FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 22, no.3, December 2009, 285-292

A Design of a Four Square Coil System for a Biomagnetic Experiment

Dejana Herceg, Anamarija Juhas, and Miodrag Milutinov

Abstract: In this paper, a square coil system similar to the Merritt four coil system is investigated. This system is used for a biomagnetic experiment at the laboratory at the Department of Biology, Faculty of Sciences, University of Novi Sad. For the experiment, it is necessary that the magnetic field inside the coils be uniform. Therefore, distribution of the magnetic flux density inside the coil system is considered. For this system, magnetic flux density produced by sinusoidal current through the coils is calculated and then uniformity of the magnetic field inside the coils is investigated. Calculated magnetic flux density is compared to the measured magnetic flux density obtained experimentally at the laboratory.

Keywords: Magnetic field, uniform field, coil system, merritt coils.

1 Introduction

ALMOST all laboratory-based experiments, which seek to assess the effect of magnetic fields on living systems, use electric coils to generate electromagnetic exposure. In the classic Helmholtz design, the coils (either square or circular) are separated at a distance such that the first and second spatial derivatives of the applied field are zero at the central point of the coil system. Subsequent work has shown that several higher-order derivatives can be zeroed using assemblies of three, four or five coils, yielding much larger volumes of uniform field space [1].

Coil systems provide uniform magnetic field only in limited volume at their centre. In order to maximize the uniformity region of magnetic field inside a coil

---

Manuscript received on November 15, 2009. An earlier version of this paper was presented at the 9th International Conference on Applied Electromagentics PES 2009, August 30 - September 2, 2009, 18000 Nis, Serbia.

The authors are with the Faculty of Technical Sciences, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia (e-mails: `[vuletic, ajuhas, miodragm]@uns.ac.rs`).

system, systems with more coils are used. Merritt coil system [2] with four coils provides larger volume of uniformity than two Helmholtz coils.

An existing exposure system that consists of four square coils, shown in Figure 1, has been analyzed. We shall refer to this system as the ECS (exposure coil system). This system closely resembles the Merritt four coil system.

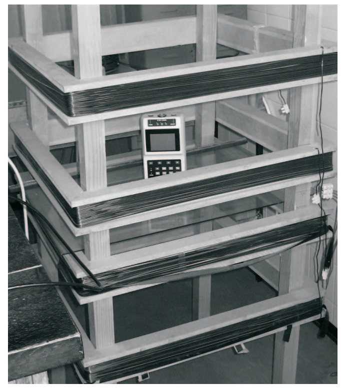

Fig. 1. Four square coil system.

Particularly, the ECS is used for a biomagnetic experiment. For the experiment, it is necessary that the magnetic field inside the coils be uniform. The aim of this paper is to determine the magnetic field and its uniformity inside the ECS.

The magnetic flux density is calculated, first analytically and then using the finite element method (FEM). Obtained results are compared to the results from [1], [2]. Calculated results are also compared to the measured results. Measurements were performed using the Narda EFA-300 EM field analyzer [3].

The ECS consists of the four coils of the same size, but different number of turns. Two outer coils have 26 turns of lacquer copper wire, whereas two inner coils have 11 turns of the same wire. Dimensions of the coils in the Cartesian coordinate system are depicted in Figure 2. The dimensions are expressed in terms of variables d and z_i , $i = 1, 2, 3, 4$.

Inaccuracies in coil position are the most important parameter that deteriorates the coil system performance [2]. Coil positions in the Merritt coil system and the ECS are given in Table 1.

Table 1. Coil positions in the Merritt coil system and the ECS.

	Merritt	ECS
z_1	$-0.5055d$	$-0.5042d$
z_2	$-0.1281d$	$-0.1292d$
z_3	$0.1281d$	$0.1292d$
z_4	$0.5055d$	$0.5042d$

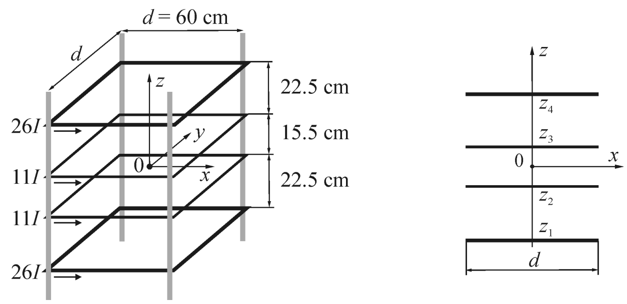

Fig. 2. Configuration of the ECS.

The ECS is built around a wooden frame. For that reason, the magnetic permeability of free space ($\mu_0 = 4\pi 10^{-7}$H/m) is used in calculations for the whole domain.

