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# Modulation-Mode and Power Assignment in Broadband MIMO Systems

#### Andreas Ahrens and César Benavente-Peces

**Abstract:** Since the capacity of multiple-input multiple-output (MIMO) systems increases linearly with the minimum number of antennas at both, the transmitter as well as the receiver side, MIMO schemes have attracted a lot of attention. However, non-frequency selective MIMO links have reached a state of maturity. By contrast, frequency selective MIMO links require substantial further research, leading in this contribution to a joint optimization of the number of activated MIMO layers and the number of bits per symbol along with the appropriate allocation of the transmit power under the constraint of a given fixed data throughput. Our results show that in order to achieve the best possible bit-error rate, not necessarily all MIMO layers have to be activated.

**Keywords:** Multiple-Input Multiple-Output System, Singular-Value Decomposition, Bit Allocation, Power Allocation, Wireless Transmission.

#### 1 Introduction

In order to comply with the demand on increasing available data rates in particular in wireless technologies, systems with multiple transmit and receive antennas, also called MIMO systems (multiple-input multiple-output), have become indispensable and can be considered as an essential part of increasing both the achievable capacity and integrity of future generations of wireless systems [3, 4].

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This paper was presented in parts at the International Conference on Wireless Information Networks and Systems (WINSYS) [1] as well as at the Personal, Indoor and Mobile Radio Communications Symposium (PIMRC) [2].

In general, the most beneficial choice of the number of activated MIMO layers and the number of bits per symbol along with the appropriate allocation of the transmit power offer a certain degree of design freedom, which substantially affects the performance of MIMO systems. The well-known water-filling technique is virtually synonymous with adaptive modulation [5–10] and it is used for maximizing the overall data rate. However, delay-critical applications, such as voice or video streaming transmissions, may require a certain fixed data rate. For these fixed-rate applications it is desirable to design algorithms, which minimize the bit-error rate (BER) at a given fixed data rate.

Assuming perfect channel state information, the channel capacity can only be achieved by using water-pouring procedures. However, in practical application only finite and discrete rates are possible. Therefore in this contribution the efficiency of fixed transmission modes is studied regardless of the channel quality. However, non-frequency selective MIMO links have attracted a lot of research and have reached a state of maturity [11]. By contrast, frequency selective MIMO links require substantial further research, where spatio-temporal vector coding (STVC) introduced by RALEIGH seems to be an appropriate candidate for broadband transmission channels [12, 13]. Here it can be shown that multipath propagation is no longer a limiting factor in data transmission [14].

Against this background, the novel contribution of this paper is that we demonstrate the benefits of amalgamating a suitable choice of activated MIMO layers and number of bits per symbol along with the appropriate allocation of the transmit power under the constraint of a given data throughput.

The remaining part of this paper is organized as follows: Section 2 introduces the system model and the considered quality criteria are briefly reviewed in section 3. The proposed solutions of bit and power allocation are discussed in section 4, while the associated performance results are presented and interpreted in section 5. Section 6 provides some concluding remarks.

### 2 Broadband MIMO System Model

When considering a frequency selective SDM (space division multiplexing) MIMO link, composed of  $n_{\rm T}$  transmit and  $n_{\rm R}$  receive antennas, the block-oriented system is modelled by

$$\mathbf{u} = \mathbf{H} \cdot \mathbf{c} + \mathbf{w} \quad . \tag{1}$$

In (1), **c** is the  $(N_T \times 1)$  transmitted signal vector containing the complex input symbols transmitted over  $n_T$  transmit antennas in K consecutive time slots, i. e.,  $N_T = K n_T$ . This vector can be decomposed into  $n_T$  antenna-specific signal vectors

 $\mathbf{c}_{\mu}$  according to

$$\mathbf{c} = \left(\mathbf{c}_1^{\mathrm{T}}, \dots, \mathbf{c}_{\mu}^{\mathrm{T}}, \dots, \mathbf{c}_{n_{\mathrm{T}}}^{\mathrm{T}}\right)^{\mathrm{T}} . \tag{2}$$

In (2), the  $(K \times 1)$  antenna-specific signal vector  $\mathbf{c}_{\mu}$  transmitted by the transmit antenna  $\mu$  (with  $\mu = 1, \dots, n_T$ ) is modelled by

$$\mathbf{c}_{\mu} = \left(c_{1\mu}, \dots, c_{k\mu}, \dots, c_{K\mu}\right)^{\mathrm{T}} . \tag{3}$$

