SOME COMPARATIVE CONSIDERATIONS OF REVENUE, ECONOMY, PROFIT, AND PROFITABILITY FUNCTIONS

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Abstract. Optimizing a certain emergent form of results and investments, involved in indicators of economy and profitability and, consequently, maximizing their relation, are both in the function of optimal efficiency of enterprise. Hence, it is especially important to track down the functional interdependence between revenue, economy, profit, and profitability, expressed in the function of market accepted product scope. Theoretical, economic, and methodological aspects of comparative consideration of revenue, economy, profit, and profitability functions and their interdependence are in the focus of the authors' analysis.

Numerous and various indicators of level of success in achieving the set objectives of an enterprise indicate problems complexity in estimating efficiency of its business activities. Each indicator expresses an aspect of business success of enterprise and partially measures its total efficiency. Market valorization of business success emphasizes the importance of economy and profitability criteria in the set of partial criteria of enterprise's efficiency. The economy, as an economically relevant relation between produced value and costs of its production, expresses the level of utility of investments in the form of cost. The general concept of profitability is based on the attitude that it is measured by the profit and committed resources (capital) ratio, as an economically relevant expression of enterprise's efficiency. Optimizing a certain form of results and investments, involved in indicators of economy and profitability and, consequently, maximizing their relation is in the function of optimal efficiency of enterprise. Therefore, it is important to track down the functional interdependence between produced value (revenue), economy, profit, and profitability, expressed in the function of product scope.

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1. ECONOMIC ASPECT OF COMPARATIVE CONSIDERATIONS

The principle of economy, as a partial economic principle, is determined according to meaning of economically relevant relations between produced value and cost of production factors. If it represents a request for producing certain value with minimal valued consumption of production elements, it follows that economic success, achieved in this way, is expressed as a quotient between produced value and costs of its production \[1, p. 247\]. The level of economy, of course, will be higher if the span between the value and the cost of production elements is higher. This creates possibilities for increasing consumption in future periods and achieving a higher level of utility of investments in the form of cost. The utility is reflected in the ability of an enterprise to perform its business activities and continually develop itself. The enterprise's development, viewed in the context of economy, is possible if there is a positive difference between produced and consumed values, which is manifested in the form of accumulation. This also expresses a certain level of accumulative capability of enterprise, which, on the other hand, expresses the economic essence of economy, as the measure of enterprise's efficiency.

The level of economy, as a quotient between produced value and cost of production elements is, practically, a function of the elements' size. The determination of their quantitative proportions defines the character of their functional interdependence. Proceeding from the assumption about certain functional dependence between produced value and cost of production elements, with no methodological problems in value expressing, the solution for maximal or optimal economy could be found. However, the fact is that it is very difficult to determine the theoretical value, since it is often unknown to the enterprise. Consequently, and starting from the market conditions, for analytical purposes it is important to determine possible functional dependence between cost \(T\) and revenue \(C\), that is, to track down cost on realized revenue.

If production in the indicator of economy is expressed by its market price, or revenue, then economy \(E\) is determined as a quotient between revenue \(C\) and cost of reproduction \(T\), which can be expressed as

\[
E = \frac{C}{T}. \quad (1.1)
\]

The economy expressed in this way is often used in economic practice. Methodologically, this is a way of solving the problem of unique expression of heterogeneous production and, additionally, every enterprise has information about the price of its products. Economically, this expression of economy does not change its economic essence since it expresses accumulative ability of the enterprise and utility of investments in the form of cost, including both production and exchange phase. In spite of it, under the relatively stable business conditions and a stable level of market price of products and services, the economy expressed as (1.1) gives important information for determining product scope.

\[1\] The economy, expressed in this way, is analyzed in more details in the authors' paper on "Theoretical and Methodological Aspects of the Economy Functions" ("Teorijsko-metodološki aspekti funkcije ekonomičnosti"), *Ekonomsko teme*, 3/2002, pp.101-113.
which is needed for profit maximization. This is a solid basis for making numerous relevant business decisions, both in current period and in dynamics.