Because of the linearity of the system with regard to the current intensity, analytical calculations and computational simulations of magnetic flux density are only computed for the sinusoidal current with rms $I = 1$A. The frequency of the current is 50Hz.

2 Uniformity of Magnetic Flux Density

2.1 Analytical results

The magnetic flux density of the ECS in an arbitrary point can be calculated analytically using Biot-Savart law. The expressions for all three spatial components from coil system were derived in closed form. Figure 3 presents uniformity and the rms value of the z-component of the magnetic flux density in the plane passing through the central point of the coil system (plane $y = 0$). Since currents in all coils are in phase, the magnetic flux density vector is linearly polarized.

The rms value of the z-component of the magnetic flux density along the z-axis can be calculated using

$$B_z(z) = \sum_{i=1}^{4} \frac{2\sqrt{2}\mu_0 N_i I}{\pi d \left(1 + \frac{4(z-z_i)^2}{d^2}\right)\sqrt{1 + \frac{2(z-z_i)^2}{d^2}}}, \tag{1}$$

with $N_1 = N_4 = 26$ and $N_2 = N_3 = 11$ turns in coils.

The magnetic flux density at the central point of the ECS, denoted by B_0, has only the z-component. Therefore, B_0 could be obtained from (1) for $z = 0$. For $d = 0.6$m the magnetic flux density at the origin is $B_0 = 77.84\mu$T, for the ECS, and $B_0 = 77.75\mu$T for the Merritt four coil system.

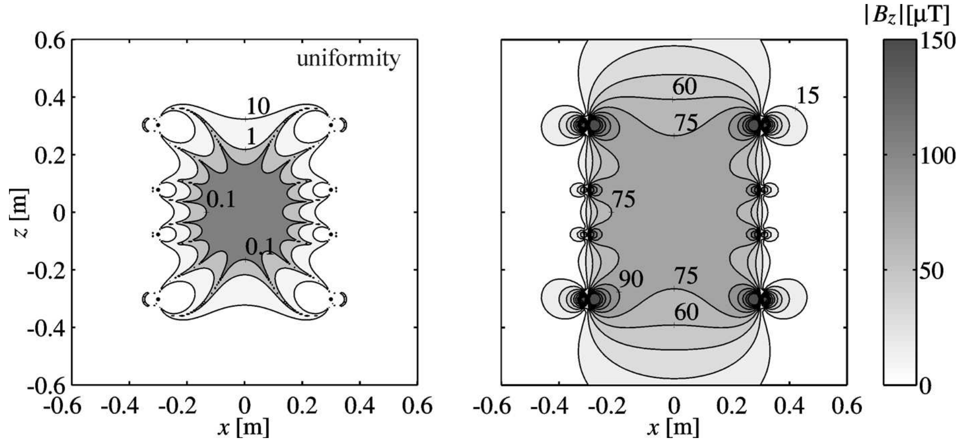

Fig. 3. The ECS: a) Regions of field uniformity, and b) rms value of the z-component of the magnetic flux density in the plane $y = 0$.

Field uniformity, u, within the coils can be described using (2), with B_0 as a reference:

$$u = \frac{|B_z - B_0|}{B_0}, \qquad (2)$$

where B_z denotes the z-component of magnetic flux density. Figures 3a and 4a present field uniformity for the ECS and Merritt coil system, respectively. Contour lines show the boundaries for the 0.1%, 1% and 10% variations from uniform field defined by (2).

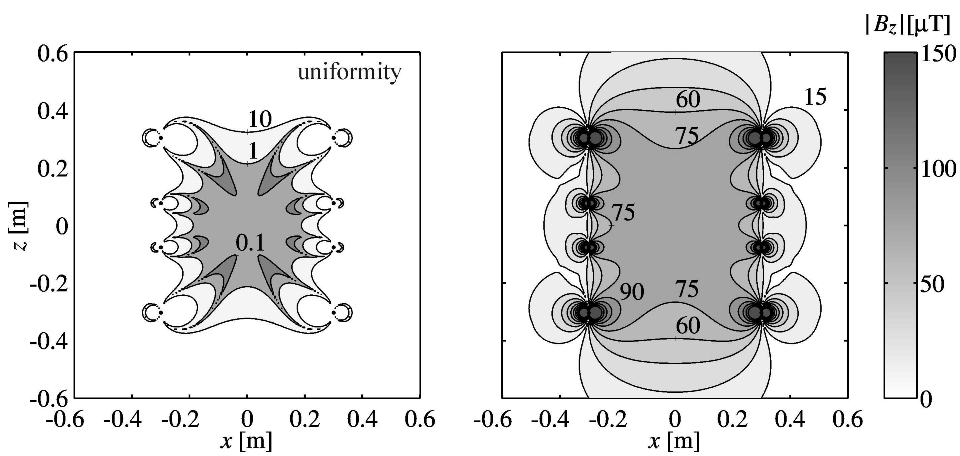

Fig. 4. The Merritt system: a) Regions of field uniformity, and b) rms value of the z-component of the magnetic flux density in plane $y = 0$.

