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Carbon Nano Tube (CNT) Multiplexers for Multiple-Valued Computing

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Abstract: Novel two types of Carbon Nano Tube (CNT)-based multiplexers are introduced. The first device is a solid-state transmission gate (t-gate) multiplexer that uses CNT as a channel in the Field Effect Transistors (FET) of both n-FET and p-FET types that are used. Because of its very small size, it has been shown that a CNT-based FET switches reliably using much less power than a silicon-based device, and thus the new device will consume less power than traditional t-gate multiplexer. The second new CNT device uses the fundamental Lorentz magnetic force from the basic laws of Electromagnetics as a switching mechanism between two conducting CNTs. The implementation of multiple-valued Galois logic using the new CNT devices is shown. Since a multiplexer is a fundamental logic block, the new devices can have a wide range of applications in a wide variety of nano circuits.

Keywords: Electronic Circuits, Nanotechnology, Carbon Nano Tube (CNT), *m*-Valued Computing.

1 Introduction

Nanotechnology is a new field of research that cuts across many fields of electronics, chemistry, physics, and biology, that analyzes and synthesizes objects and structures in the nano scale (10^{-9} m) such as nanoparticles, nanowires, and Carbon Nano Tubes (CNTs) [1–7]. CNT is one of several cutting-edge emerging technologies within nanotechnology that is showing high efficiency and very wide range of applications in many different streams of science and technology [1–8]. Examples of such applications are: (1) TVs based on field-emission of CNTs that consume

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much less power, thinner, and much higher resolution than the best plasma-based TV available [1], and (2) nano-circuits based on CNTs such as CNT Field Effect Transistors (CNTFETs) that show big promise of consuming less power and to be much faster than the available silicon-based FETs [2, 4].

This paper reports novel two CNT devices that implement a fundamental building block in logic synthesis - multiplexer [8]. The use of the new devices in multiple-valued computations is shown in the case of ternary Galois logic GF(3). Although the demonstration of the use of the new CNT-based multiplexers is shown here for GF(3), implementations over higher radices of Galois logic and other algebras is similar.

Basic background on CNT is presented in Section 2. The new CNT-based multiplexers are introduced in Section 3. Multiple-valued computation using the new devices is introduced in Section 4. Conclusions are presented in Section 5.

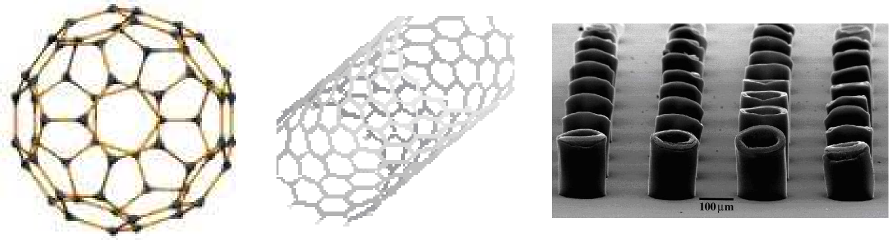
2 Carbon Nano Tubes (CNTs)

Carbon Nano Tube (CNT) has attracted attention in recent years not only for its relatively small dimensions and unique morphologies, but also for its potential of implementations in many current and emerging technologies [1–8]. CNT is made up from graphite [2]. It has been observed that graphite can be formed in nano-scale in three forms: (1) Carbon Nano Ball (CNB) (or buckyball) molecule consisting of 60 carbon atoms (C_{60}) that are arranged in the form of a soccer ball [1], (2) Carbon Nano Tube (CNT) - narrow strip of tiny sheet of graphite that comes mainly in two types [2]: (a) multi-wall CNT (MWCNT): each CNT contains several hollow cylinders of carbon atoms nested inside each other, and (b) single-wall CNT (SWCNT) that is made of just a single layer of carbon atoms, and (3) Carbon Nano Coil (CNC) [6].

CNT, which is a cylindrical sheet of graphite, is formed geometrically in two distinct forms which affect CNT properties [2]: (1) straight CNT: CNT formed as a straight cut from graphite sheet and rolled into a tube, and (2) twisted CNT: CNT formed as a cut at an angle from graphite sheet and rolled into a tube.

