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STYLIZED FACTS OF ASSET RETURNS: CASE OF BELEX

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Abstract. *This paper represents a study of the time series of stock market indexes at Belgrade Stock Market. Statistical analysis of this data is performed and a set of stylized empirical facts is presented. Various statistical properties of stock market indexes returns are described: lack of normality, correlation in the return series, and some non-linear tests. Our description emphasizes properties common to a wide variety of emerging and developing markets. We show statistical properties of Belgrade stock market, which significantly invalidate many of the common statistical approaches used to study financial data sets and examine some of the statistical problems encountered in each case.*

Key Words: *Financial Time Series, Financial Market, Belgrade Stock Exchange.*

1. INTRODUCTION

In recent years, the interest of physicists in interdisciplinary research has been constantly growing and also in the area of what is today named econophysics[1,2]. Apparently, the application of statistical physics methods to economics promises fresh insights into problems traditionally not associated with physics. Both statistical mechanics and economics study big ensembles: collections of atoms or economic agents, respectively. Besides, long-run time-series properties of equity prices, with particular attention to whether stock prices are very important in understanding and modeling financial market dynamics. These markets, though largely varying in details of trading rules and traded goods, are characterized by some generic features of their financial time series.

Theory of Random Walks claims that successive price changes ($P_t - P_{t-1}$) or price returns (P_t / P_{t-1}) are some kind of a random walk, and that cannot be determined from historical price information. Under this introduction of price returns, many models have been proposed and studied, but two of them are commonly used nowadays. The first and most common one, called the Bachelier-Osborne model and elaborated in 1959, states that

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price returns have a constant finite volatility over a given period of time ("time lag τ"), e.g. one day, one week, one month, etc [12,13,14]. Results of this theory in a log-normal distribution for price returns and a volatility is proportional to the square root of the time lag. In contemporary works it is considered that price returns do not follow a Gaussian distribution, since they exhibit kurtosis and fat tails: dramatic falls and spectacular jumps appear with higher frequency than predicted by a normal distribution. Hence, the idea of infinite volatility came into scope of modern investigations. It was introduced by Mandelbrot [16] and leads to stable Pareto-Levy distributions that can exhibit fat tails. Unfortunately, the hypothesis of infinite volatility supposes that the variance increases indefinitely with sample size, which is not verified by empirical data. Variance first increases then reaches a bound.

The premise often assumed is that prices change randomly and that returns are identically distributed following a normal distribution. This is very convenient from theoretical standpoint even if quite unrealistic. The main reason for that premise seems to be related to the fact that the normal distribution is characterized by only two parameters: its mean and its standard deviation. Actually, there is a large consensus among researchers on several stylized facts about the statistical properties of financial time series returns. For instance, it is a well-known fact that returns distributions are not normal but rather thick-tailed (leptokurtic). Absence of linear dependence does not rule out non-linear dependence. In fact, non-linear dependence may exist in a series in which we previously determined lack of linear dependence. If present, non-linear dependence would contradict the random walk model and the financial market weak form efficiency hypothesis. Checking if autocorrelation coefficients are not statistically different from zero is not enough. It is therefore necessary to test for the non-linearity of returns.

The basic random walk premise is that price movements are totally random, i.e. a random walk is a random process consisting of a sequence of discrete steps of fixed length totally independent one from another. We will commonly use the log return, mainly for two reasons. First, financially, it corresponds to the continuously compounded return of the asset. And secondly, numerically, to bound price domain to positive values. Obviously, any hypothesis about the independence and identical distribution of price changes is directly applicable to price returns and log returns.

The hypothesis of independent price returns is extremely important - and controversial - since it underlies all of the theory of random walks, and also all of the models developed around it. E. Fama discussed abundantly this hypothesis in his paper "The Behavior of Stock-Market Prices" [6] and stateed that the independence of price returns is the result of a noisy price mechanism. By noise, one should understand the psychology of different traders and the uncertainty or disagreement about the intrinsic value of the security, which depends on new information arrived or about to arrive. If successive bits of new information arise independently across time and if noise or uncertainty concerning intrinsic value does not tend to follow any consistent pattern, then successive price returns in a common stock will be independent. A third and crucial condition for independence of price returns is the existence of "superior traders", i.e. traders who will detect abnormalities on stock prices - departure of the security price from its intrinsic value - and will correct them by buying (resp. selling) the security if it is underestimated (resp. overestimated). If there are enough such traders, then the price will tend to stabilize around its intrinsic value, reducing risks of bubbles or crashes.

