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On the Restrictive Channel Thickness Estimation

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Abstract: Channel routing is an important phase of physical design of LSI and VLSI chips. The channel routing method was first proposed by Akihiro Hashimoto and James Stevens [1]. The method was extensively studied by many authors and applied to different technologies. At present there are known many effective heuristic algorithms for channel routing. A. LaPaugh [2] proved that the restrictive routing problem is NP-complete. In this paper we prove that for every positive integer k there is a restrictive channel C for which $\tau(C) > \varphi(HG) + L(VG) + k$, where $\tau(C)$ is the thickness of the channel, $\varphi(HG)$ is clique number of the horizontal constraints graph HG and $L(VG)$ is the length of the longest directed path in the vertical constraints graph VG.

Keywords: Routing problem, channel routing, channel thickness.

1 Introduction

Channel routing is one of the most important phases of routing problem in LSI and VLSI chips' physical design.

Most of modern routing systems are mainly based on channel routers. These systems employ the "divide-and-conquer" strategy, where the layout routing problem splits into channel routing problems. Channel routers are widely used in the design of custom chips as well as uniform structures such as gate-arrays and standard cells. In the design of gate arrays, after completing placement and global routing phases, the channel router is launched to perform final interconnection within individual wiring bays. Similarly, channel routers are used to realize interconnection of standard-cell based designs. In custom layout of VLSI chips channel routers are used to set up final interconnection between macro blocks.

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Since channel routers perform detailed (final) interconnections in layout design, the general routing strategies and algorithms are substantially dependent on technological restrictions. Different technologies produce different approaches to the problem.

In this paper we mainly concentrate on the channel thickness bounds for classical channel routing problem. The classical model (also known as the Manhattan model) presents to be a rectangular space between two parallel rows of pins (terminals). The locations of these pins are fixed on the vertical lines of the grid. Two layers are available for routing: one exclusively for horizontal segments and the other for vertical segments.

Vias are used for each layer change.

2 Channel Routing Problem

To formulate the problem more precisely we need the following definitions. A channel C is a pair of vectors of nonnegative integers: $T=(t(1),t(2),\dots,t(n))$and $B=(b(1),b(2),\dots,b(n))$. We assume that these numbers are the labels of points that are located at the top and at the bottom of an $n\times m$ rectangle grid under condition that any positive integer with an entry in T or B has at least another entry, as it is shown in Fig. 1.

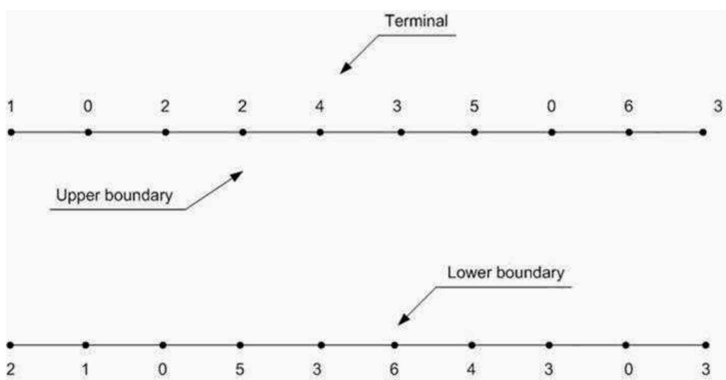

Fig. 1. A channel and its associated net list.

They represent the netlist. Terminals (points) with the same positive label i must be connected by net i. Terminals numbered as zero are called vacant terminals. A vacant terminal does not belong to any net, and therefore requires no electrical connection. With every net i we associate an interval $I(i)$, where the left (right) point of $I(i)$ is the minimal (maximal) column number j such that $t(j)$ or $b(j)$ equals i. Let us consider the restrictive channel routing problem, when the number of horizontal tracks on which any net can be positioned is 1. In this case wire geometry is very simple. Every net is implemented as a single horizontal segment with vertical branches connecting it to the pins; this is illustrated in Fig. 2.

