FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 20, no. 2, August 2007, 187-202

A New and Improved Model of a Lead Acid Battery

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Abstract: This paper presents a new and improved model of a lead acid battery that takes into account if the battery is in discharging state, in charging state or in the rest period. The parameters of the model depend upon the changes in the received or delivered battery current. The method to obtain the model parameters and experimental results are also presented.

Keywords: Battery model, discharging state, charging state, rest period.

1 Introduction

Battery is one of the most important parts in an electric vehicle or hybrid electric vehicle and also in a conventional (gasoline) vehicle. Although batteries seem to act like simple electrical energy storage devices, when they deliver and accept energy, they undergo thermally dependent electrochemical processes that make them difficult to model. Thus, the electrical behaviour of a battery is a nonlinear function of a variety of changing parameters.

Lead acid batteries are among the most used devices to store and deliver energy. There are also other types of batteries such as: Nickel-Metal Hybrid, Lithium-Ion, Nickel-Cadmium.

To estimate the behaviour of a system receiving energy from a battery, an equivalent circuit or a model of the battery is needed.

The most commonly used model of a battery is the Thevenin model [1–4]. It contains an ideal DC voltage source, in series with a resistor and a parallel resistor-capacitor group. However, this model has some drawbacks that the models that followed tried to eliminate.

Manuscript received on April 27, 2007.

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Reference [2] presents a battery model based on the Thevenin model, where the charging and discharging processes of the battery are considered separately. For this purpose, the parallel group contains a resistor for the charging state, a resistor for the discharging state and a capacitor. Furthermore, the values of the DC voltage source and the two resistors depend on the state of charge (SOC) of the battery.

In [5] a battery model is presented that is valid for NiMH batteries, for time intervals of a few seconds and for a constant SOC. The model contains an ideal DC voltage source in series with a resistor and two parallel resistor-capacitor groups. Thus, the transient processes that occur when the load is switched on or off can be better modeled. However, the model does not take into account if the battery is in the charging/discharging state or in the rest period.

The Advanced Simulator for Vehicle (ADVISOR) package program [6] contains several sophisticated battery models. These models have the disadvantage that a longer time is required for computation due to the high order of the models.

This paper introduces a model that allows fitting to the voltage waveforms across battery terminals obtained experimentally, as will be seen in section 4. The model is valid for lead acid batteries and takes into account if the battery is in charging/discharging state, or in the rest period. The parameters of the model depend on the discharging or the charging current magnitude. The equations for computing all the model parameters are presented and derived. The parameters derived from experiments were used to build the battery model. The output voltage of the model (the simulation voltage) was compared with the one obtained by experiments to validate the model.

2 The Proposed Model of the Lead Acid Battery

The structure of the proposed model has been derived by studying the data gathered for a lead acid battery in four different experiments.

This model is a dynamic one, that is it takes into account the changes in the received or delivered battery current. It is valid for time intervals of a few minutes, so the SOC of the battery can be approximated as being constant. The model comprises four operating modes: (a) charging, when the battery receives current, (b) rest period after charging, (c) discharging, when the battery delivers current and (d) rest period after discharging. During the rest periods the battery neither receives nor delivers current. In each of the four operating modes, the voltage across the battery can be approximated by a sum of two exponentials. The equivalent time constant is different for each operating mode.

The most important parameter of a battery is the open circuit voltage (OCV).

OCV is the voltage across the battery after a long enough rest period. Every battery model has to contain the OCV.

Fig.1 presents the proposed model of the lead acid battery. It contains an ideal DC voltage source U_0 (its value being OCV) connected in series with resistor R and two branches of two parallel resistor-capacitor groups. C_1, R_1, R_3 and C_2, R_2, R_4 are used when the battery is in discharging state, that is $i(t) < 0$, and also when the battery is in the rest period after discharging. In this case, diode D_1 is on and diode D_2 is off. C_3, R_5, R_7 and C_4, R_6, R_8 are used when the battery is in charging state, that is $i(t) > 0$, and also when the battery is in the rest period after charging. Now D_1 is off and D_2 is on.

In the discharging state, switches SW_1 and SW_2 are closed, switches SW_3 and SW_4 are open, and the two capacitors C_1 and C_2 start to charge. When the capacitors are completely charged, the voltage across the battery will be constant, if the discharge conditions remain unchanged (the current $i(t)$ is constant or the resistive load is constant). When the load is disconnected, that is the current through the battery is zero and the battery is in the rest period after discharging, switches SW_3 and SW_4 are closed and switches SW_1 and SW_2 are open. Thus, the two capacitors start to discharge. When the capacitors are completely discharged, the voltage across the battery will be U_0. The use of switches in the model is justified because the time constants of the two transient processes are different. Thus, when the capacitors are charging, resistors R_1 and R_2 are used, and when the capacitors are discharging, R_3 and R_4 are used.