2. Stylized facts of asset returns

We analyze three financial time series: the BELEXfm (Belgrade Stock Exchange Free Market) Composite Index, the BELEX15 (Belgrade Stock Exchange 15) Index and the Republic of Serbia Bond A2007. BELEXfm and BELEX15 are composite stock market indexes that reflect the aggregate movements of the Belgrade stock market. BELEXfm is the first index created by BSE. The main purpose of BELEXfm is to measure changes of prices of shares with at least one transaction realized on the free exchange market. BELEX15 is the first index from the recently created new group and it includes shares of 15 companies, following the movements of the most liquid shares, which are traded in the continuous method. BELEX 15 is market capitalization weighted index. Since the Serbian financial market is a very thin one, individual stock prices do not seem to be a good choice. The historical data covers a time period of 283 trading days (from 4 October 2005 to 20 November 2006) for BELEX15, 306 trading days (from 1 September 2005 to 20 November 2006) for BELEXfm, and 469 trading days (from 10 January 2005 to 20 November 2006) for A2007. Table 1 gives a selection of descriptive statistics for the daily log returns of the three series and are plotted on Figures 1.

Table 1. Descriptive Statistics of logarithmic returns

	Mean	Median	Min	Max	Stan. Dev.	Skewness	Kurtosis
BELEXfm	0.00065	0.00057	-0.01439	0.01308	0.00384	-0.23870	5.38944
BELEX15	0.00140	0.0012	-0.03089	0.02759	0.00811	0.05216	4.00941
A2007	0.00023	0.00011	-0.01385	0.01016	0.00182	-0.42937	13.98931

2.1. Nonnormality

Measures of central tendency (arithmetic mean and median) are very close to zero. Thus the standard assumption of the Random Walk model that the expected value of daily returns equals zero is met. As for the standard deviation, the A2007 had the lowest (0.2%) and the BELEX15 the highest (0.8%), more than double the BELEXfm. In order to assess the normality of logarithmic returns, the results show that all three series returns show strong departure from normality, as the coefficients of skewness and kurtosis are statistically different from those of a normal distribution. All three series have asymmetric tails: BELEX15 is skewed to the right and BELEXfm and A2007 are skewed to the left. All series are clearly leptokurtic (the sample kurtosis is much greater than 3), which justifies the assumption of fat-tailed distributions.

Table 2. Normality test

	Jarque-Bera(5%)	p-value	Kolmogorov-Smirnov(5%)	p-value
BELEXfm	73.29791	0.00000	0.49478	0.00000
BELEX15	11.43483	0.00329	0.48899	0.00000
A2007	2343.68016	0.00000	0.49595	0.00000

Table 2 shows the Jarque-Bera, Kolmogorov-Smirnov normality test and their p-value for each of the logarithmic daily returns on the three series considered in this paper. Normality test shows that their unconditional distribution is not Normal. Both tests strongly reject the null hypothesis of normality for all series. The null of the Jarque-Bera test is rejected at the 0 per cent level for BELEXfm and A2007 and 0.003 for BELEX15, and the null hypothesis of the Kolmogorov-Smirnov test is rejected at the 0 per cent level for all series.

We could conclude that emerging market returns are not normally distributed. Even in developed markets stock returns have been found to be either leptokurtic or platykurtic, [4-6]. When there is a strong deviation from normality, correlation analysis should be done using nonparametric testing methods [7], such as the runs test, since they do not assume a specific distribution. However, we notice despite leptokurtosis and skewness, near normality can still be assumed for the sake of statistical analysis, as long as the number of observations is large. In this situation, we actually went on to perform parametric serial correlation tests even though the normality assumption had been rejected, the justification being that these tests "[help] in detecting the presence of higher order serial correlation which is difficult to detect by merely using the runs tests" [7]. For the same reason serial correlation tests are conducted in this study.

