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BASIC MATHEMATICAL MODELS IN ECONOMIC-ECOLOGICAL CONTROL

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Abstract. *This paper is devoted to the application of the general principles of mathematical modeling to such a specific area as economic-environmental interaction. For the description of some of these models we use results and algorithms from [5] and [6]. In [7], [8] and [9] some of these mathematical models are used in the analysis of Pareto optimality and external effects in ecology.*

Key Words: *Economic-ecological system, mathematical models, deterministic and stochastic models, continuous and discrete models, linear and nonlinear models, differential and integral models.*

1. INTRODUCTION

Economic-ecological system (EES) is an economic system considered jointly with the ecosystem of a region. The EES notion includes two-way interactions between economics and environment (ecosystem) and supposes presence of a human control in the system.

Modeling provides a preliminary explanation and prediction of EES behavior and adds new theoretical information about the nature, since there is always a gap between real influence on the nature and theoretical understanding of that influence. Therefore**,** all possible variants of EES control should be modeled for the purpose of decreasing undesirable ecological consequences.

Mathematical modeling has particular importance among modeling methods. The advantages of the modeling as compared to a real experiment are: 1) relatively low cost of modeling, 2) easy model modification, 3) possibility of multiple experiments with changed parameters 4) taking into account the prehistory of ecosystem's evolution that is important for modeling of irreversible processes.

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Modeling ought to begin at an earlier stage of study, so far as the analysis of numerical experiments suggests what kind of additional information is needed and what should be changed to achieve a better accordance with a real-life picture.

A mathematical model should not be a copy of the real world, it is always a simplification which assists in revealing a principal process which takes place in reality.

In a decision-making process we have always used models because we have not possessed absolute knowledge of reality. Ideal models of the future first emerge in the human brain (mental models). Mathematical modeling methods are supplemented with mental modeling, and what is important is that a mathematical model cannot be better than a mental one on the basis of which it is created. Formal models are secondary with respect to the mental models but cannot substitute them.

2. Analysis of economic-ecological control problems

We start from the analysis of systems under study and their substantial problems. In this section, some general notes are made about goals, peculiarities and techniques of economic-ecological systems modeling.

2.1. Features of economic-ecological interaction

Mankind can not refuse a transformation of the natural environment but ecologically careful acting must compensate for the negative ecological and human activity, especially, of science and technological change cases an unreasonable and ecologically dangerous use of technology leads to ecological problems rather than the technology itself. Technology creates a possibility that can be released under certain conditions. These conditions, in turn, begin to affect the direction of technological change having created an ecologically dangerous feedback.

All consequences of human activity can be classified in accordance with their environmental and ecological impacts as: negative, neutral and positive.

Negative ecological impacts. Some achievements of science and technology (synthetics, pesticides and others) play a negative role with respect to environment until an efficient means for their neutralization is created.

Such negative scientific achievements as radio nuclides were caused by badly formulated society goals rather than by the technological change. Many particular technical problems can be resolved today but their solutions cause negative ecological consequences just because of local character of the problems statement. Contribution of science and engineering into resolving ecological problems is determined to a large extent by goals raised in human society.

Negative ecological impacts can be divided into: real negative consequences of human activity and potential ecological dangers.

Pollution in the environment, increasing levels of radiation, soil erosion and others are real negative ecological consequences of human activity. Other scientific achievements (nuclear power, mining mineral resources, and urbanization) are fraught with potential risk.

Potential ecological dangers can also be subdivided into two categories: 1) possible future dangers that can appear if modern tendencies of technical and economic development remain (these are exhaustion of traditional natural resources, destruction of ozone layer and others); 2) the dangers that are possible at any time (such as intensive radiation pollution because of nuclear power stations).

The potential ecological dangers are often more complicated and important than real ones. The real negative ecological consequences can be reduced (there were first successes in prevention of environmental pollution) while the potential dangers are revealed suddenly, as a rule, and they have a tendency to accumulate and increase when a range of human activity is growing. Chernobyl disaster is an example of turning a potential danger into a real one.

Positive ecological impacts. Some scientific achievements (electronics, computers, automation, biotechnology, and space exploration) give an opportunity to reduce the total negative ecological impact of human activity.

People interact with nature in substance, power and information aspects. The information interaction inherent in modern technology is most ecologically advantageous. At this point, ecological role of computers should be emphasized because computers allow us to treat such inexhaustible resource as information.

Thus, further development of technology can eliminate or reduce a part of its own negative ecological consequences.