Figure 1 shows a typical carbon nano-ball (buckyball), single-wall CNT (SWCNT), and Scanning Electron Microscopy (SEM) image of chemical vapor deposition (CVD) grown array of multi-walled CNT (MWCNT) towers [9–11].

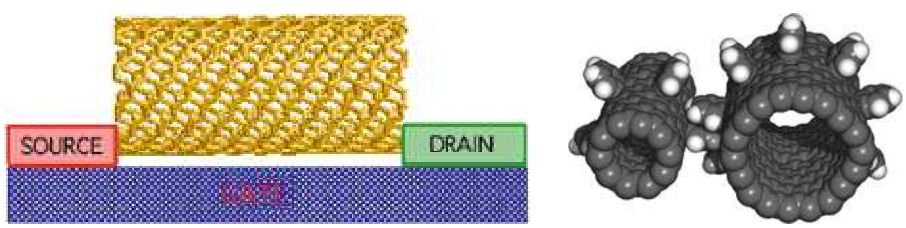
CNT technology has been implemented in many new exciting applications. This includes (1) TVs based on field-emission of CNTs that consume much less power, thinner, and much higher resolution than the best plasma-based TV available [1], (2) nano-circuits based on CNTs such as CNT Field Effect Transistors



(a) (b) (c)

Fig. 1. (a) carbon nano ball (buckyball), (b) single-wall CNT (SWCNT), and (c) image from Scanning Electron Microscopy (SEM) of chemical vapor deposition (CVD) grown array of multi-walled CNT (MWCNT) towers.

(CNTFETs) that consume less power and much faster than the available silicon-based FETs [2,4], (3) Carbon Nano Coils [6] that can be used as inductors in nano-filters and as nano-springs in nano dynamic systems, and (4) CNT rings. CNT has also promising potential uses in several applications such as: (1) CNT probes, (2) new composite materials, (3) CNT data storage devices capable of storing 10^{15} bytes/cm^2, (4) drug delivery systems, (5) nano lithography, and (6) CNT gears [12] in which large gears drive small gears or vice versa. Figure 2 shows some of CNT applications in electrical systems as a channel in a FET [2,4,12], and in mechanical systems as a potential nano gear made of two different size CNTs [12].



(a) (b)

Fig. 2. (a) CNT as a channel in FET, and (b) potential nano gear using two CNTs of different sizes.

CNT growth, as observed using (1) Transmission Electron Microscopy (TEM), (2) Atomic Force Microscopy (AFM), and (3) Scanning Electron Microscopy (SEM), requires processes with correct conditions and materials. Several methods for growing CNTs exist [2]: (1) a big spark between two graphite rods, few millimeters apart, that are wired to a power supply: a 10^2 Ampere spark between the two rods vaporizes carbon into hot plasma which partially re-condenses into the form of CNT, (2) chemical vapor deposition (CVD) of a hot gas such as methane: a substrate is placed in an oven, then the oven is heated to approximately 600 degrees

Celsius and slowly methane is added. As methane decomposes, it frees carbon atoms that partially re-compose into the form of 0.6-1.2 nm in diameter SWCNTs, and (3) a laser blast of a graphite target: laser pulses blasts a graphite rod which generates hot carbon gas from which CNT forms.

Although CNT has been grown into several forms, CNT use is still limited as compared to other wide spread technologies. This is mainly due to: (1) it is still difficult to exactly control CNT growth into desired forms, and (2) CNT growth is still very expensive due to the low yield of CNTs that meet desired geometrical specifications (cf. Property #9 in Table 1).

Table 1. Carbon Nano Tube (CNT) properties.