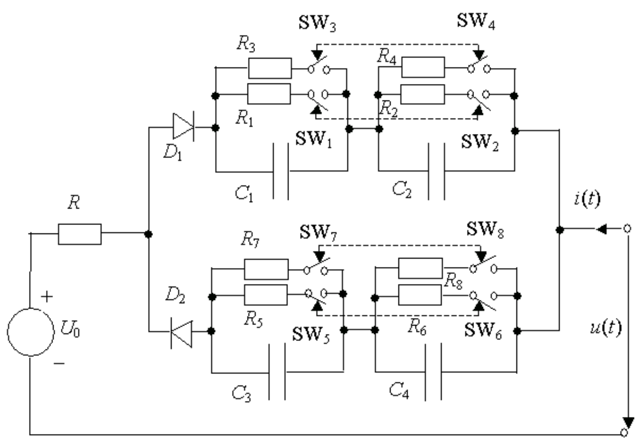

Fig. 1. The proposed battery model.

If the battery is in charging state, capacitors C_3 and C_4 start to charge. The behaviour of the model is similar to that of the discharging state. This model assumes that the battery enters a state (charging or discharging) only after the capacitors used in the other state have been completely discharged. In other words, the transition from one state to the other is done only after long enough rest periods.

In the following, the equations that describe the model behaviour will be presented.

2.1 The discharging state of the battery

Assuming that switch K is closed at $t = 0$ (fig.2) and the battery discharges through a constant load, R_{ext}, the voltage across the battery can be expressed by

$$u(t) = -i(t)R_{ext}, \tag{1}$$

In order to obtain the expression of $i(t)$, the Laplace transform $I(p)$ of $i(t)$ will be derived. If the voltages on the two capacitors are zero at $t = 0$, $I(p)$ is expressed by

$$I(p) = -\frac{U_0}{p} \cdot \frac{1}{Z(p)}, \tag{2}$$

where $Z(p)$ is the operational impedance of the circuit,

$$Z(p) = R + \frac{R_1 \frac{1}{pC_1}}{R_1 + \frac{1}{pC_1}} + \frac{R_2 \frac{1}{pC_2}}{R_2 + \frac{1}{pC_2}} + R_{ext}. \tag{3}$$

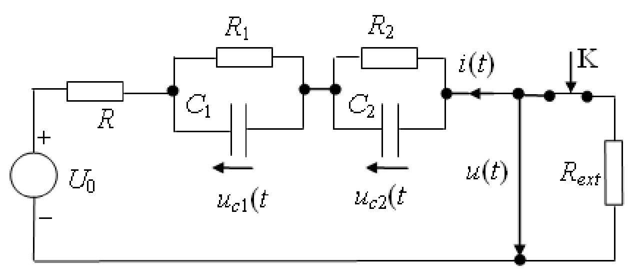

Fig. 2. The circuit for analyzing the discharging state of the battery.

By using the inverse Laplace transform, the following expression of $i(t)$ is obtained:

$$i(t) = -\frac{U_0}{R + R_{ext} + R_1 + R_2}\left(1 + \frac{R_1}{R + R_{ext}}\, e^{-\frac{t}{\tau_{1p}}} + \frac{R_2}{R + R_{ext}}\, e^{-\frac{t}{\tau_{2p}}}\right), \tag{4}$$

where τ_{1p} and τ_{2p} are:

$$\tau_{1p} = C_1 \frac{R_1(R + R_{ext})}{R_1 + R + R_{ext}}, \tag{5}$$

and,

$$\tau_{2p} = C_2 \frac{R_2(R + R_{ext})}{R_2 + R + R_{ext}}. \tag{6}$$

Finally, according to (1) and (4), the following expression of $u(t)$ is obtained:

$$u(t) = \frac{U_0 R_{ext}}{R + R_{ext} + R_1 + R_2}\left(1 + \frac{R_1}{R + R_{ext}}\, e^{-\frac{t}{\tau_{1p}}} + \frac{R_2}{R + R_{ext}}\, e^{-\frac{t}{\tau_{2p}}}\right). \tag{7}$$

In order to determine the parameters of the model, it is necessary that the two time constants τ_{1p} and τ_{2p} differ by at least an order of magnitude [5].