2.2. Goals of economic-ecological control

In a broad sense the main goal of economic-ecological control consists of harmonization of relations between human activity and natural environment, creation of propitious natural conditions for human existence, rational planning of biosphere. The following ecological strategies of relations between society and nature can be separated:

- *defensible* (creation of various purification tools and constructions, development of captive technologies, and so on),
- *correlative* (co-ordination between production and natural ecological processes),
- *strategy of technologization of natural processes* (exploitation of natural processes as technological ones).

Three main global ecological problems can be highlighted chronologically: 1. shortage of food (it has always been very important); 2. exhaustion of natural resources (arose in the XIX century); 3. pollution of the environment (arose in the XX century).

All usually considered applied ecological and environmental problems are a reflection of some of the above problems on the local or regional level.

Interconnection and interlacing of ecological problems do not permit solving a particular problem without encountering others. Reducing one negative consequence entails, as a rule, increasing the others. Therefore, there is no sense in achieving a complete and final solution for a particular ecological problem. It is only worth speaking about working-out recommendations and techniques for shifting a whole complex of particular ecological problems towards an optimal interrelation between man and the environment. Let us consider an example of the situation mentioned above.

Example: Rational exploitation of an agricultural ecosystem: Getting the highest possible yield leads to the creation of artificial single-crop ecosystems. However, such man-made biogeocoenosis are not as stable as the natural multi-species ones, they are more vulnerable to vermin, diseases and climatic conditions. To increase their stability it is necessary to use pesticides. Ecosystems themselves develop towards maximal stability with minimal productivity, and any increase in their efficiency demands expenses. Thus, the goal should be achieving optimal productivity of an ecosystem rather than maximum one with taking into account economic expenses, instability and contamination of the ecosystem.

On the whole, a decision choice in EES development requires co-coordinating different control goals and achieving a compromise between them. People play a key role in achieving such compromise and can not be substituted by formalized methods (including mathematical ones).

2.3. Components of economic-ecological control

Any control system includes three basic functional components: **measuring** (monitoring), **modeling** and **controlling** components. These three parts are inseparably linked and can not work without each other.

EES modeling and controlling tools become senseless without a developed measuring part. Environmental monitoring is the first and probably the most expensive part of the EES control. In the literature dedicated to economic-ecological control, principal attention is given to elaboration of monitoring systems.

Environmental monitoring is a multipurpose information system for observation of the biosphere, assessment and forecast of its state, evaluation of human influence on the environment, and bringing to light the factors and sources of such influence. It includes three levels: bioecological monitoring (observation of environmental state from the viewpoint of its influence on man), geoecological monitoring (observation of ecosystem's evolution), and monitoring of biosphere (observation and forecast of the change in the biosphere on the whole).

The system of monitoring can cover local regions (local monitoring) or whole countries (national monitoring). The concept of global monitoring (for the whole globe) is also meaningful.

The concept of monitoring implies observation and prediction functions rather than decision-making. A more general concept is a decision-support (or control) system. It implies a complex of hardware, software, mathematical, information and organizational means intended for efficient management of an economic-environmental system under control.

On the other hand, the absence of the modeling component turns an EES control system into a kind of information system. It is necessary to emphasize the importance of mathematical modeling in a broad sense as a basis of EES control decision-support.

Modeling of EES control has two aspects:

- modeling of current state and forecast of ecosystem functioning,
- modeling of control decisions themselves.

These problems are solved by means of various theoretical and mathematical methods.

Mathematical modeling of large-scale systems like economic-ecological systems is a complicated scientific and technical process. In this paper, we mainly limit our attention to the following two stages: SP (statement of a substantial problem) and MP (statement of a mathematical problem).

Interaction between these stages is illustrated in the following scheme:

SP (substantial problem)	$\Rightarrow$ **MP (mathematical problem)**
Description of system dynamics	$\rightarrow$ Mathematical model of the system;
Goals of control (and modeling)	$\rightarrow$ Objective functional (criterion);
Identification of current state	$\rightarrow$ Initial (and boundary) conditions;
Control influences	$\rightarrow$ Sought-for model variables (controls);
Given parameters	$\rightarrow$ Given (known) model variables (characteristics).

Note that the concept of mathematical model (MM) is not identical to the mathematical problem (MP). After a MM has been created, there is still a lot of work needed to analyze and select goals and constraints of the problem, given and desired characteristics of the process, and so on.

The structure and basic characteristics of the mathematical problems (MP) applied to EES control are illustrated further in the study.