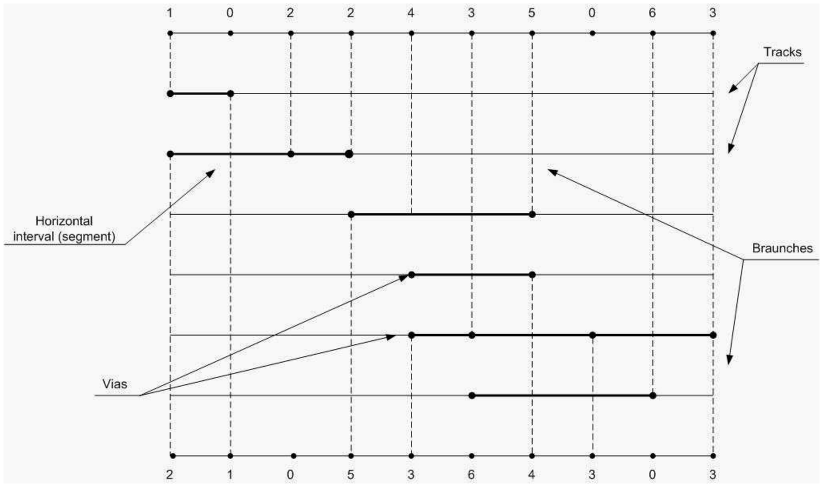

Fig. 2. Terminology for channel routing problem.

The problem of restrictive routing is to determine the number of a horizontal track for every net. Simplicity and the usage of minimal number of vias are the main advantages of restrictive routing. In two-layer routing, all the horizontal wires lay out on the tracks of one layer and all the vertical wires on the tracks of the other one. If two horizontal intervals of different nets do not overlap, then they could be assigned to the same track. If two horizontal intervals of different nets overlap, then they must be assigned to different tracks. Thus, there are horizontal constraints on nets, whether or not they can be assigned to the same tracks.

Also, any two net must not overlap at a vertical column. It is clear that the interval of a net, connected to the upper terminal at a given column, must be placed above the interval of another net, connected to the lower terminal at that column (see Fig. 2). Therefore, there are also vertical constraints between nets. Hence, we may associate two constraints graphs to any channel routing problem, one to model the horizontal constraints and the other to model vertical ones. For both graphs every net is represented by a vertex.

The horizontal constraints graph denoted by $HG = (V,E)$ is an undirected graph, where a vertex $i \in V$ represents net i and $(i,j) \in E$ if the horizontal intervals of net i and net j overlap. So it is an interval graph by construction (Fig.3).

The vertical constraints graph, denoted by $VG = (V,E')$ is a directed graph,

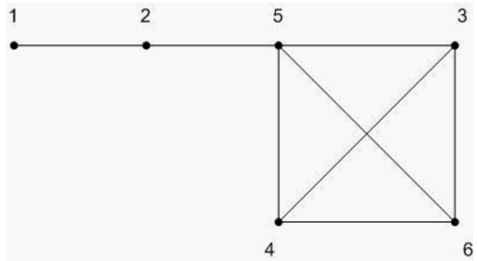

Fig. 3. The horizontal constraints graph HG for the channel (Fig. 1).

where each node corresponds to net i, and each column j, $(j = 1,2,\ldots,n))$ such that $t(j)$ and $b(j)$ are distinct nonzero numbers introduces a directed edge from node $t(j)$ to $b(j)$. That is, for any two nets having pins in the same column, on the opposite sides of the channel there will be a directed edge between corresponding vertices of the graph $VG = (V,E')$. Therefore, if there is a directed cycle in graph VG, the routing requirements cannot be realized without splitting some nets. For some cases, the restrictive routing problem is quite unrealizable. For example the restrictive channel $T = (1,2)$ and $B = (2,1)$ is not routable. It is easy to see that the vertical constraint implies the horizontal constraint, however, the converse is not true. Fig.4 shows the vertical constraints graph for the channel routing problem, given in Fig. 1. These two graphs allow us to view the channel routing problem as a graph theoretic problem.

