t) denotes the realized values of (Y_t).

A practical illustration of this estimation procedure is given in Table 2, which shows, firstly, the gross national income in constant prices from 2002 (column labeled GNP), the level of gross investment and expenditure on final consumption of the Republic of Serbia, based on data of the Statistical Office of the Republic of Serbia [10] in the period 1997-2009. In the second part of the same table the estimation of appropriate parameters β i γ, is made with the modalities established for all standard statistical assessments: the average value as a measure of central tendency, then the standard deviation and coefficient of variation, which are measures of dispersion. Based on the assessed value β and γ, using the equality (21), is obtained through the empirical parameter α of logistic model (22). Its value $\hat{\alpha} = 1.0108$, based on previous theoretical considerations, indicates a stable, "not chaotic" dynamics of growth of national income.

Table 1. Estimated parameters values of growth of national income

Estimations	GNP (10^9 RSD)	Investments (10^9 RSD)	Consumption (10^9 RSD)	Parameters $\hat{\beta}$	$\hat{\gamma}$
Mean	1 133.23	222.86	1 122.29	0.8639	0.9908
Standard deviation	857.75	215.82	834.25	0.5843	0.0385
Variation (%)	75.70	96.84	45.92	67.63	3.89

Note that, on the other hand, the estimation of the parameter α can be done through directly, on the basis of equality (25). Using empirical data, as the value of gross national product of Serbia in the same period, but expressed in millions of dollars, we get the "regression" estimate $\hat{\alpha}' = 1.3824$ which has the same interpretation as the previously obtained "empirical" estimate $\hat{\alpha}$. Graphical representation of the dynamics of gross national income, as the empirical time series and modeled values of the corresponding logistic model (22), obtained for the estimated value $\hat{\alpha}'$, is given in Fig. 4. We notice that the apparent high correlation of these two series, where the deviations are considered random, i.e. they are attributed to fluctuations of "noise" (ξ_t), which is defined by additive decomposition (23).

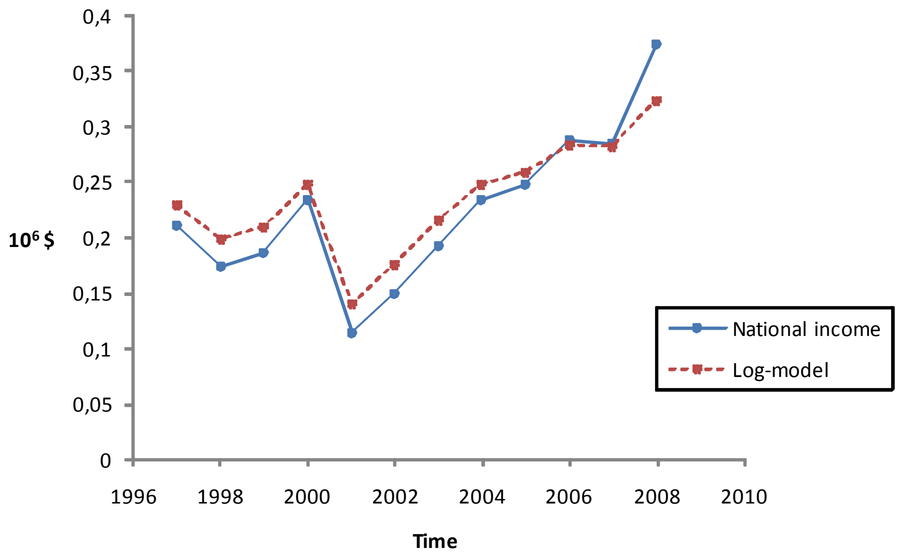

Fig. 4. Comparative chart of empirical and modeled values of national income in the Republic of Serbia in the period 1997-2009.

5.2 Analysis of the dynamics in the Serbian stock market

Now we will describe one of the possible applications of the modified stochastic model of chaos, presented in the work of Stojanović and Božinović [7], where, the Baker's operator (8) is used as a basic functional model. As an empirical series we will take the official data from the Belgrade Stock Exchange (BSE), namely, price and volume of trading companies:

- "Alpfa plam" (Vranje);
- "DIN" (Niš);
- "Hemofarm" (Vršac);
- "Metalac" (Gornji Milanovac).

Now, if we denote the price with S_n and the total volume of stock trading at a given point in time n=1,...,N, with Q_n, one can notice a certain degree of negative correlation between these two sets, expressed by the simple linear correlation coefficient ρ, as it is shown in Table 3. Of course, this is a consequence of the famous law of demand and price value, or reduction in interest of customers to buy shares when their prices increase, and vice versa. However, our idea is to express this correlation, if possible, in a new, more complete way, using an adequate model of chaos. For this purpose, we will observe a ratio of the sequences defined above, i.e. a sequence

$$Y_n = \frac{S_n}{Q_n}, \; n = 1,\dots,N. \tag{26}$$

Furthermore, the series (Y_n) can be standardized by introducing a series

$$X_n = \frac{Y_n}{\max\{Y_1,\dots,Y_n\}}, \; n = 1,\dots,N. \tag{27}$$

The realized values of sequence (X_n), in the case of the above mentioned companies, are shown in Fig. 5. It is easily observed that this realization has a dynamic structure which, in certain circumstances, can be interpreted with Baker's chaotic model. Namely, it is obvious that it is valid $X_n \in [0,1]$ $X_n \in [0,1]$ for an arbitrary n=1,..., N and, using the theoretical considerations which were presented in the previous section, we can make the construction of the appropriate model. Of course, our main task will be primarily based on the estimating the unknown parameter of the Baker's model $a \in (0,1)$.