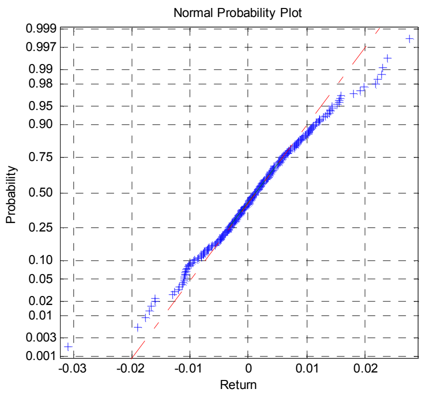
Normal Probability Plots

A normal probability plot is a useful graph for assessing of data, and comes from a normal distribution. Many statistical procedures make the assumption that the underlying distribution of the data is normal, so this plot can provide some assurance that the assumption of normality is not being violated, or provide an early warning of a problem with your assumptions. We could notice that on the Fig. 1 there is clear evidence that the underlying distribution is not normal.

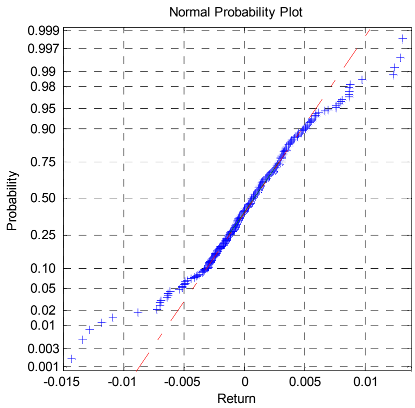
Quantile-Quantile Plots

A quantile-quantile plot is useful for determining whether two samples come from the same distribution (whether normally distributed or not).

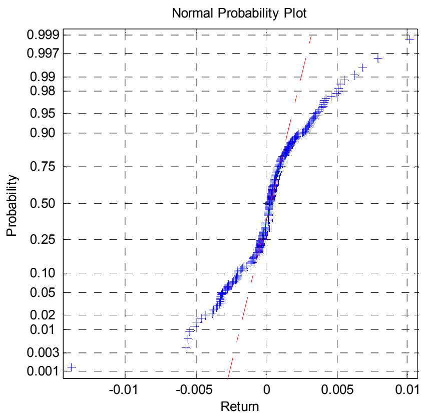
Even though the parameters and sample sizes are different, the straight line relationship shows that the two samples come from the similar kind of distribution. Like the normal probability plot, the quantile-quantile plot has three graphical elements. The set consist of numerous pluses are the quantiles of each sample. By default the number of pluses is the number of data values in the smaller sample. The solid line joins the 25th and 75th percentiles of the samples. The dashed line extends the solid line to the extent of the sample.