When $t = 0$, the maximum value of $u(t)$ is obtained:

$$U_{max,\, d} = u(0) = \frac{U_0 R_{ext}}{R + R_{ext}}. \tag{8}$$

When t is large enough such as and $e^{\,-t/\tau_{1p}} \approx 0$ and $e^{\,-t/\tau_{2p}} \approx 0$, $u(t)$ reaches its minimum value:

$$U_{min,\, d} = \frac{U_0 R_{ext}}{R + R_{ext} + R_1 + R_2}. \tag{9}$$

By using (9), a simpler expression of $u(t)$ can be obtained:

$$u(t) = U_{min,\, d} + U_{1,\, d} e^{\,-\frac{t}{\tau_{1p}}} + U_{2,\, d} e^{\,-\frac{t}{\tau_{2p}}}, \tag{10}$$

where $U_{1,\, d}$ is

$$U_{1,\, d} = \frac{U_{min,\, d} R_1}{R + R_{ext}}, \tag{11}$$

and $U_{2,\, d}$ is

$$U_{2,\, d} = \frac{U_{min,\, d} R_2}{R + R_{ext}}. \tag{12}$$

In the following, the equations for the rest period after a discharging state will be presented. Assuming that switch K is opened at $t = 0$ (fig.2) and R_1 and R_2 are replaced by R_3 and R_4, the voltage across the battery can be expressed by:

$$u(t) = U_0 + U_{c1}\, e^{-\frac{t}{\tau_{1s}}} + U_{c2}\, e^{-\frac{t}{\tau_{2s}}}, \tag{13}$$

where τ_{1s} and τ_{2s} are

$$\tau_{1s} = R_3 C_1, \tag{14}$$

and,

$$\tau_{2s} = R_4 C_2, \tag{15}$$

and U_{c1} and U_{c2} are the voltages across capacitors C_1 and C_2 at $t = 0$, both being negative. When the battery delivers current, and the transient process of charging C_1 and C_2 has ended, the voltages across the two capacitors have maximum values as follows:

$$U_{c1} = U_{c1max} = I_{min,\,d} R_1, \tag{16}$$

and

$$U_{c2} = U_{c2max} = I_{min,\,d} R_2, \tag{17}$$

where $I_{min,\,d} = -U_{min,\,d}/R_{ext}$.

The minimum value of $u(t)$ is obtained at $t = 0$ and is denoted $U_{jump,\,d}$,

$$U_{jump,\,d} = U_0 + U_{c1} + U_{c2}. \tag{18}$$

The maximum value of $u(t)$ is U_0, and is obtained when $e^{-t/\tau_{1s}} \approx 0$ and $e^{-t/\tau_{2s}} \approx 0$. Because the value of $u(t)$ before the start of the rest period is

$$u(t) = U_{min,\,d} = U_0 + I_{min} R + U_{c1} + U_{c2}, \tag{19}$$

and based on (18) and (19), the following equation can be written

$$U_{jump,\,d} - U_{min,\,d} = -I_{min,\,d} R. \tag{20}$$

2.2 The charging state of the battery

Assuming that switch K is closed at $t = 0$ (fig.3), and the battery charges from the power supply whose output voltage is U_s, the voltage across the battery can be expressed by

$$u(t) = U_s - i(t) R_{ext}, \tag{21}$$

where $i(t)$ is (the voltages across the two capacitors are zero at $t = 0$):

$$i(t) = \frac{U_s - U_0}{R + R_{ext} + R_5 + R_6}\left(1 + \frac{R_5}{R + R_{ext}}\, e^{-\frac{t}{\tau_{3p}}} + \frac{R_6}{R + R_{ext}}\, e^{-\frac{t}{\tau_{4p}}}\right), \tag{22}$$

and the time constants τ_{3p} and τ_{4p} are similar to τ_{1p} and τ_{2p}, according to (5) and (6).