The  $(N_R \times 1)$  received signal vector  $\mathbf{u}$ , defined in (1), can again be decomposed into  $n_R$  antenna-specific signal vectors  $\mathbf{u}_v$  (with  $v = 1, ..., n_R$ ) of the length  $K + L_c$ , i. e.,  $N_R = (K + L_c) n_R$ , and results in

$$\mathbf{u} = \left(\mathbf{u}_{1}^{\mathrm{T}}, \dots, \mathbf{u}_{\nu}^{\mathrm{T}}, \dots, \mathbf{u}_{n_{\mathrm{D}}}^{\mathrm{T}}\right)^{\mathrm{T}} . \tag{4}$$

By taking the  $(L_c+1)$  non-zero elements of the resulting symbol rate sampled overall channel impulse response between the  $\mu$ th transmit and  $\nu$ th receive antenna into account, the antenna-specific received vector  $\mathbf{u}_{\nu}$  has to be extended by  $L_c$  elements, compared to the transmitted antenna-specific signal vector  $\mathbf{c}_{\mu}$  defined in (3). The  $((K+L_c)\times 1)$  signal vector  $\mathbf{u}_{\nu}$  received by the antenna  $\nu$  (with  $\nu=1,\ldots,n_R$ ) can be constructed, including the extension through the multipath propagation, as follows

$$\mathbf{u}_{V} = (u_{1V}, u_{2V}, \dots, u_{(K+L_{c})V})^{\mathrm{T}} .$$
 (5)

Similarly, in (1) the  $(N_R \times 1)$  noise vector w results in

$$\mathbf{w} = \left(\mathbf{w}_1^{\mathsf{T}}, \dots, \mathbf{w}_{\nu}^{\mathsf{T}}, \dots, \mathbf{w}_{n_{\mathsf{R}}}^{\mathsf{T}}\right)^{\mathsf{T}} . \tag{6}$$

The vector **w** of the additive, white Gaussian noise (AWGN) is assumed to have a variance of  $U_R^2$  for both the real and imaginary parts and can still be decomposed into  $n_R$  antenna-specific signal vectors  $\mathbf{w}_V$  (with  $V = 1, \dots, n_R$ ) according to

$$\mathbf{w}_{v} = (w_{1v}, w_{2v}, \dots, w_{(K+L_{c})v})^{\mathrm{T}} . \tag{7}$$

Finally, the  $(N_R \times N_T)$  system matrix **H** of the block-oriented system model, introduced in (1), results in

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \dots & \mathbf{H}_{1n_{\mathrm{T}}} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{n_{\mathrm{R}}1} & \dots & \mathbf{H}_{n_{\mathrm{R}}n_{\mathrm{T}}} \end{bmatrix} , \qquad (8)$$

and consists of  $n_R \cdot n_T$  single-input single-output (SISO) channel matrices  $\mathbf{H}_{\nu\mu}$  (with  $\nu = 1, ..., n_R$  and  $\mu = 1, ..., n_T$ ). The system description, called spatiotemporal vector coding (STVC) was introduced by RALEIGH and can also be seen

as an extension of the data directed estimation (DDE) introduced by HsU [15]. Each of theses matrices  $\mathbf{H}_{v\mu}$  with the dimension  $((K+L_c)\times K)$  describes the influence of the channel from transmit antenna  $\mu$  to receive antenna v including transmit and receive filtering. The channel convolution matrix  $\mathbf{H}_{v\mu}$  between the  $\mu$ th transmit and vth receive antenna is obtained by taking the  $(L_c+1)$  non-zero elements of resulting symbol rate sampled overall impulse response into account and results in:

$$\mathbf{H}_{\nu\mu} = \begin{bmatrix} h_0 & 0 & 0 & \cdots & 0 \\ h_1 & h_0 & 0 & \cdots & \vdots \\ h_2 & h_1 & h_0 & \cdots & 0 \\ \vdots & h_2 & h_1 & \cdots & h_0 \\ h_{L_c} & \vdots & h_2 & \cdots & h_1 \\ 0 & h_{L_c} & \vdots & \cdots & h_2 \\ 0 & 0 & h_{L_c} & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & h_{L_c} \end{bmatrix} . \tag{9}$$

Throughout this paper, it is assumed that the  $(L_{\rm c}+1)$  channel coefficients, between the  $\mu$ th transmit and  $\nu$ th receive antenna have the same averaged power and undergo a Rayleigh distribution. Furthermore, a block fading channel model is applied, i. e., the channel is assumed to be time invariant for the duration of one SDM MIMO data vector.