The general concept of profitability of an enterprise is based on the attitude that it is measured by relation between profit $D$ and committed resources (capital) $S$, as an economic relevant expression of the enterprise's efficiency [1, p. 292] Therefore, profitability $R$ can be expressed as

$$R = \frac{D}{S}$$

Representing certain relation between effects and investments, the indicators of profitability, as a rule, depend on the achieved results of business activities in the form of achieved profit and rational use of invested resources. The indicators are often regarded as relative measures, in the form of percent of profit per invested sources unit value, or as percentage return of enterprise. Starting with profit, as the motive of performing business activities, it is natural that a market oriented enterprise tends to maximize it with minimal resource investment, so it uses the indicators of profitability to estimate achieved business success.

All activities of the enterprise permeate the indicators of profitability. Hence, in an economic analysis, the profitability is the most important expression in relation to the three basic criteria of expressing and evaluating economic efficiency or quality of performing business activities (productivity, economy, profitability). Although the profitability criterion is mainly characterized as a partial measure of efficiency, it is, in some extent, an aggregate expression with attributes of total business success measure. The profitability, as a rule of business behavior and measure of rational business functioning, has a great importance both for enterprise and society as a whole. A higher level of profitability sets the condition for extended reproduction and, consequently, more successful satisfaction of common social needs. All of this makes the profitability a complex and universal measure of efficiency in managing enterprise as well as an indicator of macroeconomic development.

There is a high level of interdependence between economy and profitability, as two partial measures of business efficiency of an enterprise [1, p. 296.] The level of economy significantly influences the level of profitability and, conversely, the level of achieved profitability has a great impact upon the level of economy, especially in business activity dynamics. The relationship between these indicators is also reflected in functional interdependence between their elements. Each change in costs - total and per product unit, influences the level of economy and profitability. The character of changes is the same, but the effects of their influence are different. Namely, cost reduction is mostly related to economy and profitability increasing. Conversely, each cost increase leads to economy and profitability decrease. However, costs, in their total value, are the denominator in economy expression, but, in profitability expression, costs are both a factor of numerator - profit and a denominator - average committed resources (capital).

\[\text{The profitability is analyzed in more details in the co-author's paper on "Some Theoretical and Methodological Aspects of the Rentability Function" ("Neki teor ijsko-metodolo{ki aspekti funkcije rentabilnosti"), Ekonomske teme, 3/2003, pp. 111-120.}\]
Changes in the product scope have direct impact upon the level of economy through unit cost dynamics. These changes, however, have indirect impact upon profitability - they influence the level of both costs and revenue. Changes in the product prices only influence profitability, if the economy is expressed by unit cost. However, if the economy is expressed by a quotient between revenue and cost of production elements, the changes in product prices are a factor both of economy and profitability and induce changes in their expressions in the same direction, but in different proportions, since the revenue is an element of profit in the profitability expression.

The analysis of economy and profitability functions indicates contribution of certain elements, factors, activities, and participants in creating indicators of economy and profitability. The determined elements of economy and profitability functions have important managerial and control role. The managerial role is related to the goal of analysis of function flow since the control role is primarily reflected in controlling success in the processes of function optimization.

The flow of economy and profitability functions is projected by extrapolation on a certain future period, proceeding dynamics of activities and factor influences from the previous business period. The importance of the functions will be greater if it is possible to make a comparative analysis with other enterprises, especially with the average one or the best competitive enterprise in branch. The economy and profitability functions provide the basis for certain changes in indicators of economy and profitability, as well as changes in their elements and factors, which have to be realized for improving economic power of enterprise and achieving its continual growth and development.

2. METHODOLOGICAL ASPECT OF COMPARATIVE CONSIDERATIONS

Let A be an empirical (discrete) set of values
\[ A = \{ (p_i, q_i) | p_i \in [0, b], b > 0, q_i > 0, i = 1, 2, \ldots, n \} , \]
where \( p_i \) is the price and \( q_i \) is the demand for product \( Q \), which determines proposed dependence between these economic dimensions. Let us denote with
\[ q = f(p) \]
the demand function, obtained by smoothing out these data. Taking into consideration that demand function (2.1) is bijective one, the function has its inverse demand function
\[ p = \varphi(q) , \]
so that total revenue \( C \) could be expressed as the function of product scope (demand) \( q \) and we obtain that
\[ C = q\varphi(q) = C(q) . \]
For the inverse demand function, expressed by relation (2.2), it is valid
\[ (\forall q \in [0, b]) \; \varphi(q) > 0 \quad \text{and} \quad (\forall q \in (0, b)) \; \varphi'(q) < 0 , \]
where \([0, b], b > 0\), is the final interval from the set of real numbers, which corresponds to empirical set of values \( A \), and on which the demand function is defined [2, p. 304].
If the demand function has been defined in segment \([0, b] \), \( b > 0 \), then the total revenue function \( C \), expressed by formula (2.3), is defined in the same segment. For demand \( q_0 \in (0, b) \), where