Thus, using (21) and (22), $u(t)$ can be expressed by

$$u(t) = U_{max,\,c} - U_{1,\,c}\, e^{-\frac{t}{\tau_{3p}}} - U_{2,\,c}\, e^{-\frac{t}{\tau_{4p}}}, \tag{23}$$

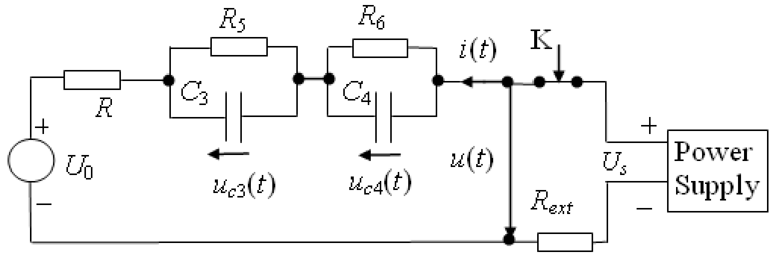

Fig. 3. The circuit for analyzing the charging state of the battery.

where $U_{max,\,c}$, that is obtained from $u(t)$ when t is large enough such as $e^{-t/\tau_{3p}} \approx 0$ and $e^{-t/\tau_{4p}} \approx 0$, has the expression

$$U_{max,\,c} = U_s - \frac{(U_s - U_0)R_{ext}}{R + R_{ext} + R_5 + R_6}, \tag{24}$$

and $U_{1,\,c}$ and $U_{2,\,c}$ are expressed by the following equations

$$U_{1,\,c} = (U_s - U_{max,\,c})\frac{R_5}{R + R_{ext}}, \tag{25}$$

and,

$$U_{2,\,c} = (U_s - U_{max,\,c})\frac{R_6}{R + R_{ext}}. \tag{26}$$

The minimum value of $u(t)$ is obtained with eq. (23) when $t = 0$:

$$U_{min,\,c} = u(0) = U_{max,\,c} - U_{1,\,c} - U_{2,\,c} = U_s - \frac{(U_s - U_0)R_{ext}}{R + R_{ext}}. \tag{27}$$

In the following, the equations for the rest period after a charging state will be presented. Assuming that switch K will open at $t = 0$ (fig.3) and R_5 and R_6 are replaced by R_7 and R_8, the voltage across the battery can be expressed by:

$$u(t) = U_0 + U_{c3}\, e^{-\frac{t}{\tau_{3s}}} + U_{c4}\, e^{-\frac{t}{\tau_{4s}}}, \tag{28}$$

where τ_{3s}, τ_{4s}, U_{c3} and U_{c4} are similar to τ_{1s}, τ_{2s}, U_{c1} and U_{c2}, according to equations (14)-(17). The voltages U_{c3} and U_{c4} are positive. The maximum value of $u(t)$ is obtained at $t = 0$ and is denoted $U_{jump,\,c}$,

$$U_{jump,\,c} = U_0 + U_{c3} + U_{c4}. \tag{29}$$

The minimum value of $u(t)$ is U_0 and is obtained when $e^{-t/\tau_{3s}} \approx 0$ and $e^{-t/\tau_{4s}} \approx 0$.

In fig.4, two examples with theoretical waveforms for $u(t)$ and $i(t)$ are presented (the first for discharging and the second for charging state).

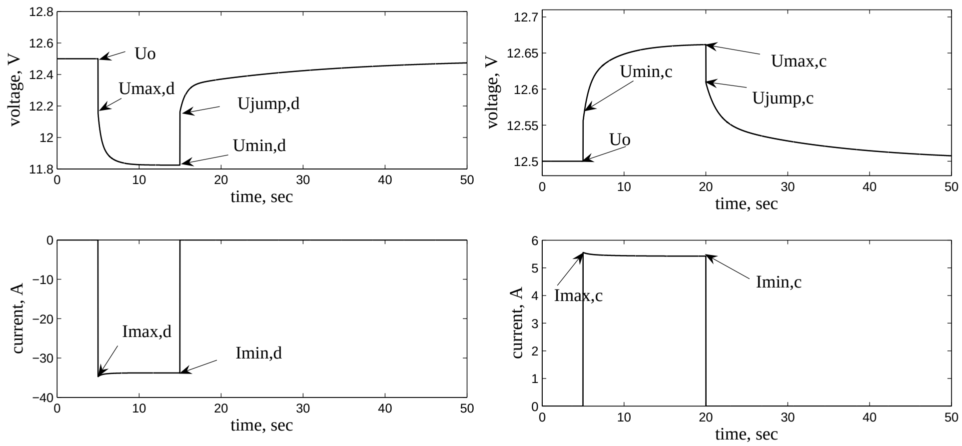

Fig. 4. The waveforms for discharge and charge the battery.