#	Property	Single-Walled CNT	By Comparison
1	Size	0.6 - 1.8 nm in diameter	Electron beam lithography can create lines 50 nm wide, and a few nm thick
2	Density	1.33 - 1.40 g/cm^3	Aluminum has a density of 2.7 g/cm^3
3	Tensile Strength	$\approx 45 \times 10^9$ Pascals	High-strength steel alloys break at $\approx 2 \times 10^9$ Pascals
4	Resilience	Can be bent at large angles and re-straightened without damage	Metals and carbon fibers fracture at grain boundaries
5	Current Carrying Capacity	$\approx 1 \times 10^9$ A/cm^2	Copper wires burn out at $\approx 1 \times 10^6$ A/cm^2
6	Field Emission	Can activate phosphors at 1 - 3 V if electrodes are spaced 1 micron apart	Molybdenum tips require $\approx$50 - 100 V/micrometer with very limited lifetimes
7	Heat Transmission	$\approx$ 6,000 W/m·K at room temperature	Nearly pure diamond transmits $\approx$3,320 W/m·K
8	Temperature/Thermal Stability	Stable up to 2,800 C in vacuum, and 750 C in air	Metal wires in microchips melt at $\approx$ 600 - 1,000 C
9	Cost	$\approx$1,500 $/g	Gold sells for $\approx$10 $/g
10	Preservation of the Quantum Property of Electron Spin	Optimal; Very High	Low in regular conductors
11	Power Consumption	Very low	Higher in metal wires
12	Speed	Very high; $\geq 1 \times 10^{12}$ Hz nanoscale switch	$\geq$1,000 times as fast as processors available today
13	Electron Scattering; Resistance	Almost none	Comparatively high
14	Energy Band Gaps	Easily tunable; Depends on CNT diameter, and thus wide range of band gaps can be obtained; $\approx$0 (like a metal), as high as band gap of Silicon, and almost anywhere in between	No other known material can be so easily tuned

The wide usability of CNTs in so many applications is due to the unique structural properties they possess. Table 1 summarizes most of these properties as compared to traditional counterparts [2]. For example, property #6 is used in the recently developed highly-efficient CNT-based TV [1] and is used in newly developed prototype vacuum-tube lamps in six colors that are twice as bright as conventional light bulbs, longer-lived and at least ten times more energy-efficient [2]. Properties #5, #8, #11, and #13 are qualifying the CNT for very promising future use in highly-efficient power transmission. Property #10 will be used in using the CNT for reliable quantum-based computations. Property #1 is very useful for using CNTs as nanowires that would result in decreasing the total size of areas (and volumes) occupied by wires and interconnects in ICs. Property #4 is useful in building circuits and structures that has to maintain stress without structural damages. Property #14 qualifies CNTs to be used in a very wide of applications that require wide range of energy band gaps from conductor state to the semiconductor state.

3 New CNT Multiplexers (CNTMUXs)

This Section introduces newly invented Carbon Nano Tube Multiplexers (CNTMUXs) [8]. Sub-Section 3.1 introduces solid-state CNTMUX, and Sub-Section 3.2 introduces magnetic field CNTMUX.

3.1 Solid-State CNTMUX

Figure 3 shows the new solid-state CNTMUX [8]. CNT is used as a channel in a Field Effect Transistor (FET) [2, 4] as was illustrated in Fig. 2(a).

Figure 3(a) shows the solid-state structure of CNTMUX. The use of CNT as a channel in a FET has been shown [2,4] and a simple inverter, that works exactly the same way as an ordinary CMOS inverter, has been built using such a scheme [4]. Here, we use the same principle of using a CNT as a channel but in a different device that uses two CNT n-FETs and two CNT p-FETs with different topologies of interconnects that are used for inputs A and B, controls C and C', and output F.

In Fig. 3(a), silicon (Si) is used as back-gate in the device, silicon dioxide (SiO_2) is used as an insulator, and gold is used as electrodes (conductors). Four metallic catalyst islands with each pair located at the facing ends of each pair of gold electrodes can be also used to grow CNTs between each pair of gold electrodes [2, 4], rather than just placing CNTs in contact with the gold electrodes. PMMA is a cover that protects anything beneath it from being exposed to oxygen (O_2), where oxygen is used to convert n-CNTFET to p-CNTFET [4].