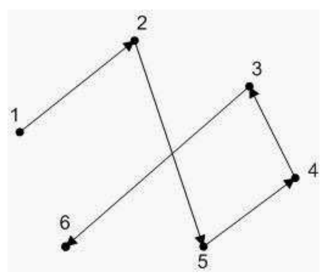

Fig. 4. The vertical constraints graphs for channel (Fig.1.).

The thickness of the channel C denoted by $\tau(C)$ must be determined by the router, i.e. we are allowed to add horizontal tracks to the rectangle, but vertical columns must remain intact. The channel router is provided to minimize the number of tracks, in other words to route within a channel of minimal thickness.

The horizontal constraints graph HG plays an important role in determining the channel thickness. No two nets which have a horizontal constraint may be assigned to the same track. Hence the maximum clique number $\varphi(HG)$ of graph HG, which is the maximum number of pairwise intersected intervals, forms a lower bound for channel thickness, i.e $\varphi(HG) \leq \tau(C)$.

This trivial lower bound is important and sometimes is the only lower bound available.

Let us consider the effect of directed path in the vertical constraints graph VG on the channel thickness. The length of the longest directed path $L(VG)$ of graph VG presents another lower bound for channel thickness: $L(VG) \leq \tau(C)$. This is due to the fact that no two nets in a directed path may be routed on the same track. For other lower bounds of channel thickness the interested reader can refer to [3].

3 Main Result

Obviously the existence of "good" (i.e. easily computable) lower and upper bounds for the channel thickness is very important, because they can help to estimate the chips' area and the quality of placement. So, it is natural to ask the question: *is it possible to derive an upper bound for channel thickness in terms of easily-computable $\varphi(HG)$ and $L(VG)$*? The answer to this question for restrictive channel routing is negative, since, as it will be demonstrated bellow, $\varphi(HG)$ and $L(VG)$ are not the only factors determining the channel thickness.

Note that for restrictive channel routing horizontal wire segments positioned on the same track do not intersect. This means that assignment of wire segments to tracks is reduced to proper coloring of the horizontal constraints graph HG, and vice versa. Since HG is an interval graph, one can use the well known left edge algorithm to color its vertices. As it is mentioned above, when the vertical constraints graph VG has directed cycles, then the restrictive solution does not exist. But even if graph VG is acyclic, the restrictive channel routing problem is far from being simple. A. LaPaugh [2] proved that the restrictive channel routing problem is NP-complete. It is easy to see that the restrictive routing problem is reduced to the special coloring of vertical constraints graph VG, and vice versa.

A special k-coloring of $VG = (V, E')$ is a labeling $f : V \rightarrow \{1, 2, \ldots, k\}$. Here the labels are colors. A special k-coloring f is proper, if $(i, j) \in E$ implies $f(i) \neq f(j)$ and if $(k, m) \in E'$ implies $f(k) < f(m)$. The graph VG is called special k-colorable if it has a proper special k-coloring. The special chromatic number $\chi'(V, G)$ is the minimum k such that VG is special k-colorable.

A. Frank [4] conjectures that for every channel

$$\tau(G) \leq L(VG) + \varphi(HG) + 2,$$

where $L(VG)$ is the length of the longest directed path in VG and $\varphi(HG)$ is the clique number of HG.

Theorem 1. For every positive integer k there is a restrictive channel C for which $\tau(C) > L(VG) + \varphi(HG) + k$.

Proof. Let us consider a restrictive channel the initial part of which is shown in Fig. 5. Fig. 6 illustrates graph VG for the mentioned part of restrictive channel. The channel is constructed step by step (the vertices added at each step are encircled by a dashed curve in Fig. 6).

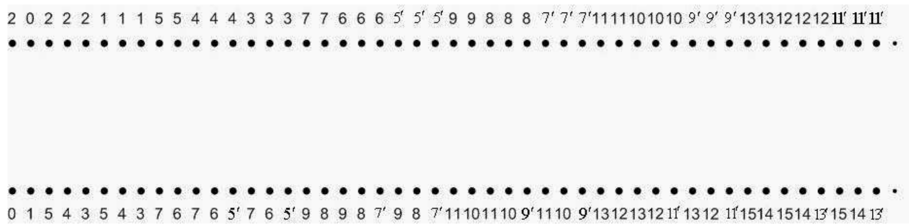

Fig. 5. Restrictive channel.