3 Determination of Model Parameters

The parameters of the battery model can be determined by measuring the current through the battery and the voltage across the battery during the following sequence: discharging, rest, charging and again rest. For this purpose, an experimental bench was set up, its structure being presented in fig.5. It contains the battery, a resistive load, a power supply, two relays and a resistive shunt that is used to measure the current through the battery. The voltage across the battery and the voltage across the shunt are acquired with a National Instruments PCI 6023 data acquisition board. The PCI 6023 contains a 12-bit analog to digital converter with 5V input range.

The value of the resistive shunt was 0.01Ω. The voltage across the battery is applied to the Ch 0 input of the data acquisition board by means of a resistive divider, omitted for simplicity from fig. 5.

The sampling frequency was 50 Hz. This relatively high value is necessary only for fast regions of $u(t)$, namely close to U_{max}, U_{min} and U_{jump}, according to fig.4. The acquired data were filtered with a 5-th order low pass digital Butteworth filter, having a cutoff frequency of 5 Hz.

By appropriately driving the relays, the battery can be put in one of four following states: charging, rest period after charging, discharging, and rest period after discharging.

A Bosch 5D battery with a capacity of 55 Ah and a nominal voltage of 12 V was used throughout the tests.

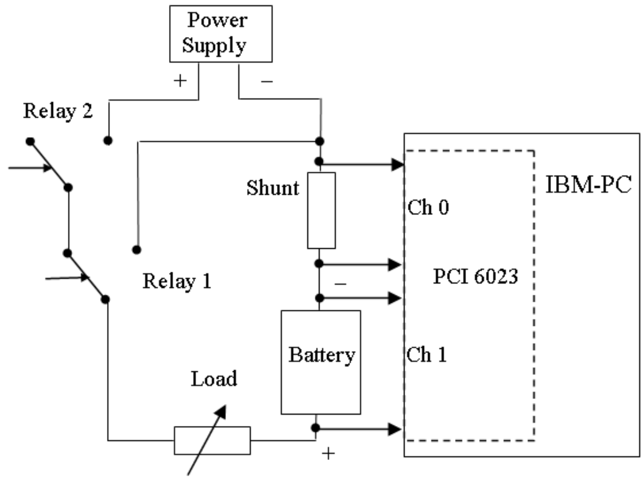

Fig. 5. The experimental setup.

By processing the voltage and the current corresponding to the four operating modes, the model parameters will be computed.

In the discharging state, the length of the acquisition was 50 s. The two relays were driven such as the battery delivered current between $t = 5$ s and $t = 15$ s. In the charging state, the length of the acquisition was 250 s. The two relays were driven such as the battery received current between $t = 5$ s and $t = 100$ s. Then, by using the acquired voltage waveforms, the following values are extracted in discharging state: U_0, $U_{max,\,d}$, $U_{min,\,d}$ and $U_{jump,\,d}$, and the similar values in charging state.

3.1 Determination of parameters for the discharging state

The resistive load R_{ext} is obtained by using equation (1), where t can have any value between 5 and 15 s.

Then, from equation (8), R is computed as

$$R = \frac{U_0 R_{ext}}{U_{max,\,d}} - R_{ext}. \tag{30}$$

In the following, the elements R_1, R_2, C_1 and C_2 must be computed. For this purpose, at first, the voltage coefficients $U_{1,\,d}$ and $U_{2,\,d}$, and then τ_{1p} and τ_{2p} must be computed, by using equation (10). In order to reduce the complexity of these operations, the following condition has been imposed in the model: $R_1 = R_2$. This condition leads to $U_{1,\,d} = U_{2,\,d}$, and in equation (10) when $t = 0$, there is only one unknown parameter:

$$U_{1,\,d} = U_{2,\,d} = (U_{max,\,d} - U_{min.\,d})/2. \tag{31}$$

A similar condition is applied to the charging part of the model, that is $R_5 = R_6$.

In order to compute the time constant τ_{2p}, a value of $u(t)$ at a time instant large enough such as $e^{-t/\tau_{1p}} \approx 0$ is considered: $u_1 = u(t)$ at $t = 7.5$ s. Then, by using equation (10), τ_{2p} can be computed as:

$$\tau_{2p} = \frac{-2.5[\mathrm{s}]}{\ln \dfrac{u_1 - U_{min,\,d}}{U_{2,\,d}}}. \tag{32}$$