The fabrication of each sub-device in Fig. 3(a) is done as follows [4]: Initially the two CNTFETs are p-type. After vacuum annealing both CNTFETs are converted to n-type. The two CNTFETs are exposed to oxygen (10^{-3} Torr of oxygen

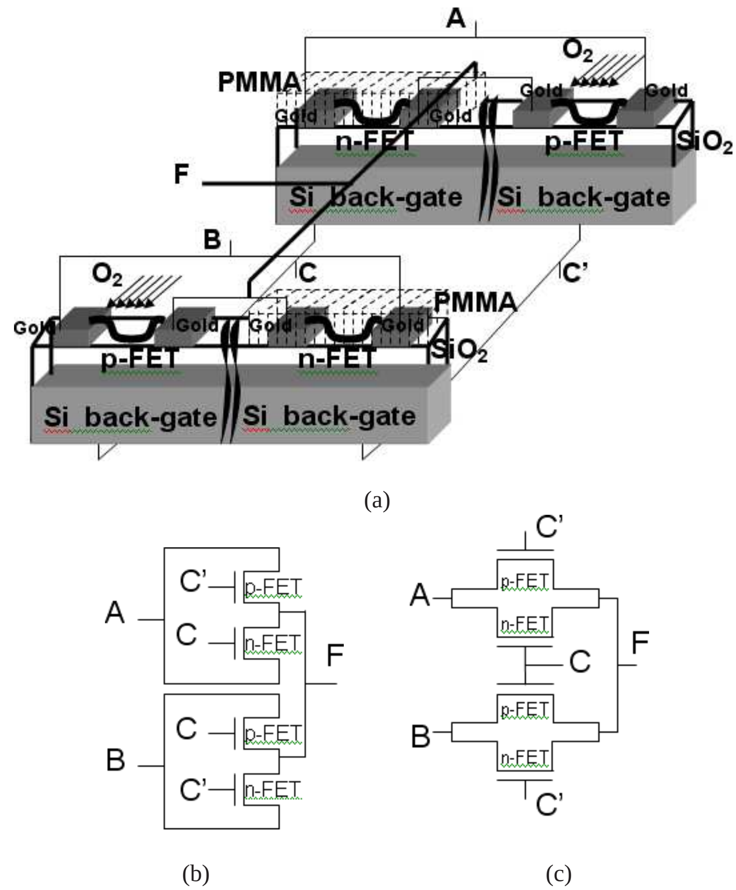


Fig. 3. Solid-state intermolecular CNTMUX: (a) the device structure, (b) a simplified schematic of the interconnected two transmission gates (t-gates) each composed of a single n-FET and a single p-FET, and (c) total device symbol.

for three minutes), and the unprotected n-CNTFET converts back to the original p-type, while the protected CNTFET remains n-type. (Another method to form p-type and n-type CNTFETs has been reported as follows [4]: CNT channel dopped with potassium (K) produces an n-type CNTFET, while a CNT without dopping produces p-type CNTFET. Dopping Produces n-type CNTFET by shifting the Fermi energy level to the conduction band that results in an increase of electron concentration in the conduction band which increases the conductance of the FET for a given positive gate voltage.)

The function of the device in Fig. 3(a) can be analyzed as shown in Figs. 3(b) and 3(c) as follows [13]: if C = 1 then the upper transmission gate (t-gate) is activated and A is passed to F while the lower t-gate is deactivated and B is not passed to F, and if C = 0 then the lower t-gate is activated and B is passed to F while the upper t-gate is deactivated and A is not passed to F. Thus, the new solid-state CNTMUX functions exactly as a 2-to-1 multiplexer (MUX).

Intra-molecular CNTFET technology [4] can be used instead of each sub-device in Fig. 3(a) (i.e., inter-molecular CNTFET [4]), in which a single CNT bundle is placed on top of three gold electrodes to produce an n-type and a p-type CNTFETs on the same substrate using the exact procedure that is used for the inter-molecular CNTFET in Fig. 3(a) [4].

3.2 Magnetic CNTMUX

Figure 4 shows the new magnetic CNTMUX [8]. The numbers on the parts of the device in Figure 4 indicate the following: (1, 2, 3, 4, 5, 6) are CNTs, (7, 8, 9, 10, 11) are electrical current directions, (12) is the body of the device which is an electrical insulator such as glass, SiO2, plastics, or any other type of electrical insulator, and (13) is a hollow space with structural walls in two opposite sides (sides of CNTs #1 and #2) made of an electrical insulator.