The interference between the different antenna's data streams, which is introduced by the off-diagonal elements of the channel matrix  $\mathbf{H}$ , requires appropriate signal processing strategies. Common strategies for separating the data streams are linear equalization at the receiver side or linear pre-equalization at the transmitter side. Unfortunately, linear equalization suffers from noise enhancement and linear pre-equalization of the transmit signal from an increase in the transmit power. Both schemes only offer poor power efficiency. Therefore, other signal processing strategies have attracted a lot of interest. A popular technique is based on the singular-value decomposition (SVD) [16] of the system matrix  $\mathbf{H}$ , which can be written as  $\mathbf{H} = \mathbf{S} \cdot \mathbf{V} \cdot \mathbf{D}^H$ , where  $\mathbf{S}$  and  $\mathbf{D}^H$  are unitary matrices and  $\mathbf{V}$  is a real-valued diagonal matrix of the positive square roots of the eigenvalues of the matrix  $\mathbf{H}^H \mathbf{H}$  sorted in descending order<sup>1</sup>. The SDM MIMO data vector  $\mathbf{c}$  is now multiplied by the matrix  $\mathbf{D}$  before transmission. In turn, the receiver multiplies the received vector  $\mathbf{u}$  by the matrix  $\mathbf{S}^H$ . Thereby neither the transmit power nor the noise power is enhanced. The overall transmission relationship is defined as

$$\mathbf{y} = \mathbf{S}^{\mathrm{H}} \left( \mathbf{H} \cdot \mathbf{D} \cdot \mathbf{c} + \mathbf{w} \right) = \mathbf{V} \cdot \mathbf{c} + \tilde{\mathbf{w}}. \tag{10}$$

<sup>&</sup>lt;sup>1</sup>The transpose and conjugate transpose (Hermitian) of  $\mathbf{D}$  are denoted by  $\mathbf{D}^{T}$  and  $\mathbf{D}^{H}$ , respectively.

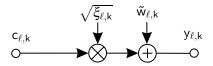


Fig. 1. Resulting layer-specific SDM MIMO system model (with  $\ell = 1, 2, ..., L$  and k = 1, 2, ..., K)

As a consequence of the processing in (10), the channel matrix **H** is transformed into independent, non-interfering layers having unequal gains.

# 3 Quality Criteria

In general, the quality of data transmission can be informally assessed by using the signal-to-noise ratio (SNR) at the detector's input defined by the half vertical eye opening and the noise power per quadrature component according to

$$\rho = \frac{(\text{Half vertical eye opening})^2}{\text{Noise Power}} = \frac{(U_{\text{A}})^2}{(U_{\text{R}})^2} , \qquad (11)$$

which is often used as a quality parameter [17]. The relationship between the signal-to-noise ratio  $\rho = U_{\rm A}^2/U_{\rm R}^2$  and the bit-error probability evaluated for AWGN channels and *M*-ary Quadrature Amplitude Modulation (QAM) is given by [18,19]

$$P_{\rm BER} = \frac{2}{\log_2(M)} \left( 1 - \frac{1}{\sqrt{M}} \right) \, {\rm erfc} \left( \sqrt{\frac{\rho}{2}} \right) \ . \tag{12}$$

When applying the proposed system structure, the SVD-based equalization leads to different eye openings per activated MIMO layer  $\ell$  (with  $\ell = 1, 2, \dots, L$ ) at the time k (with  $k = 1, 2, \dots, K$ ) within the SDM MIMO signal vector according to

$$U_{\mathbf{A}}^{(\ell,k)} = \sqrt{\xi_{\ell,k}} \cdot U_{\mathbf{s}\ell} \quad , \tag{13}$$

where  $U_{s\ell}$  denotes the half-level transmit amplitude assuming  $M_{\ell}$ -ary QAM and  $\sqrt{\xi_{\ell,k}}$  represents the corresponding positive square roots of the eigenvalues of the matrix  $\mathbf{H}^H\mathbf{H}$  (Fig. 1). Together with the noise power per quadrature component, the SNR per MIMO layer  $\ell$  at the time k becomes

$$\rho^{(\ell,k)} = \frac{\left(U_{\rm A}^{(\ell,k)}\right)^2}{U_{\rm R}^2} = \xi_{\ell,k} \frac{\left(U_{\rm s}\ell\right)^2}{U_{\rm R}^2} \ . \tag{14}$$

Using the parallel transmission over  $L \le \min(n_T, n_R)$  MIMO layers, the overall mean transmit power becomes  $P_s = \sum_{\ell=1}^{L} P_{s\ell}$ , where the number of readily separable

layers<sup>2</sup> is limited by  $\min(n_T, n_R)$ . Considering QAM constellations, the average transmit power  $P_{s\ell}$  per MIMO layer  $\ell$  may be expressed as [21,22]

$$P_{\rm s}\ell = \frac{2}{3}U_{\rm s}^2(M_\ell - 1) \ . \tag{15}$$

Combining (14) and (15), the layer-specific SNR at the time k results in

$$\rho^{(\ell,k)} = \xi_{\ell,k} \frac{3}{2(M_{\ell} - 1)} \frac{P_{s\ell}}{U_{R}^{2}} . \tag{16}$$