According to equation (12) the value of R_2 will be computed as

$$R_2 = \frac{U_{2,\,d}(R + R_{ext})}{U_{min,\,d}}, \tag{33}$$

and, according to equation (6) the value of C_2 will be computed as

$$C_2 = \frac{\tau_{2p}(R_2 + R + R_{ext})}{R_2(R + R_{ext})}. \tag{34}$$

In order to compute the time constant τ_{1p} a value of $u(t)$ at a time instant small enough such as $e^{-t/\tau_{1p}} \neq 0$ is considered: $u_2 = u(t)$ at $t = 6$ s, and then, based on equation (10), τ_{1p} can be computed as:

$$\tau_{1p} = \frac{-1[\mathrm{s}]}{\ln \dfrac{u_2 - U_{min,\,d} - U_{2,\,d}e^{-\frac{1[\mathrm{s}]}{\tau_{2p}}}}{U_{1,\,d}}}. \tag{35}$$

Finally, C_1 can be computed with the following equation, similar to eq. (34):

$$C_1 = \frac{\tau_{1p}(R_1 + R + R_{ext})}{R_1(R + R_{ext})}, \tag{36}$$

where $R_1 = R_2$.

In the following, the determination of the parameters when the battery is in the rest period after a discharging state is presented.

In this case, what have to be found are the values of time constants τ_{1s} and τ_{2s} and then the values of R_3 and R_4, because the capacitors have the same values as in the discharging state.

Before computing these values, two verifications can be done. First, the value of resistor R can be computed from equation (20) by:

$$R = \frac{U_{jump,\,d} - U_{min,\,d}}{I_{min,\,d}} \tag{37}$$

and the sum of the maximum voltages on the capacitors can be computed from equation (18) as:

$$U_{c1} + U_{c2} = U_0 - U_{jump,\,d}. \tag{38}$$

Then, the value of R from (37) can be compared with that obtained with equation (30), and the sum of the two voltages from (38) with that based on equations (16) and (17). If these values are close enough, the behaviour of the battery corresponds to the model.

In order to compute the time constant τ_{2s}, a value of $u(t)$ at a time instant large enough such as $e^{-t/\tau_{1s}} \approx 0$ is considered: $u_3 = u(t)$, at $t = 30$ s. Then, by using equation (13), τ_{2s} can be computed as:

$$\tau_{2s} = \frac{-15[\mathrm{s}]}{\ln \dfrac{u_3 - U_0}{U_{c2}}}. \tag{39}$$

The value of R_4 can be computed as

$$R_4 = \frac{\tau_{2s}}{C_2}. \tag{40}$$

In order to compute the time constant τ_{1s}, a value of $u(t)$ for a time moment small enough such as $e^{-t/\tau_{1s}} \neq 0$ is considered: $u_4 = u(t)$ at $t = 16$ s. Then, by using equation (13) τ_{1s} can be computed as:

$$\tau_{1s} = \frac{-1[\mathrm{s}]}{\ln \dfrac{u_4 - U_0 - U_{c2}\, e^{-\frac{1[\mathrm{s}]}{\tau_{2s}}}}{U_{c1}}}. \tag{41}$$

The value of R_3 can be computed as

$$R_3 = \frac{\tau_{1s}}{C_1}. \tag{42}$$

3.2 Determination of parameters for the charging state

In this case the procedure is similar to that for the discharging state, but the equations for the charging state are used. In the following, the most important equations are presented.

The resistive load R_{ext} is obtained using equation (21), where t can have any value between 5 and 100 s. Then, from (27), R is computed as

$$R = \frac{R_{ext}(U_{min,\,c}) - U_0}{U_s - U_{min,\,c}}. \tag{43}$$

In order to compute the time constant τ_{4p}, a value of $u(t)$ at a time instant large enough such as $e^{-t/\tau_{3p}} \approx 0$ is considered: $u_1 = u(t)$ at $t = 65$ s. Then, by using equation (23), τ_{4p} can be computed as:

$$\tau_{4p} = \frac{-60[\mathrm{s}]}{\ln \dfrac{U_{max,\,c} - u_1}{U_{2,\,c}}}. \tag{44}$$

In according to equation (26) the value of R_6 will be computed as

$$R_6 = \frac{U_{2,\,c}(R + R_{ext})}{U_s - U_{max,\,c}}. \tag{45}$$

In order to compute the time constant τ_{3p} a value of $u(t)$ at a time instant small enough such as $e^{-t/\tau_{3p}} \neq 0$ is considered: $u_2 = u(t)$ at $t = 10$ s, and then, based on equation (23), τ_{3p} can be computed as:

$$\tau_{3p} = \frac{-5[\mathrm{s}]}{\ln \dfrac{U_{max,\,c} - u_2 - U_{2,\,c}\, e^{-\frac{5[\mathrm{s}]}{\tau_{4p}}}}{U_{1,\,c}}}. \tag{46}$$