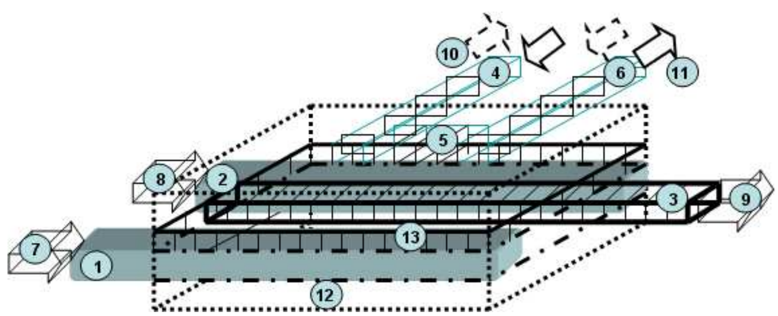


Fig. 4. The device structure of the magnetic CNTMUX.

The device in Fig. 4 operates as follows (numbers of the parts in the device in Fig. 4 are used): electrical current (7) is input B and can have two levels x and y that indicate logics 0 and 1, respectively. Electrical current (8) is input A and can have two levels x and y that indicate logics 0 and 1, respectively. The two electrical currents (7, 8) are flowing in the directions indicated in Fig. 4 in CNTs (1) and (2), respectively.

The flowing of these currents will produce magnetic fields around CNTs (1, 2) according to Amperes law in Maxwells equations [14] (in MKS measurement system):

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \tag{1}$$

where $\nabla \times \vec{B}$ is the curl of the magnetic field, μ_0 is permeability, ε_0 is permittivity, $\vec{J}$ is current density, and $\partial\vec{E}/\partial t$ is the time derivative of the electric field. The direction of such magnetic fields follow the conventional right-hand thumb rule [14].

CNT (3) initially is in contact with either CNT (1) or CNT (2), and thus a current would be running in CNT (3) in the same direction as currents (7, 8). An electrical current is pumped through CNTs (4, 5, 6), and this current can be pumped either in clock wise (CW) direction (10) or counter clock wise (CCW) direction (11). The electrical current (10) flowing in CNTs (4, 5, 6) will produce a magnetic field according to Maxwells equations in a direction according to the right-hand thumb rule, and if current (11) flows in CNTs (4, 5, 6) a magnetic field will be produced in an opposite direction of that which is produced by current (10). The direction of the current flowing in CNTs (4, 5, 6) and thus the direction of the magnetic fields plays the role of the control signal in a regular MUX as follows: if current (11) is flowing in CNTs (4, 5, 6) then an attractive Lorentz force occurs [14]:

$$\vec{F} = \vec{I} \times \vec{B} \tag{2}$$

where $\vec{I} \times \vec{B}$ is the cross product between current $\vec{I}$ in a conductor that lays in magnetic field $\vec{B}$, and $\vec{F}$ is the Lorentz force between two current-carrying conductors that occurs between CNT (5) and CNT (3) that causes CNT (3) to move in the space (13) towards CNT (5) and thus makes a contact with CNT (2) which means that current (8) or input A is selected to flow to the output. On the other hand if current (10) is flowing in CNTs (4, 5, 6) then a repulsive Lorentz force occurs between CNT (5) and CNT (3) that causes CNT (3) to move in the space (13) away from CNT (5) and thus makes a contact with CNT (1) which means that current (7) or input B is selected to flow to the output. Thus, the nano mechanical device in Fig. 4 implements a 2-1 logic MUX (or selector).

It has to be noted that one has to wait a period of time equals to T in order to obtain the result at the output. This time indicates the traveling (or displacement) time needed for CNT (3) to make a contact with either CNT (1) or CNT (2) depending on the direction of the electrical current flowing in CNTs (4, 5, 6).