3. Mathematical models for modeling EES

At present, two tendencies can be pointed out in applied mathematical modeling:

– The first tendency is to construct as simple models as possible and to attach them to initial data without a deep insight into the process investigated. Thus, linear equations have been used more and more widely. Such approach is rather popular in applied areas of modeling and gives good results in many cases.

– The second tendency consists of the elaboration of mathematical models that reflect an internal structure of the systems under study in a complete manner, taking into account some delicate features. It leads, as a rule, to rather complicated mathematical problems. Such models are not always convenient for use in practice. Nevertheless, their elaboration reflects an internal logic of scientific development: improvement of both pure and applied mathematics would be impossible if new models were not created.

Various mathematical tools are used in EES modeling, from linear algebraic equations to multicriteria optimization, fuzzy sets theory, expert methods, etc. A key adequacy criterion for applied mathematical models is their successful approbation on real-life objects. This, however, does not depreciate the significance of their theoretical analysis and comparison with other alternative models. In this case the efficiency criteria of models are their capability to take into consideration different control factors and aspects of the process under study.

Real life often advances research of substantially new features of systems. In doing so, it is necessary either to develop new mathematical models or to modify considerably known models (often, by using a new mathematical apparatus). Such cases will be illustrated in the next parts of the book.

Below, there are some notes about various types of mathematical models used in applied modeling. These notes are not exhaustive and reflect goals of the present monograph.

3.1. Deterministic and stochastic models

Economic-ecological systems belong to complex systems with high dimensionality and uncertainty of relationships inherent in them. Nevertheless, the most widely used models for description of general tendencies of the EES functioning and evolution are *deterministic models* rather than *probabilistic* (*stochastic*) ones. It is probably due to unjustified complexity of mathematical description using stochastic factors without substantial insight into interpretation of the essence of processes. Strictly speaking, deterministic models always operate with some averaging probabilistic performances of processes that take place in EES (the expected value of "population amount" instead of the real "population amount", and so on). At least one can recommend restricting consideration within the deterministic models of EES at the initial stage of study of the subject.

Stochastic (statistical) modeling is very useful for the analysis of repetitive processes. It requires a corresponding amount of initial data for the modeling (usually a large one). However, implementation of evolutionary processes in economics and ecology is often unique and is accompanied by a shortage of the data (especially, for large-size systems). This fact stresses the importance of construction of phenomenological models (i.e., based on substantial hypotheses) for evolutionary processes. Of course, a comprehensive analysis (including statistical one) of all available information should be provided for such systems.

3.2. Continuous and discrete models

Depending on the techniques of process description, mathematical models are subdivided into *continuous* and *discrete models* (which operate with continuous and discrete variables).

Different types of data operated distinguish these models. The discrete models operate with vectors like $x = (x_1, x_2,..., x_n) \in R^n$ whereas the continuous models operate with functions $x(t)$ of an independent continuous variable t (scalar or vector). Note that in dynamic models one of the independent variables usually means a time t defined on some interval $t \in (t_0, T)$.

A general form of discrete models is

$$F_j(x_1,x_2,...,x_n) = 0,\ j=1,...,m, \tag{1.1}$$

where $Fj(.)$ are some, in general, nonlinear functions of n scalar variables.

A general form of continuous models is

$$\Phi(x) = 0, \tag{1.2}$$

where $\Phi(.)$ is a functional of the function $x(.)$.

The functional is an operator putting a real value from R^1 for each function $x(.)$ from a certain functional space Ω. Some well-known examples of the functional spaces are:

– $C[a,b]$ - the space of all continuous functions defined on the interval $[a,b]$;

– $C^1[a,b]$ - the space of all functions with a continuous derivative defined on the interval $[a,b]$;

– $L^\infty[a,b]$ - the space of all functions defined and bounded almost everywhere on the interval $[a,b]$; and others.

As a rule, a discrete analogue can be constructed for a known continuous model, and vice versa. For the most of continuous models of economic and ecological systems considered below, their discrete analogues are known and often used. The choice between continuous and discrete models depends on model's capabilities in reflecting peculiarities of the problem and objects (processes) under investigation, as well as on the preferences of the researcher.

3.3. Linear and nonlinear models

The choice between linear and nonlinear models depends on the behavior of the process under study or its desired approximation. Sometimes a process is nonlinear, but it is convenient to describe it by a linear model because of its greater simplicity.