In order to transmit at a fixed data rate while maintaining the best possible integrity, i. e., bit-error rate, an appropriate number of MIMO layers has to be used, which depends on the specific transmission mode, as detailed in Tab. 1. In general, the BER per SDM MIMO data vector is dominated by the specific transmission modes and the characteristics of the singular values, resulting in different BERs for the different QAM configurations in Tab. 1. An optimized adaptive scheme would now use the particular transmission modes, e. g., by using bit auction procedures [23], that results in the lowest BER for each SDM MIMO data vector. This would lead to different transmission modes per SDM MIMO data vector and a high signaling overhead would result. However, in order to avoid any signalling overhead, fixed transmission modes are used in this contribution regardless of the channel quality. The MIMO layer specific bit-error probability at the time *k* after SVD is given by [17]

$$P_{\text{BER}}^{(\ell,k)} = \frac{2\left(1 - \frac{1}{\sqrt{M_{\ell}}}\right)}{\log_2(M_{\ell})} \operatorname{erfc}\left(\sqrt{\frac{\xi_{\ell,k}}{2}} \cdot \frac{U_{s\ell}}{U_{\text{R}}}\right) . \tag{17}$$

The resulting average bit-error probability at the time k assuming different QAM constellation sizes per activated MIMO layer is given by

$$P_{\text{BER}}^{(k)} = \frac{1}{\sum_{\nu=1}^{L} \log_2(M_{\nu})} \sum_{\ell=1}^{L} \log_2(M_{\ell}) P_{\text{BER}}^{(\ell,k)} . \tag{18}$$

Taking *K* consecutive time slots into account, needed to transmit the SDM MIMO data vector, the aggregate bit-error probability per SDM MIMO data vector yields

$$P_{\text{BER block}} = \frac{1}{K} \sum_{k=1}^{K} P_{\text{BER}}^{(k)}$$
 (19)

When considering time-variant channel conditions, rather than an AWGN channel, the BER can be derived by considering the different transmission block SNRs.

<sup>&</sup>lt;sup>2</sup>It is worth noting that with the aid of powerful non-linear near Maximum Likelihood (ML) sphere decoders it is possible to separate  $n_R > n_T$  number of layers [20].

throughput	layer 1	layer 2	layer 3	layer 4
8 bit/s/Hz	256	0	0	0
8 bit/s/Hz	64	4	0	0
8 bit/s/Hz	16	16	0	0
8 bit/s/Hz	16	4	4	0
8 bit/s/Hz	4	4	4	4

Table 1. Investigated transmission modes

Assuming that the transmit power is uniformly distributed over the number of activated MIMO layers, i. e.,  $P_{s\ell} = P_s/L$ , the half-level transmit amplitude  $U_{s\ell}$  per activated MIMO layer results in

$$U_{\rm s\ell} = \sqrt{\frac{3P_{\rm s}}{2L(M_{\ell} - 1)}} \ . \tag{20}$$

The layer-specific signal-to-noise ratio at the time k, defined in (14), results together with (20) in

$$\rho^{(\ell,k)} = \xi_{\ell,k} \frac{3}{2L(M_{\ell} - 1)} \frac{P_{\rm s}}{U_{\rm R}^2} = \xi_{\ell,k} \frac{3}{L(M_{\ell} - 1)} \frac{E_{\rm s}}{N_0} , \qquad (21)$$

with

$$\frac{P_{\rm s}}{U_{\rm R}^2} = \frac{E_{\rm s}}{N_0/2} \ . \tag{22}$$

Finally, the MIMO layer-specific BER at the time *k* is now given by:

$$P_{\rm BER}^{(\ell,k)} = \frac{2\left(1 - \frac{1}{\sqrt{M_{\ell}}}\right)}{\log_2(M_{\ell})} \, \text{erfc} \left(\sqrt{\frac{3\,\xi_{\ell,k}}{2\,L(M_{\ell} - 1)}} \, \frac{E_{\rm s}}{N_0}\right) \ . \tag{23}$$

The resulting average bit-error probability taking the different number of activated MIMO layer at the time *k* into account, is obtained as

$$P_{\rm BER}^{(k)} = \frac{2}{R} \sum_{\ell=1}^{L} \left( 1 - \frac{1}{\sqrt{M_{\ell}}} \right) \operatorname{erfc} \left( \sqrt{\frac{3 \, \xi_{\ell,k}}{2 \, L(M_{\ell} - 1)} \, \frac{E_{\rm s}}{N_0}} \right) , \qquad (24)$$

with

$$R = \sum_{\ell=1}^{L} \log_2(M_\ell) \tag{25}$$

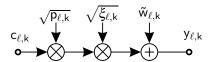


Fig. 2. Resulting layer-specific system model including MIMO-layer PA

describing the number of transmitted bits at the time *k*. Finally, the aggregate biterror probability per SDM MIMO data block can be calculated by taking (19) into account.