Let G_j denotes that subgraph of VG which is induced by all vertices of level j, $j = 0, 1, \ldots$ (The levels are shown in Fig. 6). To prove the theorem we need the following statements:

(i) $\varphi(HG) = 5$,

(ii) $L(G_0) = 1$ and $L(G_{j+1}) = L(G_j) + 1, \quad j = 0, 1, 2, \ldots,$

(iii) $\chi'(G_0) = 5$ and $\chi'(G_{j+1}) = \chi'(G_j) + 2, j = 0, 1, 2, \ldots$

By construction of the channel the vertices of the same step are pairwise adjacent in the horizontal constraints graph HG. Every directed edge connects only vertices from succesive steps, moreover the vertical constraints graph VG contains all such edges except to $(5, 5')$, $(7, 7')$, $(9, 9')$, $(11, 11')$ and so on. Since the vertical constraint implies vertical constraint, hence all directed edges of VG as undirected edges belong to HG. It is not difficult to see that the addition of every next step increases the length of the longest path by 1 in the corresponding subgraph.

This proves statements (i) and (ii). To prove (iii) first note that if there is a directed path from vertex i to vertex j, then in the special coloring of VG, the color of i must be less than the color of j. Secondly, having the optimal special coloring of G_i we can continue this coloring to obtain optimal special coloring of G_{i+1} by using two new colors. As G_0 is a clique in HG, hence $\chi'(G_0) = 5$. Every vertex of G_0 is connected by direct path to every vertex of step 3 except vertex 5 which is not connected to vertex $5'$ by a directed path, therefore the color of vertices 6 and 7 must be greater than the color of all the vertices of subgraph G_0. We can color vertex $5'$ by the color of vertex 5. Hence the optimal special coloring of G_1 is obtained by adding two new colors.

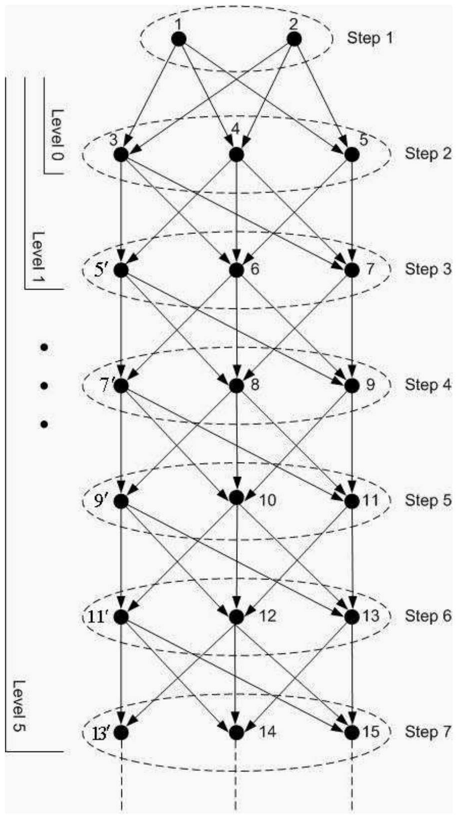

Fig. 6. Vertical constraints graph.

Using the same arguments one can obtain the optimal special coloring of G_2 from the optimal special coloring G_1 by using 2 new colors and so on. Statement (iii) is proved. Thus addition of every new step i $(i > 2)$ retains the clique number, the length of the longest path is increased by 1, but the special chromatic number is increased by 2. For the graph in Fig. 6 $\varphi(HG) = 5$, $L(VG) = 6$ and $\chi'(VG) = 15$. Taking sufficient steps for every k we can construct restrictive channel for which $\tau(C) > L(VG) + \varphi(HG) + k$. The theorem is proved.

Note that the conjecture mentioned above remains open for nonrestrictive channels.

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