A **linear discrete model** is the following *system of linear algebraic equations*:

$$\Sigma a_{ij}x_j = b_i, \quad i = 1,...,m, \tag{1.3}$$

or

$$\mathrm{A}\mathbf{x} = \mathbf{b}, \tag{1.4}$$

where $\mathbf{x}=(x_1, x_2, ..., x_n)\in \mathrm{R}^n$, $\mathbf{b}=(x_1,x_2,...,x_m)\in \mathrm{R}^m$, and $A = \{a_{ij}\}$ is a *matrix* of the dimension mx n.

The model (1.3) is a convenient and completely investigated mathematical object. If $m=n$ and the determinant $\det A \neq 0$, then the system (1.3) has a unique solution x (under given A and b) that can be found by very fast algorithms.

A general theory of nonlinear discrete equations does not exist, and solving a concrete system of nonlinear equations (1.1) often runs into great theoretical or numeric difficulties. The solution may be non-unique or non-existent in general.

A **linear continuous model** is the model (1.2) with a linear functional Φ(.), i.e. such that preserves the *linear operations of addition and scalar multiplication* for all elements x, y of the functional space Ω:

$$\Phi(x+y) = \Phi(x) + \Phi(y), \qquad \Phi(ax) = a\Phi(x), \text{ for } a\in \mathrm{R}^1.$$

In **nonlinear continuous models** the functional Φ(.) is nonlinear.

3.4. Differential and integral models

Depending on the type of the functional Φ(.), *differential* and *integral models* are separated in the continuous models (1.2).

Integral models are more general but differential models are simpler and more efficient in analytical and numerical study, so they are more common. The general selection rule is: if a process can be efficiently described by a differential model with a required accuracy, there is no sense to construct and use an integral model.

Integral models. Let us restrict ourselves to the case of a linear model and one-dimensional independent variable t. It is known that, in general case, any linear functional $\Phi(x)$ with respect to the function $x(t)$, $t\in[t_0,T]$, can be described by the following integral

$$\Phi(x) = \int_a^b K(r)x(r)dr,$$

where $K(r)$ is a given function. Then an integral model of a linear process can be described as:

$$\int_a^b K(t,r)x(r)dr = f(t), \quad t\in[a,b], \tag{1.5}$$

where $f(t)$ is a given function.

The formula (1.5) describes the Fredholm integral equation of the first kind with respect to the sought-for $x(t)$, $t\in[t_0,T]$. The Fredholm integral equation of the second kind is

$$x(t)=\int_a^b K(t,r)x(r)dr+f(t), \quad t\in[a,b]. \tag{1.6}$$

After discretization by the variable t, the model (1.5) gives the equation (1.4). The analogy between continuous integral models and their discrete analogues (system of equations (1.3)) is very useful for better understanding and interpretation of a model. However, the theory of linear continuous models is much more complex compared to linear discrete models. In particular, a big difference exists between integral equations of the first and the second kind.

If we consider a dynamical process, then its current state depends on the past states only (it cannot depend on the future) and, hence, $K(t,r)=0$ for $r>t$. In this case we obtain from (1.5) the following model:

$$\int_a^x K(t,r)x(r)dr=f(t), \quad t\in[a,b], \tag{1.7}$$

that determines the Volterra integral equation of the first kind with respect to the sought-for x. The Volterra integral equation of the second kind is defined analogously to (1.6).

Once again, despite the similarity between the Fredholm and Volterra equations, their properties are quite different.

Differential models. They are represented by a functional connection between a sought-for function and some of its derivatives. Such models were first developed for the description of *dynamical processes* (i.e., processes developed in a time t). They describe a special class of such processes *(non-spatial processes, processes without after-effect, dynamic processes)* when the dynamics of future development depends on the current state of the process only. Such approximation appears to be good enough for many physical, mechanical, economic and other real processes. The reason is that various filial perturbations are quickly damped in real processes and may be secluded from a process model.

The example: a trajectory of a thrown ball is usually determined by the ball mass and vector velocity and external conditions (wind, and other), however, a completely exact model of the ball dynamics has to take into account the ball size and shape (as well as the player's fingers position and so on).

4. MODELS OF CONTROLLABLE DYNAMICAL SYSTEMS

In this section, the emphasis is placed on the deterministic models of *dynamical systems* (DS) in continuous time t. We understand the notion of DC in a wide sense (as an inertial system, system with memory, etc.) rather than in its strong sense (as *dynamic system*). The traditional mathematical tools for such DS description are differential and integral equations.

Here, the often-used term "control" is used in a pure mathematical sense and corresponds to "a control function" or "a set of control functions".