4 Multiple-Valued Computations Using the New CNT Devices

Multiple-valued computing [15, 16] will be illustrated using the CNTMUXs from Section 3 for the case of ternary radix Galois field GF(3). Although the demonstration of the use of the new CNT-based multiplexers is for the case of GF(3), implementations over higher radices of Galois logic follow the same proposed method.

GF(3) addition and multiplication tables are shown in Figure 5.

	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

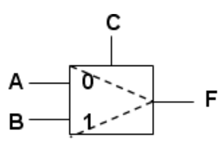
(a)

	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

(b)

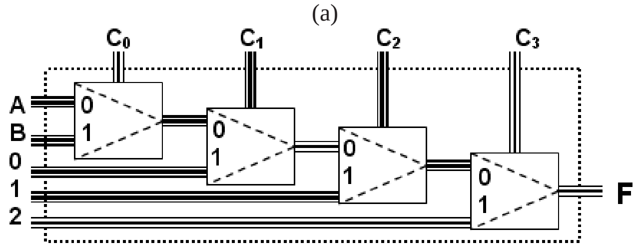
Fig. 5. (a) GF(3) addition, and (b) GF(3) multiplication.

As was demonstrated in Sects. 3.1 and 3.2, CNTMUX can be constructed using methods in Figs. 3 and 4. These CNTMUXs perform as a selector; a single input is transmitted to the output from several inputs by setting a control line to a specific state. A multiplexer-based circuit that implements Figs. 5(a) and 5(b) is shown in Fig. 6, where Fig. 6(a) can be any of the 2-input single output CNTMUXs from Figs. 3 and 4. The internal nano interconnects in Fig. 6(b) can be implemented using CNTs as well (cf. Fig. 8), where $\equiv$ means a metallic CNT used as a nanowire [2].



A	B	C	F
a	b	0	a
a	b	1	b

(a)



(b)

Fig. 6. (a) Multiplexer (MUX) symbol that can be implemented using Fig. 3 or Fig. 4 CNTMUXs where {a, b} are any input values, and (b) circuit that uses MUX from Fig. 6(a) to implement GF(3) addition and multiplication tables from Fig. 5.

In Fig. 6(b), A and B are two ternary input variables that can take any value from the set $\{0,1,2\}$, inputs $\{0,1,2\}$ are constant inputs, and inputs C_k $(k = 0,1,2,3)$ are two-valued control variables that take values from the set $\{0,1\}$. Note that Fig. 6(b) implements Figs. 5(a) and 5(b) by using the appropriate values of control variables C_k that select the variable inputs $\{A,B\}$ and constant inputs

$\{0,1,2\}$. Table 2 shows an example for the implementation of Figs. 5(a) and 5(b) using Fig. 6(b).

Table 2. An example for the implementation of Figs. 5(a) and 5(b) using Fig. 6(b), where $+$ means GF(3) addition from Fig. 5(a), $*$ means GF(3) multiplication from Fig. 5(b), $+\mathrm{C}_k$ means control variable C_k to implement $+$ operation, and $*\mathrm{C}_k$ means control variable C_k to implement $*$ operation.

A	B	$+C_0$	$+C_1$	$+C_2$	$+C_3$	$*C_0$	$*C_1$	$*C_2$	$*C_3$	$+$	$*$
0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	0
0	2	1	0	0	0	0	0	0	0	2	0
1	0	0	0	0	0	1	0	0	0	1	0
1	1	0	0	0	1	0	0	0	0	2	1
1	2	0	1	0	0	1	0	0	0	0	2
2	0	0	0	0	0	1	0	0	0	2	0
2	1	0	1	0	0	0	0	0	0	0	2
2	2	0	0	1	0	0	0	1	0	1	1

Since many multiple-valued circuits over GF(3) will be synthesized using the addition and multiplication operations from Figs. 5(a) and 5(b), the circuit in Fig. 6(b) can be used in multiple-valued implementations whenever GF(3) addition and multiplication are applied, and the internal nano interconnects can be implemented using metallic CNTs [2]. A simple example is presented in Example 1.

Example 1. Let us implement the ternary function $F = x_1x_2x_4 + x_3x_2 + x_1x_3$ using the addition and multiplication operations that are realized using Fig. 6(b), where $\equiv$ means a metallic CNT used as a nanowire [2].