In order to compute the time constant τ_{4s}, a value of $u(t)$ at a time instant large enough such as $e^{-t/\tau_{3s}} \approx 0$ is considered: $u_3 = u(t)$ at t=190 s. Then, by using equation (28), τ_{4s} can be computed as:

$$\tau_{4s} = \frac{-90[\mathrm{s}]}{\ln \dfrac{u_3 - U_0}{U_{c4}}}. \tag{47}$$

In order to compute the time constant τ_{3s}, a value of $u(t)$ at a time instant small enough such as $e^{-t/\tau_{3s}} \neq 0$ is considered: $u_4 = u(t)$ at t=105 s. Then, by using equation (28), τ_{3s} can be computed as:

$$\tau_{3s} = \frac{-5[s]}{\ln \dfrac{u_4 - U_0 - U_{c4}\, e^{-\frac{5[s]}{\tau_{4s}}}}{U_{c3}}}. \tag{48}$$

4 Experimental Results

In order to obtain the parameters of the battery model, the equations from previous section have been applied to the measurement data. Then, a model with the obtained parameters was achieved, and the output voltage was computed in the same conditions as in the practical case (the same R_{ext}, the same time moments for driving the relays, the same U_s). In order to validate the model, the measured voltage was compared to the simulated one.

In the discharging state, the model parameters have been determined for several initial discharging currents (I_{max}), corresponding to different load resistors R_{ext}.

Table 1 presents the model parameters for various range of $i(t)$.

Table 1. Some experimental results for model parameters in the discharging state.

I_{max}, I_{min} [A]	U_0 [V]	R [mΩ]	τ_{1p} [s]	τ_{2p} [s]	τ_{1s} [s]	τ_{2s} [s]	R_1, R_2 [mΩ]	R_3 [mΩ]	R_4 [mΩ]	C_1 [F]	C_2 [F]
$-50, -47,5$	12.50	8.7	0.39	1.37	0.63	19.1	5.6	8.7	75.9	72.7	252
$-43.1, 41.6$	12.50	10.3	0.48	1.72	1.08	22.7	5.4	11.8	69.6	91.9	327
$-34.4, -33.6$	12.50	9.2	0.60	2.02	1.03	22.7	6.2	10.5	68.4	98.9	332
$-30.3, -29.7$	12.48	10.1	0.58	2.33	0.82	20.2	7.0	9.8	59.4	84.4	340
$-21.7, -21.3$	12.48	11.1	0.69	3.33	1.02	23.4	8.4	12.2	58.1	83.6	403
$-18.3, -17.8$	12.47	10.3	0.70	3.76	0.86	19.6	9.6	11.7	49.9	74.4	399

Based on the data in table 1, the following conclusions can be drawn. The value of R is about 10 mΩ . Its accuracy is affected by the value of U_{max} according to (30). The values of time constants τ_{1p} and τ_{2p} depend on the value of the discharging current. Both of them decrease as the discharging current increases. However, the variation of τ_{2p} is faster. The same dependence holds for R_1 and R_2. Therefore, the value of C_2 depends strongly on the value of the discharging current while the value of C_1 depends only slightly.

The values of time constants τ_{1s} and τ_{2s} do not depend on the value of the discharging current. Therefore, the value of R_4 depends on C_2 and value of R_3 depends on C_1.

Figure 6 presents the measured and the simulated (with dotted line) battery voltage for two of the cases in table 1. A good similarity between the waveforms can be observed.

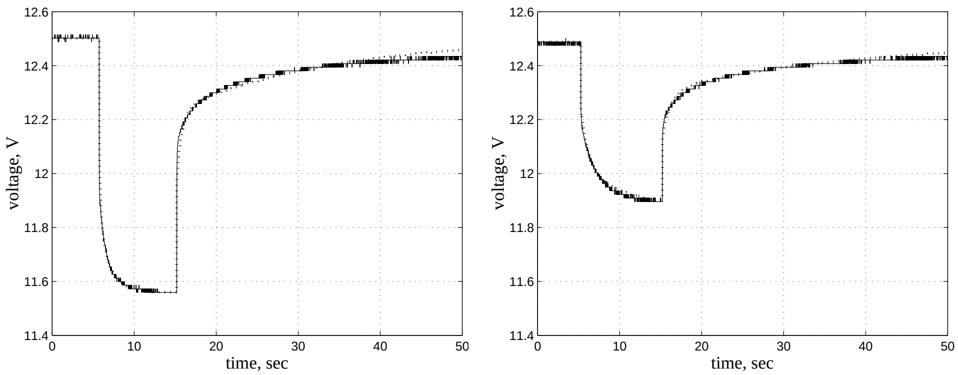

Fig. 6. The waveform voltage for $I_{max} = -50$A, and, respective, for $I_{max} = -21.7$A