a) BELEXfm

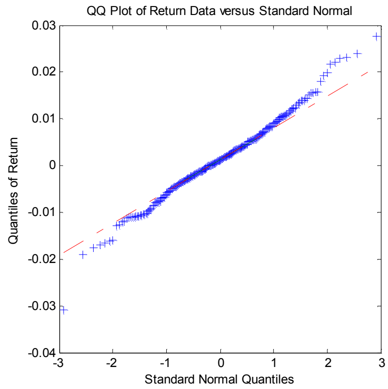


b) BELEX15

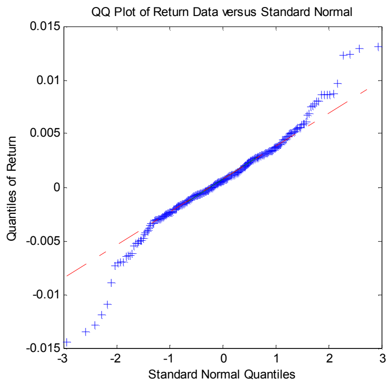


c) A2007

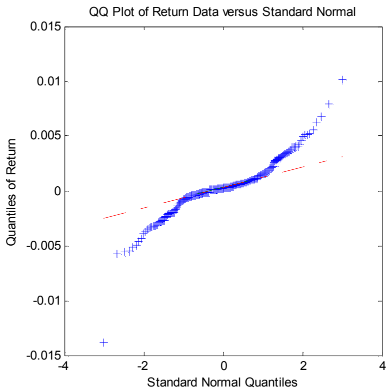
Fig. 1. Normal probability plots



a) BELEXfm

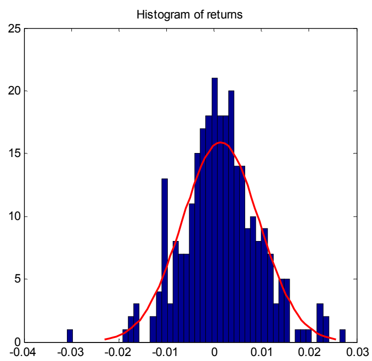


b) BELEX15

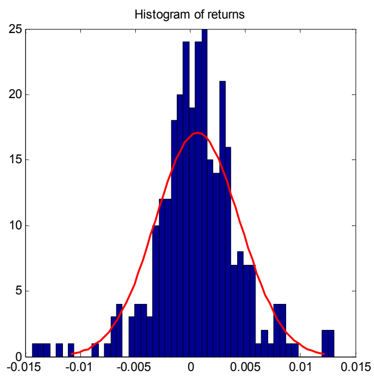


c) A2007

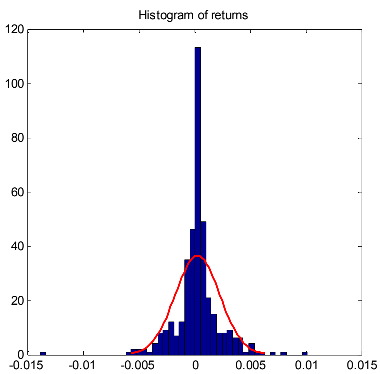
Fig. 2. Q-Q Plots



a) BELEXfm



b) BELEX15



c) A2007

Fig. 3. Histogram of normalized logaritmic returns

Normalized logarithmic returns

Probability distributions for asset returns often exhibit fatter tails than the standard normal, or Gaussian, distribution. The fat tail phenomenon is known as excess kurtosis. Time series that exhibit a fat tail distribution are often referred to as leptokurtic. The red (dashed) line in the following figure illustrates excess kurtosis. The blue (solid) line shows a Gaussian distribution.

It is now well known that log returns exhibit two specific kinds of departure from a Gaussian: kurtosis and fat tails. A high kurtosis means that the model distribution is more peaked than a Gaussian around the mean. Fat tails mean that crashes and huge increases appear far more often than predicted by the normal law.

2.2. Correlation of Financial Time Series

In the case when we treat a financial time series as a sequence of random observations, this random sequence, or stochastic process, may exhibit some degree of correlation from one observation to the next. One can use this correlation structure to predict future values of the process based on the past history of observations. Exploiting the correlation structure, if any, allows one to decompose the time series into a deterministic component (i.e., the forecast), and a random component (i.e., the error, or uncertainty, associated with the forecast).

Correlation in the return series

The autocorrelation function computes and displays the sample ACF of the returns, along with the upper and lower standard deviation confidence bounds, based on the assumption that all autocorrelations are zero beyond lag zero.

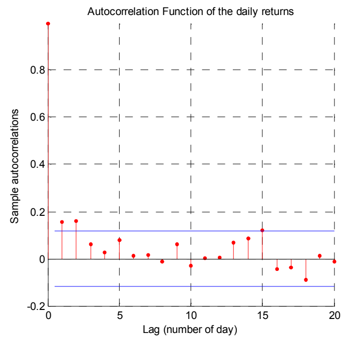
Figure 4 shows the autocorrelation and partial autocorrelation functions for the logarithmic returns.

Both partial correlation coefficients and the autocorrelation coefficients show that there is some dependency in stock returns. All the correlation coefficients are positive for the first lag, except for A2007. All series have significantly correlated returns for both the first and the second lag. There is no notable difference between the results from partial correlation analysis and autocorrelation analysis.

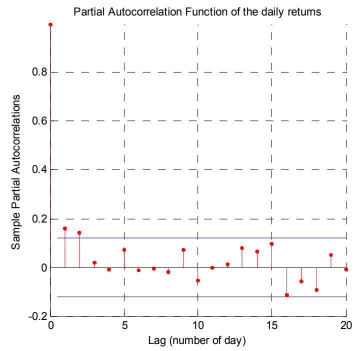
3. Non-linear Tests

Ljung-Box Q-statistic lack-of-fit hypothesis test

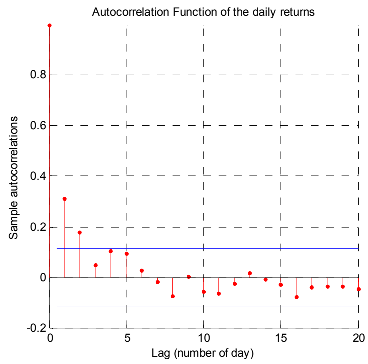
In this study, we quantify the preceding qualitative checks for correlation using formal hypothesis tests, such as the Ljung-Box-Pierce Q-test and Engle's ARCH test. Autoregressive conditional heteroescedasticity (ARCH) models were developed by Engle [10] who also proposed a test that explicitly examines for non-linearity in the second moment i.e. the Engle test examines evidence for non-linearity in the variance.