## 4 Adaptive MIMO-layer Power Allocation

In systems, where channel state information is available at the transmitter side, the knowledge about how the symbols are attenuated by the channel can be used to adapt the transmit parameters. Power allocation can be used to balance the biterror probabilities in the activated MIMO layers. Adaptive power allocation (PA) has been widely investigated in the literature [5, 6, 9, 17, 24, 25].

The BER of the uncoded MIMO system is dominated by the specific layers having the lowest SNR's. As a remedy, a MIMO-layer transmit PA scheme is required for minimizing the overall BER under the constraint of a limited total MIMO transmit power. The proposed PA scheme scales the half-level transmit amplitude  $U_{s\ell}$  of the  $\ell$ th MIMO layer by the factor  $\sqrt{p_{\ell,k}}$ . This results in a MIMO layer-specific transmit amplitude of  $U_{s\ell}\sqrt{p_{\ell,k}}$  for the QAM symbol of the transmit data vector transmitted at the time k over the MIMO layer  $\ell$  (Fig. 2). Applying MIMO-layer PA, the half vertical eye opening per MIMO layer  $\ell$  at the time k becomes

$$U_{\rm PA}^{(\ell,k)} = \sqrt{p_{\ell,k}} \cdot \sqrt{\xi_{\ell,k}} \cdot U_{\rm s\,\ell} \quad . \tag{26}$$

Now the layer-specific signal-to-noise ratio, defined in (21), is changed to

$$\rho_{\text{PA}}^{(\ell,k)} = \frac{\left(U_{\text{PA}}^{(\ell,k)}\right)^2}{U_{\text{R}}^2} = p_{\ell,k} \cdot \frac{3\,\xi_{\ell,k}}{L(M_\ell - 1)} \frac{E_{\text{s}}}{N_0} = p_{\ell,k} \cdot \rho^{(\ell,k)} \quad . \tag{27}$$

Using (12) and (27), along with the MIMO detector's input noise power, the resultant layer-specific BER at the time k can be calculated according to

$$P_{\text{BERPA}}^{(\ell,k)} = \frac{2\left(1 - \frac{1}{\sqrt{M_{\ell}}}\right)}{\log_2(M_{\ell})} \operatorname{erfc}\left(\sqrt{\frac{3p_{\ell,k}\xi_{\ell,k}}{2L(M_{\ell} - 1)}} \frac{E_s}{N_0}\right) . \tag{28}$$

Finally, the BER at the time k, applying MIMO-layer PA, results in

$$P_{\text{BERPA}}^{(k)} = \frac{2}{R} \sum_{\ell=1}^{L} \left( 1 - \frac{1}{\sqrt{M_{\ell}}} \right) \operatorname{erfc} \left( \sqrt{\frac{3 p_{\ell,k} \xi_{\ell,k}}{2 L (M_{\ell} - 1)}} \frac{E_{\text{s}}}{N_{0}} \right) . \tag{29}$$

The aggregate bit-error probability per SDM MIMO data block can be calculated by taking (19) into account.

Applying MIMO-layer PA, the information about how the symbols are attenuated by the channel, i. e., the singular-values, has to be sent via a feedback channel to the transmitter side and leads to a high signalling overhead that is contradictory to the fix transmission modes that require no signalling overhead. However, as shown in [26] a vector quantizer (VQ) can be used to keep the signalling overhead moderate. Besides, a VQ for the power allocation parameters instead of the singular values guarantees a better adaption at a given codebook size, since the power level vectors has less or equal dimensions than the singular-value vectors [26]. Moreover, its elements are much smaller digits ranged from 0 to 1, rather than from 0 to  $+\infty$  in the singular-value vector case. Hence, the entropy of the power level vectors is smaller, which benefits the quantization accuracy and the feedback overhead.