\[
q_0 = -\frac{\varphi(q_0)}{\varphi'(q_0)} > 0, \tag{2.5}
\]

since, according to relation (2.4), \( \varphi(q_0) > 0 \) and \( \varphi'(q_0) < 0 \), the total revenue function \( C \) has its maximum, i.e. it achieves maximal revenue \([2, pp. 310-312]\)

\[
C_{\text{max}} = C(q_0) = -\frac{(\varphi(q_0))^2}{\varphi'(q_0)} > 0. \tag{2.6}
\]

According to (2.5) and (2.6), point \( M_C(q_0, C(q_0)) \) is the point of maximum on the graph of revenue function \( C \), expressed by formula (2.3).

The graph of revenue function expressed by formula (2.3) is shown in Figure 1.

Let \( B \) be an empirical (discrete) set of values

\[
B = \{(q_i, T_i) \mid q_i \in [0, b], b > 0, T_i > 0, i = 1, 2, \ldots, n \},
\]

where \( q_i \) is the scope of product \( Q \) and \( T_i \) denotes the total cost of product \( Q \), which determines proposed dependence between these economic dimensions. Smoothing out the data, we obtain the total cost function

\[
T = T(q). \tag{2.7}
\]

For the total cost function, expressed by formula (2.7), it is valid

\[
(\forall q \in [0, b]) \ T(q) > 0 \text{ and } (\forall q \in (0, b)) \ T'(q) > 0, \tag{2.8}
\]

where \([0, b], b > 0\), is the final interval from the set of real numbers, in which the cost function is defined, and \( T'(q) \) is the derivative of the function.

Starting from the total revenue function, expressed by (2.3), and the total cost function, expressed by (2.7), the economy function can be obtained, namely
which is defined in interval \([0,b]\), \(b > 0\) [3, pp. 107-109]. For demand \(q_1 \in (0,b)\), where

\[
q_1 = \frac{\phi(q_1)T(q_1)}{\phi(q_1)T'(q_1) - \phi'(q_1)T(q_1)} > 0,
\]

since, according to (2.4) and (2.8), \(\phi(q_1) > 0\), \(T(q_1) > 0\), \(\phi'(q_1) < 0\), \(T'(q_1) > 0\), maximal economy is achieved

\[
E_{\text{max}} = G(q_1) = \frac{\phi^2(q_1)}{\phi(q_1)T'(q_1) - \phi'(q_1)T(q_1)} > 0.
\]

On the basis of (2.10) and (2.11), point \(M_0(q_1,G(q_1))\) is the point of maximum on the graph of economy function, expressed by formula (2.9).

For the total revenue function, expressed by formula (2.3), and the economy function, expressed by formula (2.9), we conclude that they are defined in the same interval \([0,b]\), \(b > 0\). Graphically, these functions could be expressed in the same coordinate system. For that purpose, we will test sizes of demand \(q_0\) and \(q_1\).

Let us prove inequality \(q_1 \neq q_0\). The proof is presented indirectly. Namely, let us assume contrary, i.e.

\[
q_1 = q_0. \tag{2.12}
\]

From relations (2.12), (2.5), and (2.10), we obtain

\[
\frac{\phi(q_1)}{\phi'(q_1)} = \frac{\phi(q_1)T(q_1)}{\phi(q_1)T'(q_1) - \phi'(q_1)T(q_1)} \Rightarrow T'(q_1) = 0,
\]

this is in contradiction with (2.8), where \(T'(q_1) > 0\). Accordingly, assumption (2.12) brings to contradiction, which proves inequality \(q_1 \neq q_0\).