In the charging state, the model parameters have been determined also for several initial charging currents (I_{max}), corresponding to different load resistors (R_{ext}). Table 2 presents these parameters.

Table 2. Some experimental results for model parameters in the charging state.

I_{max}, I_{min} [A]	U_0 [V]	R [mΩ]	τ_{3p} [s]	τ_{4p} [s]	τ_{3s} [s]	τ_{4s} [s]	R_5, R_6 [mΩ]	R_7 [mΩ]	R_8 [mΩ]	C_3 [F]	C_4 [F]
7.93, 7.12	12.55	12.7	2.91	15.88	2.89	19.5	44.5	40.9	51	70.8	383
7.61, 6.71	12.55	13.2	3.81	22.21	3.78	33.4	48.9	44.5	67.8	84	493
6.98, 6.17	12.55	14.4	4.92	20.75	5.35	44.5	49.9	50.1	98.9	106	450
6.39, 5.44	12.55	11.9	5.81	21.48	4.62	43.4	51.7	38	96.7	121	449
5.61, 4.78	12.55	13.6	6.01	26.04	5.88	58.3	53.1	48	110	122	528
5.15, 4.54	12.56	12.5	5.10	24.38	6.48	52.0	53.8	63.5	106	102	497

In this case, in comparison with the discharging state, the time constants are much larger. Therefore, the resistor values are much higher than those for the discharging state. All time constants decrease as the charging current increases. However, the variation of τ_{4p} is slower. The capacitor values are comparable with the ones in the discharging state.

The value of R is about 13 mΩ , being a bit higher than the one in the discharging state.

Figure 7 presents the measured and the simulated (with dotted line) voltage for two of the cases in table 2. A good similarity between the waveforms can be observed in this case, too.

Even if the model parameters depend on the discharging or charging current, in order to implement the battery model, the average values of those from table 1

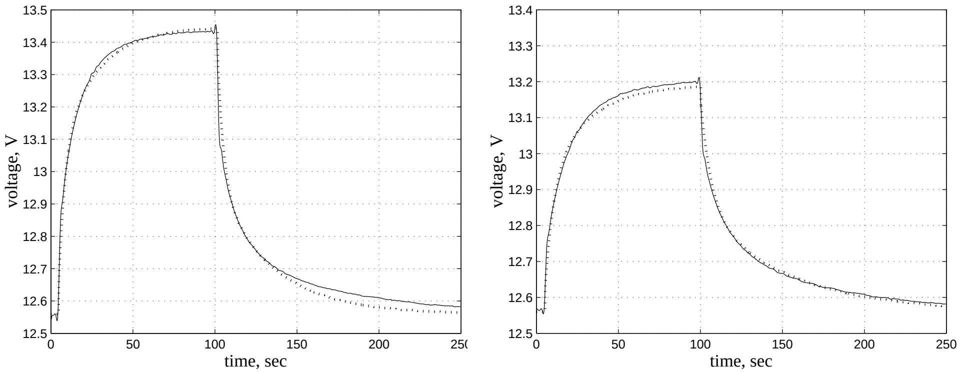

Fig. 7. The waveform voltage for $I_{max} = 7.61$A, and, respective, for $I_{max} = 5.14$A

and 2 can be used, if the variation range of the discharging or charging current is known.

5 Conclusions

In this paper, a new model of the lead acid battery was presented. In addition to other existing models, the presented model contains elements that take into account the different behaviour of the battery in charging/discharging state or in the rest period. One drawback of the model is the dependence of model parameters on the charging/discharging current. Future work will focus on a dynamic model of the battery that accepts fast transitions between charging and discharging states. The dependence of model parameters to the SOC and state of health (SOH) of the battery should also be determined.

Acknowledgment

The authors wish to thank dr. eng. Andrei Cimponeriu from Siemens VDO Automotive SRL for his help, ideas and advice during this research. They would also like to appreciate the logistic support of Siemens VDO Automotive SRL.

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