The aim of the forthcoming discussions is now the determination of the values  $\sqrt{p_{\ell,k}}$  for the activated MIMO layers. A common strategy is to use the Lagrange multiplier method in order to find the optimal value of  $\sqrt{p_{\ell,k}}$  for each MIMO layer  $\ell$  and time k needed to transmit the SDM MIMO data vector. The Lagrangian cost function  $J(p_{1,k}, \dots, p_{L,k})$  may be expressed with (29) as

$$J(p_{1,k},\cdots,p_{L,k}) = P_{\text{BERPA}}^{(k)} + \lambda B_L ,$$
 (30)

where  $\lambda$  is the Lagrange multiplier [9]. The parameter  $B_L$  in (30) describes the boundary condition taking the transmit power restriction per time slot into account and results in

$$B_L = \sum_{\ell=1}^{L} p_{\ell,k} - L = 0 . {31}$$

Unfortunately, the Lagrange multiplier method often leads to excessive-complexity optimization problems [17]. Therefore, suboptimal power allocation strategies having a lower complexity are of common interest [9, 17]. A natural choice is to opt for a PA scheme, which results in an identical signal-to-noise ratio

$$\rho_{\text{PA equal}}^{(\ell,k)} = \frac{\left(U_{\text{PA equal}}^{(\ell,k)}\right)^2}{U_{\text{R}}^2} = p_{\ell,k} \cdot \rho^{(\ell,k)}$$
(32)

for all activated MIMO layers at the time k, i. e., in

$$\rho_{\text{PA equal}}^{(\ell,k)} = \text{constant} \quad \ell = 1, 2, \dots, L$$
(33)

The power to be allocated to each activated MIMO layer at the time k can be shown to be calculated as follows [17]:

$$p_{\ell,k} = \frac{(M_{\ell} - 1)}{\xi_{\ell,k}} \cdot \frac{L}{\sum_{\nu=1}^{L} \frac{(M_{\nu} - 1)}{\xi_{\nu,k}}} . \tag{34}$$

The only difference between the optimum PA and the equal SNR PA is the consideration of the factor  $(1 - 1/\sqrt{M_\ell})$  by the optimum PA. Taking (34) and (20) into account, for each symbol of the SDM MIMO data vector, transmitted at the time k over the number of activated MIMO layers, the same half vertical eye opening of

$$U_{\text{PA equal}}^{(\ell,k)} = \sqrt{p_{\ell,k}} \cdot \sqrt{\xi_{\ell,k}} \cdot U_{s\ell} = \sqrt{\frac{3P_{s}}{2\sum_{\nu=1}^{L} \frac{(M_{\nu}-1)}{\xi_{\nu,k}}}}$$
(35)

can be guaranteed ( $\ell = 1, \dots, L$ ), i. e.,

$$U_{\text{PA equal}}^{(\ell,k)} = \text{constant} \qquad \ell = 1, 2, \cdots, L \ .$$
 (36)

When assuming an identical detector input noise variance for each channel output symbol, the above-mentioned equal quality scenario (33) is encountered, i. e.,

$$\rho_{\text{PA equal}}^{(\ell,k)} = \frac{\left(U_{\text{PA equal}}^{(\ell,k)}\right)^2}{U_{\text{R}}^2} = \frac{E_{\text{s}}}{N_0} \frac{3}{\sum_{\nu=1}^{L} \frac{(M_{\nu}-1)}{\xi_{\nu,k}}} . \tag{37}$$

Analyzing (37), nearly the same BER can be achieved on all activated MIMO layers at a given time k. However, different BERs arise for the K consecutive time slots needed to transmit a given SDM MIMO data vector. Therefore, the BER per SDM MIMO signal vector is mainly dominated by the symbol positions having the lowest SNR's. Furthermore, taking the time-variant nature of the transmission channel into account, different BERs arise for different SDM MIMO data blocks. In order to overcome this problem, the number of transmit or receive antennas has to be increased or coding over the different data blocks should be used.

#### 5 Results

In this contribution the efficiency of fixed transmission modes is studied regardless of the channel quality. Assuming predefined transmission modes, a fixed data rate can be guaranteed. The obtained BER curves are depicted in Fig. 3 and 4 for

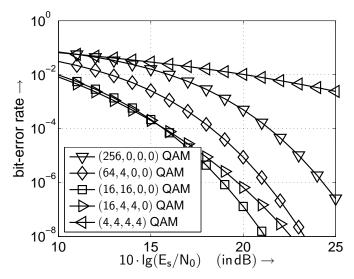


Fig. 3. BER when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over frequency selective channels with  $L_{\rm c}=1$ 

the different QAM constellation sizes and MIMO configurations of Tab. 1, when transmitting at a bandwidth efficiency of 8 bit/s/Hz within a given bandwidth<sup>3</sup>. Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it turns out that not all MIMO layers have to be activated in order to achieve the best BERs. More explicitly, our goal is to find that specific combination of the QAM mode and the number of MIMO layers, which gives the best possible BER performance at a given fixed bit/s/Hz bandwidth efficiency. The  $E_{\rm s}/N_0$  value required by each scheme at BER  $10^{-4}$  was extracted from Fig. 3 and 4 and the best systems are shown in bold in Tab. 1. Comparing the results depicted in Fig. 3 and 4, it can be seen that a high delay spread is quite beneficial for a good overall performance.