Let us prove the validity of inequality

\[
q_1 < q_0. \tag{2.13}
\]

The first derivative of economy function is

\[
E' = G'(q) = \frac{1}{T^2(q)}(\phi(q)T'(q) - q(\phi(q)T'(q) - \phi'(q)T(q))), \tag{2.14}
\]

and

\[
G'(q) < 0 \text{ for } q \in (q_1,b),
\]

that is, the economy function in interval \((q_1,b)\) is monotonously decreasing [3, p. 109]. Substituting \(q_0\), expressed by formula (2.5), in (2.14), we obtain

\[
G'(q_0) = \frac{\phi^2(q_0) T'(q_0)}{T^2(q_0) \phi'(q_0)} < 0,
\]
since, according to (2.8), \( T'(q_0) > 0 \), and, according to (2.4), \( \varphi'(q_0) < 0 \). Therefore, demand \( q_0 \), for which the total revenue is maximal, is in interval where the economy function is monotonously decreasing, i.e. \( q_0 \in (q_1,b) \), so inequality (2.13) is proved.

For demand \( q_1 \), expressed by formula (2.10), from formulas (2.11), (2.9), and (2.3), we obtain that

\[
E_{\text{max}} = G(q_1) = \frac{C(q_1)}{T(q_1)} < C(q_1),
\]

that is, the maximal economy is lower that the total revenue, achieved for demand \( q_1 \).

The graph of economy function, expressed by formula (2.9), is also shown in Figure 1, where the graph of total revenue function is.

Starting from formulas (2.3) and (2.7), the profit function could be derived, namely,

\[
D = C - T = q\varphi(q) - T(q) = D(q).
\]

Taking into consideration that functions (2.3) and (2.7) are defined in segment \([0,b]\), \( b > 0 \), then the profit function, expressed by formula (2.15), is defined in interval \((\bar{q}_1,\bar{q}_2) \subset (0,b), b > 0 \), for which it is valid

\[
(\forall q \in (\bar{q}_1,\bar{q}_2))C(q) > T(q),
\]

where \( \bar{q}_1,\bar{q}_2 \in (0,b) \) are the solutions of equation \( D(q) = 0 \).

A necessary condition for profit function (2.15) to have maximum is

\[
D'(q) = C'(q) - T'(q) = 0, \quad \text{or} \quad C'(q) = T'(q),
\]

i.e., marginal revenue has to be equal to marginal cost. If with \( \bar{q} \in (\bar{q}_1,\bar{q}_2) \) we label the solution of equation (2.17), that is,

\[
\bar{q} = \frac{\varphi(\bar{q}) - T'(\bar{q})}{\varphi'(\bar{q})} > 0,
\]

since, according to (2.4), \( \varphi'(\bar{q}) < 0 \), and \( \varphi(\bar{q}) - T'(\bar{q}) > 0 \), then a sufficient condition for profit to be maximal is \( D'(\bar{q}) < 0 \). From (2.15) and (2.18), we obtain that

\[
D_{\text{max}} = D(\bar{q}) = C(\bar{q}) - T(\bar{q}) > 0,
\]

since, according to (2.16), \( C(\bar{q}) > T(\bar{q}) \).

Consequently, on the basis of (2.18) and (2.19), point \( M_p(\bar{q},D(\bar{q})) \) is the point of maximum on the graph of profit function \( D \).

The graph of profit function, which is expressed by relation (2.15), is shown in Figure 2.

Starting from profit function (2.15) and the function of average committed resources

\[
S = \beta T(q),
\]

where \( \beta, \beta > 0 \), is the coefficient of committed resources, and \( T = T(q) \) is the total cost, we define \textit{profitability function} [4, pp. 114-117]:
The profitability function, expressed by (2.20), is defined in the same segment as the profit function, that is, in segment \((q_1, q_2) \subseteq (0, b), b > 0\). For \(q_i \in (q_1, q_2)\), where
\[
q_i = \frac{\varphi(q_i)T(q_i)}{\varphi(q_i)T'(q_i) - \varphi'(q_i)T(q_i)} > 0,
\]
since, according to (2.4) and (2.8), \(\varphi(q_i) > 0, T(q_i) > 0, \varphi'(q_i) < 0, T'(q_i) > 0\), function (2.20) has its maximum, i.e., maximal profitability is achieved, namely,
\[
R_{\text{max}} = F(q_i) = \frac{1}{\beta} \left( \frac{\varphi(q_i)}{T(q_i) - 1} \right).
\]

According to (2.21) and (2.22), point \(M_{\text{d}}(q_1, F(q_1))\) is the point of maximum on the graph of profitability function \(R\).