4.1. Differential models of DS

Denote by $u(t)$ and $x(t)$ the input and output signals of DS respectively, where t is a time. Suppose that $u=(u_1,u_2,...,u_m)$ is a m-dimensional vector and $x=(x_1,x_2,...,x_n)$ is a n-dimensional one. Then the scheme of dynamical system (DS) is:

$$\text{Input } u(r),\ r<t \Rightarrow \textbf{DS} \Rightarrow \text{Output } x(t).$$

Then a linear DC is described by the *system of ordinary differential equations* of first order:

$$dx/dt = A(t)x + G(t)u, \tag{1.8}$$

where A and G are the matrixes of corresponding dimensions, or by one of n-th order:

$$a_n(t)x^{(n)}+...+a_1(t)x^{(1)}+a_0(t)x = b_m(t)u^{(m)}+...+ b_1(t)u^{(1)}+b_0(t)u. \tag{1.9}$$

If the parameters of DS are constant (stationary DS), then the coefficients of the models (1.8) and (1.9) do not depend on t (particularly, $A(t)=A$, $G(t)=G$).

The exact analytical solution of the system (1.8) is of the form:

$$x(t) = \Phi(t,t_0)x(t_0) + \int^t_{t0}\Phi(t,r)G(r)u(r)dr, \tag{1.10}$$

where the so-called transition matrix $\Phi(t,r)$ is defined by the differential equation: $\partial\Phi(t,r)/\partial t=A(t)\Phi(t,x),$ where $\Phi(t,r)=E$ is the identity matrix.

Nonlinear DS is described by the system of nonlinear equations:

$$dx/dt = F(t, x, u), \tag{1.11}$$

where F is a n-dimensional function of $n+m+1$ variables $t, x_1, x_2,..., x_n, u_1, u_2,..., u_m$. In most cases of nonlinear ODEs an exact solution can not be found and their solving requires the use of approximate methods and computers.

The modeling of pollution propagation in the environment requires accounting for space variables along with time t. Such problems are more complex and are described by means of partial differential equations.

EES modeling often requires joint solving of a "pointed" problem of economic (technological) control described by ODEs and a space-distributed problem of pollution propagation prediction.

4.2. Explicit integral models of DS

The integral models assigning an explicit connection between DS input and output are traditionally used in automatic control theory. Thus, an arbitrary finite-dimensional linear DS is described by the *integral model* (IM) of the following form:

$$x(t) =\int^t_{-\infty} K(t,r)u(r)dr, \tag{1.12}$$

where $K(t,r)$ is termed the unit impulse response of DS.

If function $K(t,r) \neq 0$ for all $t\text{-}r > 0$, then DS is referred to as the DS with infinite memory.

In practice, the DS output $x(t)$ depends on the input signals $u(r)$ whose time instants r are distant from t at the most some time $T>0$, i.e. $K(t,r)=0$ for $t-r>T$. In this case we have *the DS with finite memory* which model is of the form:

$$x(t)=\int_{t-T}^{t} K(t,r)u(r)dr. \quad (1.13)$$

Such models are used for description of age structure of ecological populations.

The integral models of DS take into account the after-effect (persistence, contagion, hereditary effects) when the continuous sequence of the DS past states $u(r)$, $r<t$, influences on DS future evolution. These effects are not described by system of equations (for example, the solution of the system (1.9) depends on the initial DS state $x(t_0)$ only). The value T in the IM (1.13) is called the after-effect duration or the DS memory.

An integral model of elastic persistence in the form (1.12) (where x - deformation, u - strain) was introduced by Boltzman in the XIX century. Vito Volterra developed the Boltzman theory and introduced the after-effect concept for other applications, specifically, in ecology. The after-effect is, in general, defined as an arbitrary nonlinear functional of $u(t)$, $-\infty<r\leq t$.

Consider also the DS non-excited for $t<0$. Then $u(r)\equiv 0$ at $r<0$ and the DS model is of the form:

$$x(t)=\int_{0}^{t} K(t,r)u(r)dr. \quad (1.14)$$

If DS is stationary, then its output $x(t)$ depends only on the time for $t-r$ after the input instant r: $K(t,r) = K(t-r)$. Dynamics of the stationary DS is efficiently described by means of so-called transfer functions and DS natural frequency responses that represent respectively the Fourier and Laplace transforms of the unit impulse response. In the case of no stationary DS this way runs into severe difficulties and the main characteristic of DS is the unit impulse response.

The IMs (1.12)-(1.14) can describe invariable DSs as well as multivariable DSs - in the second case $x(t)$ and $u(r)$ are vector-functions and $K(t,r)$ is a corresponding matrix function (see Section 1.4.1).