2.2 FEM results

The ECS is modelled in COMSOL Multiphysics. The model is solved by FEM. The complex magnetic vector potential must satisfy the following equation:

$$\left(j\omega\sigma - \omega^2\varepsilon\right)\underline{\overrightarrow{A}} + \nabla\times\left(\mu\nabla\times\underline{\overrightarrow{A}}\right) = \underline{\overrightarrow{J}}. \tag{3}$$

To get a full description of an electromagnetic problem, boundary conditions at material interfaces and physical boundaries must be specified. In our case the boundary conditions describe the magnetic insulation and are defined by $\overrightarrow{n}\times\underline{\overrightarrow{A}} = 0$. The magnetic flux density vector is calculated using equation $\overrightarrow{B} = \text{curl}\ \overrightarrow{A}$.

Main property of the ECS is that the magnetic flux density becomes uniform in a large central region inside the coils. The results obtained by FEM are shown in Figure 5, and the central region with a uniform magnetic flux density can be observed.

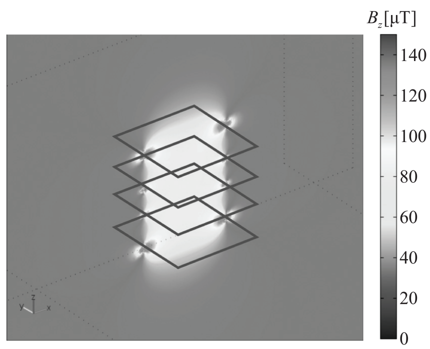

Fig. 5. The rms value of z-component of the magnetic flux density in the plane $y = 0$.

The z-component of the magnetic flux density at central point of coil system obtained by FEM simulation is $B_0 = 76.64\,\mu\text{T}$.

Although the four coil systems are relatively tolerant to small design imperfections [1], we find out that the ECS has a slightly smaller volume of the uniform field than the Merritt coil system.

3 Measurements

Measurements were performed at the laboratory of the Department of Biology, Faculty of sciences, University of Novi Sad, using Narda EFA-300 EM field analyzer [3].

Narda EFA-300 EM field analyzer with a built-in isotropic magnetic field probe was used for measurements. Measurement range of the instrument is from 100nT to 32mT for magnetic flux density. The frequency range of the instrument is from 5Hz to 32kHz.

Magnetic flux density was measured in the centre of the ECS. Table 2 shows the results obtained for four different coil current intensities. These results confirm that the system is linear with regard to the current intensity.

Table 2. Measured results for ECS.

I [A]	B_0 [μT]
1	79.7
1.25	100
1.9	150
2.5	200

Also, magnetic flux density is measured along z-axis for sinusoidal current with rms value $I = 1$A in each coil of ECS. Figure 6 shows measured results together with analytical and FEM results, presented earlier in this paper. One can see in Figure 6 that the measured magnetic flux density along z-axis from -0.1m to 0.1m reaches constant value 79.7μT. Relative differences between experimental and theoretical values of magnetic flux density are 2.39% (for analytical result) and 3.99% (for FEM result). Therefore, the calculated results were confirmed by the measurements.

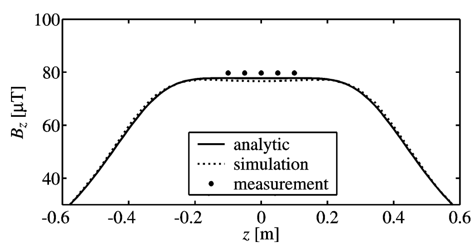

Fig. 6. Comparison of theoretical and experimental results.

In the central region of the system (approximately a cube with each side of 0.2m), both the experimental and calculated results show that the magnetic field is uniform.

4 Conclusion

In general, when large volume of uniform field is needed, it is necessary to use coil systems like Helmholtz, Merritt or similar. The square coil systems are easier to construct than Helmholtz circular coil system.

The ECS discussed above was already made. Our goal was to calculate the magnetic flux density and find out how the magnetic flux density depends on the coil current. Calculated results for exposure system are very similar to those for the Merritt-coil system.