Taking into consideration that the profit and the profitability functions are defined in the same interval, i.e. in profitability interval \((q_1, q_2) \subset (0, b)\), we will show their graphs in the same coordinate system. For that purpose, we will test sizes of demand \(q_1\) and \(q_2\).

Let us prove inequality \(q_1 \neq q\). The proof is presented indirectly. Namely, let us assume contrary, i.e.
\[
q_1 = q = \bar{q}.
\]

From relations (2.23), (2.21), and (2.18), we obtain
\[
-\frac{\varphi(q) - T'(\bar{q})}{\varphi'(\bar{q})} = \frac{\varphi(\bar{q})T(\bar{q})}{\varphi(\bar{q})T'(\bar{q}) - \varphi'(\bar{q})T(\bar{q})} \Rightarrow T'(\bar{q}) = 0,
\]
which is in contradiction with (2.8), where \(T'(\bar{q}) > 0\). Accordingly, assumption (2.23) brings to contradiction, which proves inequality \(q_1 \neq q\).

Let us prove the validity of inequality
\[
R = \frac{\varphi(q) - T(q)}{\beta T(q)} = \frac{1}{\beta} \left( \frac{\varphi(q)}{T(q) - 1} \right) = F(q).
\]

\(q_i\) are the points where the function \(R\) is defined in the same segment as the profit function, that is, in segment \((q_1, q_2) \subset (0, b), b > 0\).
Some Comparative Considerations of Revenue, Economy, Profit, and Profitability Functions

$q_1 < \bar{q}$. \hfill (2.24)

The first derivative of profitability function is

$$R' = F'(q) = \frac{1}{\beta T'^2(q)} (\varphi(q)T(q) - q(\varphi(q)T'(q) - \varphi'(q)T(q))),$$ \hfill (2.25)

where

$$F'(q) < 0 \text{ for } q \in (q_1, \bar{q}_2),$$

i.e., the profitability function in interval $(q_1, \bar{q}_2)$ is monotonously decreasing [4, p. 116].

Replacing $\bar{q}$, which is expressed by formula (2.18), in relation (2.25), we obtain

$$F'(\bar{q}) = \frac{T'(\bar{q})}{\beta T'(\bar{q})} (\varphi(\bar{q}) - 1) < 0,$$

because $\beta > 0$, $T'(\bar{q}) > 0$, $T'(\bar{q}) > 0$, and $\varphi(\bar{q}) > 0$, and $\bar{q}_1 < \bar{q} < \bar{q}_2$. Consequently, demand $\bar{q}$, for which the profit is maximal, is in interval where the profitability function is monotonously decreasing, i.e. $\bar{q} \in (q_1, \bar{q}_2)$, so inequality (2.24) is proved.

Replacing demand $q_1$, expressed by formula (2.21), in (2.20), (2.22), and (2.15), we obtain

$$R_{\text{max}} = F(q_1) = \frac{D(q_1)}{\beta T(q_1)} < D(q_1),$$

that is, the maximal profitability is lower than profit achieved for demand $q_1$.

The graph of profitability function, expressed by formula (2.20), is also shown in Figure 2, together with the graph of profit function.

For showing all analyzed functions in the same coordinate system, it is also necessary to prove:

$$q \neq q_0,$$ \hfill (2.26)

$$q < q_0,$$ \hfill (2.27)

$$R_{\text{max}} = F(q_1) < E_{\text{max}} = G(q_1).$$ \hfill (2.28)

The proof of relation (2.26) is presented indirectly. Namely, let us suppose contrary, i.e. that

$$q_0 = \bar{q}.$$ \hfill (2.29)

On the basis of relations (2.5), (2.18), and (2.29), we obtain

$$-\frac{\varphi(q_0)}{\varphi'(q_0)} - \frac{T'(q_0)}{\varphi'(q_0)} \iff 0 = \frac{T'(q_0)}{\varphi'(q_0)} \iff T'(q_0) = 0,$$

which is impossible, since, according to (2.8), $T'(q_0) > 0$. Therefore, assumption (2.29) brings to contradiction, and relation (2.26) is proved.