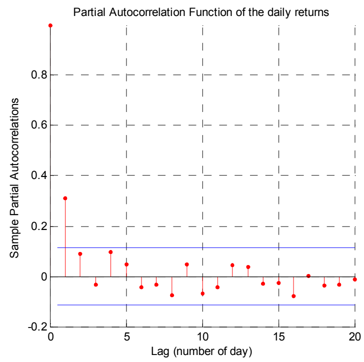
a) BELEXfm



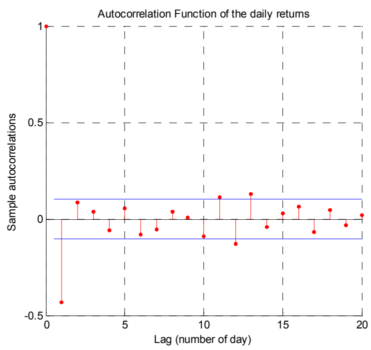
a) BELEXfm



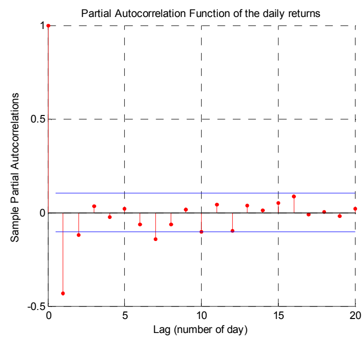
b) BELEX15



b) BELEX15



c) A2007



c) A2007

Fig. 4. Autocorrelation functions

Fig. 5. Partial Autocorrelation functions

We apply Pierce Q-test for a departure from randomness based on the ACF of the data. In this case, however, one can also use it as part of the prefit analysis because the default model assumes that returns are just a simple constant plus a pure innovations process. Under the null hypothesis of no serial correlation, the Q-test statistic is asymptotically Chi-Square distributed [17,24].

The function archtest implements Engle's test for the presence of ARCH effects. Under the null hypothesis that a time series is a random sequence of Gaussian disturbances (i.e., no ARCH effects exist), this test statistic is also asymptotically Chi-Square distributed [10].

Table 3. Ljung-Box and ARCH test

	Ljung-Box Q-statistic			ARCH		
	Test statistic	p-value	Critical value	Test statistic	p-value	Critical value
BELEXfm	49.4722	0.0000	18.3070	56.5566	0.0000	18.3070
BELEX15	19.0744	0.0393	18.3070	7.1004	0.7159	18.3070
A2007	86.2446	0.0000	18.3070	77.5545	0.0000	18.3070

After selecting an ARCH, for the log returns of the three stock exchange indexes, we applied the two tests to the residuals of the ARCH model. The results are reported in Table 3 and allow us to reject the null hypothesis in both cases. This means that we can neither conclude for the absence of an ARCH model using the Engle test (there may be nonlinearity in the variance).

The BDS Test

The BDS test will be applied to the residuals of a fitted linear model, in our case an ARIMA model, which we presumed has extracted as much linear structure as possible from the data.

Using the Box-Jenkins methodology [17] we tried to adjust an ARIMA (autoregressive integrating moving average) process for each of our chosen stock exchange indexes log returns. Briefly, we fitted an ARIMA (p, d, q) model to our time series, where p denotes the number of autoregressive terms, d the number of times the time series has to be differenced before it becomes stationary, and q is the number of moving average terms.

Table 4. BDS Statistics Residuals from the normalized logarithm returns

m=5	ε			
	0.5σ	σ	1.5σ	2σ
BELEXfm	3.8650	4.3270	5.1324	5.9142
BELEX15	3.3782	4.8933	3.8579	1.9373
A2007	16.8943	13.035	9.2066	7.8295

To compute the BDS statistic [16] recommend using ε between one-half to two times the standard deviation of the raw data ($0,5\sigma \leq \varepsilon \leq 2\sigma$) while suggesting that m = 2. For samples with less than 500 observations *m* should be set less or equal to 5.