Further improvements are possible by taking the adaptive allocation of the transmit power into account. The differences between the optimal and the suboptimal equal SNR PA, as investigated in [11], show a negligible performance gap between the optimal and the equal SNR PA. The only difference between the optimum PA and the equal SNR PA is the consideration of the factor  $(1-1/\sqrt{M_\ell})$  by the optimum PA. However, their influence, introduced by the layer-specific QAM constellation sizes, is by far too small to generate remarkable differences in the performance. Furthermore, from Fig. 5 and 6 we see that unequal PA is only effective in conjunction with the optimum number of MIMO layers. Using all MIMO lay-

<sup>&</sup>lt;sup>3</sup>The expression  $\lg(\cdot)$  is considered to be the short form of  $\log_{10}(\cdot)$ .

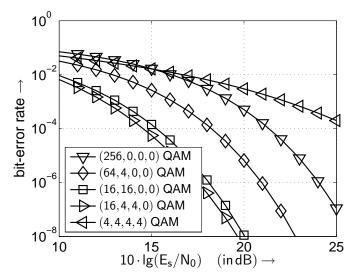


Fig. 4. BER when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over frequency selective channels with  $L_{\rm c}=4$ 

ers, our PA scheme would assign much of the total transmit power to the specific symbol positions per MIMO layer having the smallest singular values and hence the overall performance would deteriorate.

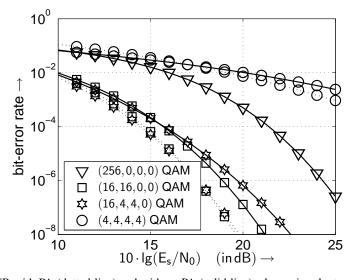


Fig. 5. BER with PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over frequency selective channels with  $L_{\rm c}=1$ 

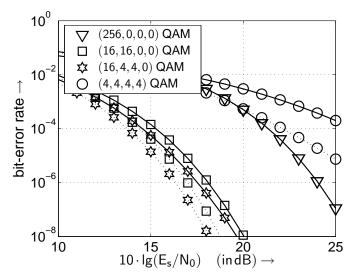


Fig. 6. BER with PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over frequency selective channels with  $L_c = 4$ 

However, the lowest BERs can only be achieved by using bit auction procedures leading to a high signalling overhead [23]. Analyzing the probability of choosing a specific transmission mode by using optimal bitloading, as depicted in Tab. 2, it turns out that only an appropriate number of MIMO layers has to be activated, e. g., the (16,4,4,0) QAM configurations. The results, obtained by using bit auction procedures justify the choice of fixed transmission modes regardless of the channel quality as investigated in the contribution.

Table 2. Probability of choosing specific transmission modes at a fixed data rate by using optimal bitloading  $(10 \cdot \lg(E_8/N_0)) = 10$  dB and  $L_c = 1$ )

mode	(16,4,4,0)	(16, 16, 0, 0)	(64, 4, 0, 0)	(4,4,4,4)
pdf	0.881	0.112	0.007	0

# 6 Conclusion

Bit and power loading in broadband MIMO systems were investigated. It turned out, that the choice of the number of bits per symbol as well as the number of activated MIMO layer substantially affects the performance of a MIMO system, suggesting that not all MIMO layers have to be activated in order to achieve the

best BERs. The main goal was to find that specific combination of the QAM mode and the number of MIMO layers, which gives the best possible BER performance at a given fixed bit/s/Hz bandwidth efficiency. The  $E_{\rm s}/N_0$  value required by each scheme at BER  $10^{-4}$  was extracted from computer simulations and the best systems are shown in bold in Tab. 1.