For proving inequality (2.27), we will analyze the first derivative of total revenue function
\[
C'(q) = \varphi(q) + q\varphi'(q) \cdot (2.30)
\]

For \( q \in (0, q_0) \) it follows \( C'(q) > 0 \), i.e., the total revenue function is monotonously increasing function. For the demand \( \overline{q} \), expressed by formula \( (2.18) \), maximal profit is achieved, and, according to \( (2.17) \), equality \( C'(\overline{q}) = T'(\overline{q}) \) follows. Since, according to \( (2.8) \), \( T'(\overline{q}) > 0 \), it also follows that \( C'(\overline{q}) > 0 \). Consequently, demand \( \overline{q} \) is in interval \( (0, q_0) \), where the total revenue function is monotonously increasing, i.e. it is valid that \( 0 < \overline{q} < q_0 \). In this way, inequality \( (2.27) \) is proved.

It is remain to prove relation \( (2.28) \). Indeed, on the basis of relations \( (2.10), (2.11), (2.21), \) and \( (2.22) \), we conclude that maximal economy and maximal profitability are achieved for the same demand \( q_1 \) and that \( R_{\text{max}} < E_{\text{max}} \).

According to relations \( (2.24) \) and \( (2.27) \), it follows

\[
q_1 < \overline{q} < q_0,
\]

where \( q_1, \overline{q}, \) and \( q_0 \) belong to interval of profitability \( (\overline{q}_1, \overline{q}_2) \subset (0, b), b > 0 \).

In accordance with previous considerations, the graphs of revenue, profit, economy, and profitability functions could be shown in the same coordinate system as illustrated in Figure 3.

3. INSTEAD OF CONCLUSION

The comparative analysis of revenue, economy, profit, and profitability functions regarding the methodological aspect and derived conclusions has appropriate and very important economic implications. For the management of an enterprise it is significant to know which accepted market scope of products will bring them maximal revenue and profit as well as maximize the indicators of economy and profitability. The fact that all these product quantities are in the interval (zone) of profitability helps managers create such plans and follow such product and business policy which will keep productive effects and business results in setting zone of profitable business activities in a relatively long period of time.
If the achieved revenue and profit are not at the level of maximum value, the enterprise management can improve them by adequate measures and instruments, primarily by increasing product volume and quality, by performing marketing activities in more qualitative ways, by lowering total and particular costs, or reorganizing the enterprise itself. This will, as a rule, result in increasing partial indicators of the enterprise's business efficiency, namely, economy and profitability. Taking into consideration the fact that the maximal value of these partial indicators is attained for the same product scope (demand), the task of the planners who seek target scope (demand) is substantially made easier. It is also easier to use "what if" analysis, that is, the analysis of economy and profitability dynamics when the current demand deviates from the target or optimal level. In that situation, the enterprise management has to decide which variant is most promising for its long-term success in performing business activities.

The market accepted scope of product would be different, depending on the priorities in the hierarchy of an enterprise's objectives. The most realized quantity of product will be required for achieving maximal revenue, until economy and profitability maximization will require a lower level of product realization. Which target scope of product that could be realized on the market will be included in business plans primarily depends on which objective of enterprise is preferred in the current business period.

REFERENCES


**NEKA UPOREDNA RAZMATRANJA FUNKCIJA PRIHODA, EKONOMIČNOSTI, DOBITI I RENTABILNOSTI**

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Optimizacija određenih pojava oblika rezultata i ulaganja, obuhvaćenih pokazateljima ekonomičnosti i rentabilnosti i, posledično, maksimiranje njihovog odnosa u funkciji je ostvarivanja optimalne efikasnosti preduzeća. Stoga je od posebnog značaja pratiti funkcionalnu međuzavisnost prihoda, ekonomičnosti, dobiti i rentabilnosti, izraženih u funkciji tržišno priznatog i prihvatljivog obima proizvoda. Teorijski, ekonomski i metodološki aspekti uporednih razmatranja funkcija prihoda, ekonomičnosti, dobiti i rentabilnosti i njihove međuzavisnosti u fokusu su analize autora rada.