Table 4 presents the results of the BDS test for the ARIMA residuals of log returns. From this point of view, we can conclude that the null hypothesis of Rt being is rejected, at the 5 percent level, for all indexes. In other words, non-linear dependence is not absent from the series returns and we must therefore conclude that the weak form efficiency hypothesis is not validated for all of the Serbian stock exchange indexes in our data set.

4. Conclusion

In this article, we have focused on certain statistical properties of logarithmic returns of three most representative time series for Belgrade stock market. As we reported here, expected value of all three returns equals zero. None of the three series studied here shows a normal distribution of returns. Besides, all three return time series are nonstationary and exhibit significant autocorrelation. Also, three logarithmic returns are not independent. The hypothesis of linearity is rejected by almost all the tests we computed. In fact, the results allow us to reject the null hypothesis of daily returns. Non-linear dependence present on those returns is therefore contradicting the random walk model supposition. Since the test results indicate that non-linear structure is present in the data, it is possible that exploitable excess profit opportunities may exist in the Serbian stock market.

Most of the stock markets in emerging and developing economies have shown themselves as inefficient, to put it mildly. Significant levels of inefficiency have been found in markets such as India, Singapore, Ghana [3], Mauritius [20,21] and Greece [21], among others; the inefficiency often arises from size of markets, thinness of trading and quality of information disclosure [7,18]. In studies devoted to the Egyptian Stock Exchange [19], it was found that the four best-known daily indices exhibited significant departure from the EMH. There was a tendency for returns to display both volatility clustering and excess kurtosis. Stated reasons for inefficiency included, but were not limited to: limited provision, and inefficient dissemination of information on performance of listed companies, the limited role of professional financial intermediaries and restrictions in the trading process which makes this market a thinly traded one.

Logarithmic returns on the Belgrade Stock Exchange exhibited strong autocorrelation. It was suggested earlier [19] that this could be evidence of time varying risk premiums or just the trading. For some researchers thin trading, which makes most of the shares 'illiquid', was cited as a cause of inefficiency, but for others it was a reason to treat the results with caution [1]. It highlighted thinness of trading as a shortcoming to be considered in adopting the results that the studying stock exchange is inefficient. We could not agree that Belgrade Stock Exchange was efficient even in the weak sense, from the same reasons which were noticed previously studying the time series of emerging and developed markets [7, 21,23]. Previous studies investigated the random walk behaviour of stock returns on four African stock markets: Egypt, Kenya, Morocco and Zimbabwe [7]. On all four markets, the hypothesis that stock returns are normally distributed was rejected. Almost half of the stocks on each of the four markets showed significant positive serial correlation and there was therefore not enough evidence to accept the hypothesis of a random walk. Even with adjusting the returns for thin-trading effect, results continued to show significant departure from the EMH.

In our case of Belgrade stock market it is realistic to suppose that like any emerging market in transitional economies, it starts from an inefficient status and moves toward efficient behaviour. They use a time varying parameter model, which can move from an indicator of inefficiency to efficiency, to assess the efficiency of the Belgrade stock market between 10 January 2005 and 20 November 2006. For the most liquid stocks, the market is initially inefficient, but it took around two and a half years to become efficient. For all the other shares, the overall performance of the market remained predictable over most of the time but there is evidence of tendency toward ongoing efficiency in the last period. One of the reasons why emerging markets are expected to show significant departures from market efficiency is the thin trading that is prevalent in these markets. It would therefore be reasonable to expect the thinly traded stocks to show significantly correlated returns.

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STILIZOVANE ČINJENICE O PRINOSU AKTIVE: SLUČAJ BELEX

Vladimir Miljković, Ognjen Radović

Ovaj rad predstavlja studiju o vremenskim serijama berzanskih indeksa sa Beogradske berze. Izvršene su statističke analize ovih podataka i prikazan je skup stilizovanih empirijskih činjenica. Opisane su različite osobine prinosa na berzanske indekse: odsustvo normalnosti, korelacija u prinosima serija, i rezultati nekih nelinearnih testova. Naši rezultati potvrđuju osobine koje su zajedničke različitim tržištima u nastajanju i tržištima u razvoju. Prikazali smo statističke osobine serija sa Beogradske berze, koje u značajnoj meri odstupaju od uobičajnog statističkog pristupa analizi finansijskih podataka i naglašavaju zapažene statističke probleme.

Ključne reči: *finansijske vremenske serije, finansijska tržišta, Beogradska berza.*