#### References

- [1] A. Ahrens and C. Benavente-Peces, "Modulation-Mode and Power Assignment in SVD-assisted Broadband MIMO Systems," in *International Conference on Wireless Information Networks and Systems (WINSYS)*, Milan (Italy), July 6–10, 2009, pp. 83–88.
- [2] —, "Modulation-Mode and Power Assignment for Broadband MIMO-BICM Schemes," in 20th Personal, Indoor and Mobile Radio Communications Symposium (PIMRC), Tokyo (Japan), Sept. 13–16, 2009.
- [3] V. Kühn, Wireless Communications over MIMO Channels Applications to CDMA and Multiple Antenna Systems. Chichester: Wiley, 2006.
- [4] L. Zheng and D. N. T. Tse, "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels." *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.
- [5] B. S. Krongold, K. Ramchandran, and D. L. Jones, "Computationally Efficient Optimal Power Allocation Algorithms for Multicarrier Communications Systems." *IEEE Transactions on Communications*, vol. 48, no. 1, pp. 23–27, 2000.
- [6] J. Jang and K. B. Lee, "Transmit Power Adaptation for Multiuser OFDM Systems." IEEE Journal on Selected Areas in Communications, vol. 21, no. 2, pp. 171–178, 2003.
- [7] J. Jang, K. B. Lee, and Y. H. Lee, "Transmit Power and Bit Allocations for OFDM Systems in a Fading Channel." in *IEEE Global Telecommunications Conference* (GLOBECOM), vol. 22, no. 1, San Francisco, 2003, pp. 858–862.
- [8] R. F. H. Fischer and J. B. Huber, "A New Loading Algorithm for Discrete Multitone Modulation." in *IEEE Global Telecommunications Conference (GLOBECOM)*, London, 1996, pp. 724–728.
- [9] C. S. Park and K. B. Lee, "Transmit Power Allocation for BER Performance Improvement in Multicarrier Systems." *IEEE Transactions on Communications*, vol. 52, no. 10, pp. 1658–1663, 2004.
- [10] Z. Zhou, B. Vucetic, M. Dohler, and Y. Li, "MIMO Systems with Adaptive Modulation." *IEEE Transactions on Vehicular Technology*, vol. 54, no. 5, pp. 1073–1096, Sept. 2005.
- [11] A. Ahrens and C. Lange, "Modulation-Mode and Power Assignment in SVD-equalized MIMO Systems." *Facta Universitatis (Series Electronics and Energetics)*, vol. 21, no. 2, pp. 167–181, Aug. 2008.

- [12] G. G. Raleigh and J. M. Cioffi, "Spatio-Temporal Coding for Wireless Communication." *IEEE Transactions on Communications*, vol. 46, no. 3, pp. 357–366, Mar. 1998.
- [13] G. G. Raleigh and V. K. Jones, "Multivariate Modulation and Coding for Wireless Communication." *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 5, pp. 851–866, May 1999.
- [14] D. Gesbert, "Multipath: Curse or Blessing? A System Performance Analysis of MIMO Wireless Systems." *Proceedings of International Zurich Seminar on Communications (IZS)*, 2004.
- [15] F.-M. Hsu, "Data Directed Estimation Techniques for Single-Tone HF Modems." in *IEEE Military Communications Conference (MILCOM)*, 1985, pp. 271–280.
- [16] S. S. Haykin, *Adaptive Filter Theory*., 2nd ed. Englewood Cliffs, N. J.: Prentice Hall, 1991.
- [17] A. Ahrens and C. Lange, "Transmit Power Allocation in SVD Equalized Multicarrier Systems." *International Journal of Electronics and Communications (AEÜ)*, vol. 61, no. 1, pp. 51–61, 2007.
- [18] I. Kalet, "Optimization of Linearly Equalized QAM." *IEEE Transactions on Communications*, vol. 35, no. 11, pp. 1234–1236, Nov. 1987.
- [19] J. G. Proakis, Digital Communications. Boston: McGraw-Hill, 2000.
- [20] L. Hanzo and T. Keller, OFDM and MC-CDMA. New York: Wiley, 2006.
- [21] G. D. Forney, R. G. Gallager, G. R. Lang, F. M. Longstaff, and S. U. Qureshi, "Efficient Modulation for Band-Limited Channels." *IEEE Journal on Selected Areas in Communications*, vol. 2, no. 5, pp. 632–647, 1984.
- [22] I. Kalet, "The Multitone Channel." *IEEE Transactions on Communications*, vol. 37, no. 2, pp. 119–124, Feb. 1989.
- [23] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.
- [24] T. Hunziker and D. Dahlhaus, "Optimal Power Adaptation for OFDM Systems with Ideal Bit-Interleaving and Hard-Decision Decoding." in *IEEE International Conference on Communications (ICC)*, vol. 1, Anchorage, Alaska (USA), May 2003, pp. 3392–3397.
- [25] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx Beamforming Design for Multicarrier MIMO Channels: A Unified Framework for Convex Optimization." *IEEE Transactions on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, Sept. 2003.
- [26] A. Ahrens and C. Lange, "Modulation-Mode and Power Assignment in SVD-assisted MIMO Systems with limited Feedback," in *First Asian Conference on e-Business and Telecommunications (CeBT)*, Changhua City (Taiwan), Feb. 9–10, 2009.