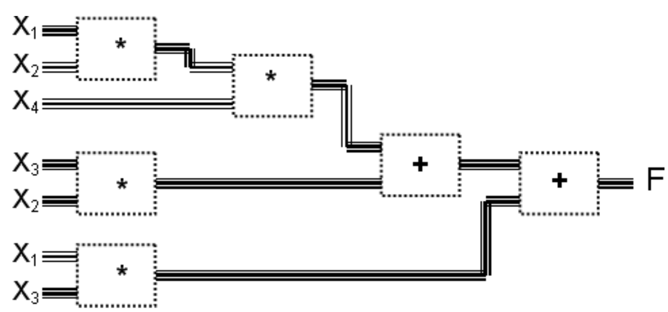
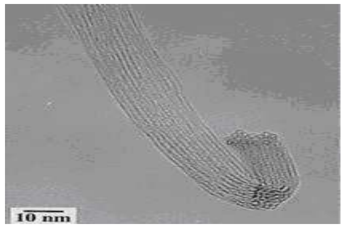


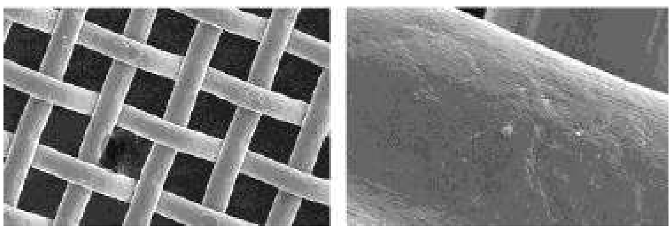
Fig. 7. GF(3) circuit to implement the ternary function $F = x_1x_2x_4 + x_3x_2 + x_1x_3$ using 2-input single output operators from Fig. 5.

For instance, the corresponding GF(3) addition and multiplication operations that are used in Fig. 7 could be implemented using Table 2 (that specifies input values to Fig. 6(b)) for specifying values to the various inputs.

As stated previously, the internal nano interconnects in Figs. 6(b) and 7 can be implemented using metallic CNTs [2], especially as CNT possesses the important properties of small size, high resilience, and very low electron scattering (cf. Properties #1, #4, and #13 in Table 1). Several efficient methods for implementing such interconnects have been reported by growing a SWCNT between two metal catalyst islands (such as iron Fe, cobalt Co, nickel Ni, yttrium Y, or molybdenum Mo). Controlling the growth of CNTs using catalysts is illustrated in Fig. 8 [17].



(a)



(b)

Fig. 8. (a) TEM image of a bundle of SWCNTs catalyzed by Ni/Y mixture, and (b) growing CNT wires on catalysts: CNT meshes on which the metal catalyst is coated.

5 Conclusions and Future Work

In this paper, two novel types of Carbon Nano Tube (CNT)-based multiplexers (MUXs) are introduced: The first CNTMUX is a solid-state transmission gate (t-gate) multiplexer that uses CNT as the channel in the Field Effect Transistors (FET) of both n-FET and p-FET types that are used, and the second CNTMUX uses the fundamental Lorentz magnetic force as a switching mechanism between two conducting CNTs. The implementation of multiple-valued Galois logic using the new CNT devices is also demonstrated.

A 2 - to - 1 multiplexer (MUX) is a basic building block of “switch logic”. The concept of the *switch logic* is that logic circuits are implemented as combination of switches, rather than a combination of logic gates as in the *gate logic*, which proves to be less-costly in synthesizing wide variety of logic circuits such as a 2 - to - 1

MUX. Since a multiplexer is of fundamental importance in logic design, the new devices can have a wide spectrum of implementations in a wide variety of nano circuits.

Future work will include items such as: (1) the modeling and simulation of the new nano devices using nano-oriented computer simulators, (2) the fabrication and test of the new MUX nano devices, (3) the integrated application of the new nano devices in system-level computer and electro-mechanical implementations, and (4) the investigation of performing CNT-based computations in low-power emerging technologies such as using *m*-valued reversible and quantum computing.

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