Although the uniform field volume obtained in ECS is slightly smaller than in the Merritt-coil system, the ECS is suitable for laboratory experiments. Calculated results were confirmed by the measurements.

5 Appendix

For a rectangular coil with side dimensions $2a$ and $2b$, as shown in Figure 7, the

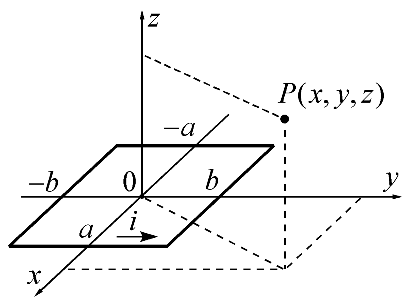

Fig. 7. Geometry for a single rectangular coil.

components of magnetic flux density vector at point $P\left(x,y,z\right)$ are

$$B_x = \frac{\mu_0 i}{4\pi}\sum_{k=1}^{k=4}\frac{(-1)^{k+1}z}{r_k\left(r_k+d_k\right)},$$

$$B_y = \frac{\mu_0 i}{4\pi}\sum_{k=1}^{k=4}\frac{(-1)^{k+1}z}{r_k\left(r_k+c_k\right)},$$

$$B_z = \frac{\mu_0 i}{4\pi}\sum_{k=1}^{k=4}(-1)^k\left(\frac{c_k}{r_k\left(r_k+d_k\right)}+\frac{d_k}{r_k\left(r_k+c_k\right)}\right),$$

where

$$\begin{aligned}
c_1 &= x+a, \quad d_1 = y+b, \quad r_1 = \sqrt{(x+a)^2+(y+b)^2+z^2},\\
c_2 &= x-a, \quad d_2 = y+b, \quad r_2 = \sqrt{(x-a)^2+(y+b)^2+z^2},\\
c_3 &= x-a, \quad d_3 = y-b, \quad r_3 = \sqrt{(x-a)^2+(y-b)^2+z^2},\\
c_4 &= x+a, \quad d_4 = y-b, \quad r_4 = \sqrt{(x+a)^2+(y-b)^2+z^2}.
\end{aligned}$$

The expressions for more coils can be easily derived using superposition theorem, and replacing z with $(z-z_i)$, where z_i determines the position of the i-th coil. For ECS $a=b=d/2$.

The expressions for the components of the magnetic flux density vector produced by a single rectangular coil can be found in previously published papers (e.g. [4]). In that paper the components of magnetic flux density vector for a rectangular loop are derived by first considering the magnetic vector potential at point $P(x,y,z)$ and then calculating the magnetic flux density vector using $\overrightarrow{B} = \operatorname{curl} \overrightarrow{A}$. Relations for B_x, B_y and B_z, presented in this appendix, are similar to those in [4]. The differences are the consequence of different definitions for c_2 and c_4.

Same expressions for B_x, B_y and B_z can be obtained from Bio-Savart law, by using the next form for the solution of the integral, which appear for any straight segment of the coil,

$$\int_{u_1}^{u_2} \frac{du}{(u^2+d^2)^{\frac{3}{2}}} = \frac{1}{\left(u_1+\sqrt{u_1^2+d^2}\right)\sqrt{u_1^2+d^2}} - \frac{1}{\left(u_2+\sqrt{u_2^2+d^2}\right)\sqrt{u_2^2+d^2}}.$$

Acknowledgement

This work was partially supported by the Serbian Ministry of Science and Technology as a part of the Project No 18043, 2008-2010.

The Narda EFA-300 EM field analyzer used in this work was kindly provided by the Provincial Secretariat for Energy and Mineral Resources, Executive Council of AP Vojvodina.

The authors would like to thank Zorana Banovački from Department of Biology, Faculty of Sciences, University of Novi Sad, for her assistance and support in the experimental work.

References

[1] J. L. Kirschvink, “Uniform magnetic fields and double-wrapped coil systems: Improved techniques for the design of bioelectromagnetic experiments,” *Bioelectromagnetics*, vol. 13, pp. 401–411, 1992.

[2] R. Merritt, C. Purcell, and G. Stroink, “Uniform magnetic field produced by three, four, and five square coils,” *Review of Scientific Instruments*, vol. 54, pp. 879–882, 1983.

[3] “Narda safety test solutions, efa-200, efa-300 field analyzers.” [Online]. Available: http://www.narda-sts.de

[4] M. Misakian, “Equations for the magnetic field produced by one or more rectangular loops of wire in the same plane,” *J. Res. Natl. Inst. Stand. Technol.*, vol. 105, pp. 557–564, 2000.