Table 2 shows the values of the parameters estimated in the case of the above listed companies. As a starting value of Newton's iterative method is taken a_0=0. Then, for the different length realization of a series (X_n), denoted with N, we get the estimated value of the first correlation coefficient $\hat{r}$, and then, using an iterative procedure (19), estimates for $\hat{a}$ which, obviously, are within the unit interval. However, it is interesting to note that for $\hat{r} \approx 0$ it follows $\hat{a} \approx 0$, which is again a consequence of the roots of the characteristic polynomial $P(a)$. Namely, as

$$\lim_{\hat{r}\to 0} P(a) = a^3 + a^2 - 4a \tag{28}$$

it is obvious that, in this case, limit polynomial has roots a=0.

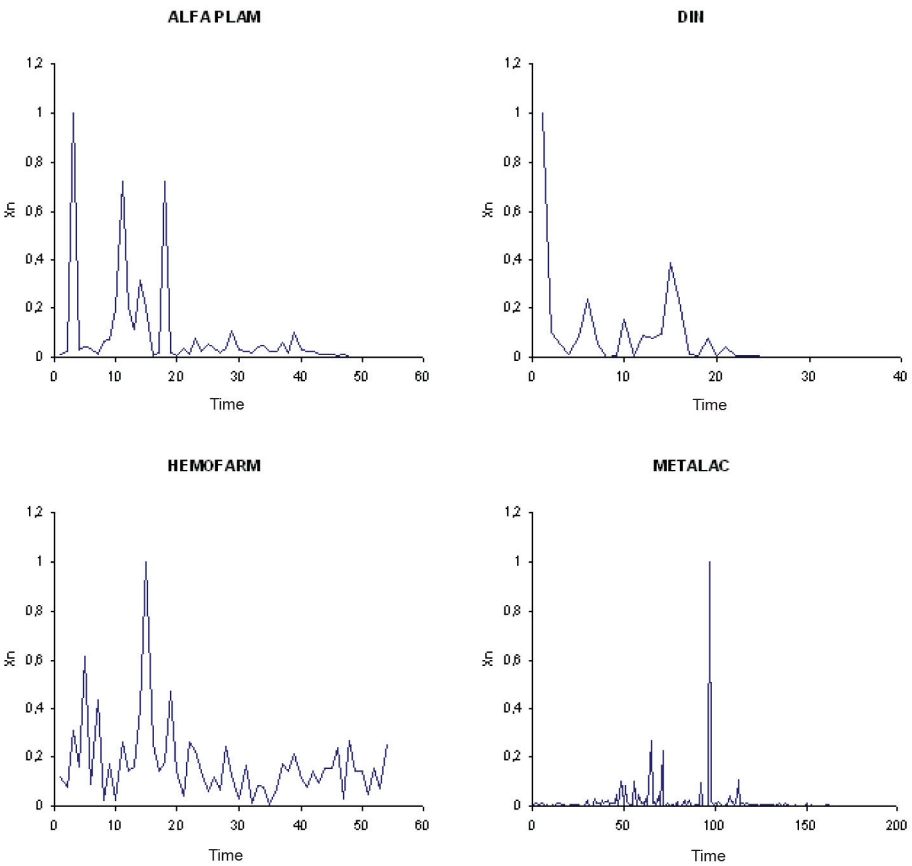

Fig. 5. The realizations of series (X_n) generated by Baker's operator.

Table 2. Estimated parameters values of Baker's model

Companies	Sample (N)	Correlation (ρ)	$\hat{r}$	$\hat{a}$
Alfa Plam, Vranje	50	-0.3992	0.04826	0.14023
DIN, Niš	33	-0.3327	0.26199	0.72931
Hemofarm, Vršac	174	-0.0882	0.10811	0.30524
Metalac, G. Milanovac	54	-0.1067	0.00959	0.02857

Therefore, as in the case of company "Metalac" from Gornji Milanovac, if the estimated value of $\hat{r}$ and $\hat{a}$ that are "close" to zero, then this fact provides a basis for a statistical test of the hypothesis that the real value of this parameter is a=0. Note that if the accuracy of this hypothesis Baker's operator is reduced to a linear continuous mapping $T(x) \equiv 0$, which eliminates the possibility of a "chaoticaly" dynamics of the observed series. Thus, we conclude that the realization of the (X_n) in the case of "Metalac" does not have an adequate term structure that could be interpreted by Baker's model. This can also be easily seen visually, as shown in Fig. 5, where one can see that most of the implementation of a series (X_n) are much smaller in size compared to the value max(X_n) = 1. On the other hand, estimated values of the parameters of other companies indicate that a>0, and the Baker's model can be used to describe the structures of observed series.

5. Conclusion

The possibility of formal defining and implementing the model of dynamic chaos, which are exposed above, are certainly only a small part in a wide range of research that is increasingly rising in importance. Chaos theory has in recent years become an indispensable application in almost all econometric analysis of financial series and other of dynamic segments in the financial markets. A more detailed overview of these concepts and applications of nonlinear stochastic models in economics, we might find written by many authors. The most significant, for example, are the works of Bollt [2], Schittenkopf, Dorffner and Dockner [6], and the monographs of Ширяев [8], and Tufillaro, Abbott and Reilly [9].

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STATISTIČKA INTERPRETACIJA I PRIMENA MODELA HAOSA U ANALIZI DINAMIKE FINANSIJSKOG TRŽIŠTA

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U ovom radu izložićemo različite aspekte u statističkom istraživanju i primeni nelinearnih dinamičkih modela tipa haosa. Najpre, koristeći formalno-deduktivan pristup, daćemo jednu od mogućih definicija i neke važnije klase ovih modela. Zatim ćemo opisati praktičnu primenu modela haosa u ispitivanju dinamike domaćeg finansijskog tržišta.

Ključne reči: *modeli dinamičkog haosa, ocene parametara, primena modela.*