The consideration of multidimensional DS corresponds to the passage from one-dimensional integrals to many-dimensional ones in the integral models (1.12)-(1.14).

4.3. Implicit integral models

Let us suppose that we know the structure of a DS and the characteristics of its elements. This usually happens to be the case for economic and technical systems. By the DS structure we mean a set of the connections among DS elements and their intensities. It is reasonable to define the structure connections of linear DS by their linear relation:

$$u(t) = Y(t)x(t)+u_0(t). \quad (1.15)$$

If these connections are non-linear, then (1.15) defines them in a first approximation. The coupling types and intensities are determined by the matrix $Y(t) = \{y_{ij}(t)\}$. If $y_{ij}(t)\neq 0$, then there is the output i feedback in the DS (positive at $y_{ij}(t)>0$ and negative at $y_{ij}(t)<0$). If the DS has no active element (for example, in economics), then $\backslash\ y_{ij}(t)\backslash\leq 1$ for all i,j. Different types of the DS structural connections can be described by choosing $y_{ij}(t)$, in particular, parallel and series connections of DS elements.

Substituting the relation (1.15) into an explicit IM, for example into (1.14), we obtain the following implicit IM of linear DS:

$$x(t)=\int^{t}_{0} K(t,r)Y(r)x(r)dr + f_0(t), \qquad (1.16)$$

where $f_0(t)=\int^{t0}_{0} K(t,r)u_0(r)dr$ is the given DS input.

From the explicit IM (1.12) we obtain the implicit IM (1.16) where $f(t)= fo(t)+\varphi(t,0)$. Here the function $\varphi(t,0) =\int^{0}_{-\infty} K(t,r)[Y(r)x(r)+ u_0(r)]dr$ characterizes the influence of DS prehistory before instant t=0 on the output signal $x(t)$ at current time t.

The model (1.16) represents the system of Volterra integral equations of the second kind (VIEs) with respect to output $x(t)$. These equations are transformed from the problem of the unite impulse response definition during the transfer from an explicit IM "input-output" to the implicit model (1.12). If a certain part of output $x_i(t)$ is given and some elements $y_{ij}(t)$ of matrix $Y(t)$ are sought, then we obtain mixed systems of VIEs of the first and second kind. If the number of sought-for functions is greater than the number of the equations (1.16), than the problem may be closed by introducing an optimization criterion which means the transfer to an optimization problem (optimal control problem) for the IM (1.16).

4.4. Integral models of DS with variable memory

Let us denote by $a(t)$ the lower integral limit in the integral models (1.12)-(1.14). Then we obtain the following uniform mathematical notation of these models:

$$x(t)=\int^{t}_{a(t)} K(t,r)u(r)dt, \qquad a(t)<t, \qquad (1.17)$$

where $a(t) = \{-\infty,t-T,0\}$.

For no stationary DS the function $a(t)$ may be an arbitrary: $-\infty<a(t)<t$. Moreover, for economic applications this function can be an unknown (sought-for) control. Then the model (1.17) is referred as the IM with variable (or controllable) memory.

Such models are used for the description of renovation processes in economic and economic-ecological systems.

5. CONCLUSION

In this paper, we would like to emphasize the similarity and basic characteristics of mathematical models that will be used for the description of various economic and ecological processes and systems in the further investigation. Thus: the Verhulst-Pearl and Lotka-Volterra models describe ecological population dynamics and innovation processes; the evolutionary equation describes population age structure and equipment replacement in production systems; the diffusion equation describes individual migration and pollution propagation in air and water medium and propagation of technological innovations.

In general, there is a deep analogy between biological and economic processes that is useful for the development of mathematical techniques in both these areas of modeling.

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OSNOVNI MATEMATIČKI MODELI U EKONOMSKO-EKOLOŠKOJ KONTROLI

Žarko Popović

Ovaj rad je posvećen primenama opštih principa matematičkog modeliranja u nekim specifičnim oblastima ekonomsko-prirodnih interakcija. Za opis nekih ovakvih modela koristimo rezultate i algoritme iz [5] i [6]. U [7], [8] i [9] neki od ovih modela korišćeni su u analizi Pareto optimalnosti i eksternih efekata u ekologiji.

Ključne reči: *ekonomsko-ekološki sistem, matematički modeli, deterministički i stohastički modeli, neprekidni i diskrene modeli, linearni i nelinearni modeli, diferencijalni i integralni modeli.*