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Second Order Statistics of the Signal in Ricean-Lognormal Fading Channel with Selection Combining

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Abstract: Level crossing rate and average fade duration of signal envelope of selection diversity are numerically determined. Statistical characterization in fading and shadowing mobile radio communication enveronment are very useful and in this paper that is computed for Ricean fading channels with lognormal shadowing. The results are presented for independent identically distributed diversity branches when selection combining technique is used.

Keywords: Level crossing rate, average fade duration, shadowed Rice fading channel, selection combining.

1 Introduction

Wireless communication systems are subjected to strong fading influence which is caused by signal multipath propagation and the appearance of shadowing. Shadowing effect is weakening signal due to specific propagation environment between transmitter and receiver, whereas multipath propagation of the signal occurs due to multiple signal reflections. Multipath propagation which is connected with receiver and/or transmitter movement leads to drastic and occasional received signal amplitude fluctuation. Fading of 30 to 40 dB in relation to mean value of received signal can occur several times in one second depending on mobile unit speed and carrier frequency. Due to the random nature of these occurrences, signal entering the receiver of the wireless telecommunication digital system is random procces. Due to randomness of these occurrences radio signal is observed on the statistical basis [1].

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One of the ways to enhance communication without increasing transmitting power and channel bandwidth are diversity techniques because they can minimize fading effects since signal fading at the same time interval on two or more paths simultaneously rarely happens. Since probability of having two deep fades from two non-correlated signals simultaneously during the same time intervals is small, fading effect can be diminished by signal combining [2]. One of the simplest methods is selection combining-SC. Major advantage of this kind of combining is smaller receiver complexity compared to other well known diversity techniques like equal gain combining-EGC, maximal ratio combining-MRC and generalized selection combining-GSC. With receivers using this combining technique of M branches we estimate current value of signal to noise ratio on all branches picking the one with highest value. Diversity receivers are used in order to diminish negative effects of fading channels and channel interference in wireless digital communication systems [3, 4]. The results of the combining is fading reduction so it is quite expected that statistical parameters change under the influence of the diversity combining.

Probability density function (pdf) and cumulative distribution function (cdf) are first order statistical parameters and can be used only for time invariable fading parameters estimation like moments (mean value, variance) or digital system error probability. Second order statistical parameters level crossing rate-LCR and average fading duration-AFD have their application in modeling and design of wireless communication systems, such as determining parameters of the equivalent code channel based on Markov's model with finite number of conditions, as well as in probability estimation of packet errors of certain lenght [5]. A very acceptable statistical model for received signal anvelope, when direct component (line of sight LoS) does not exist, was proposed by Suzuki [6, 7] and that process is product of Rayleigh and lognormal process. Suzuki model is adequate for land mobile channel but LoS is present in land mobile satellite channels and then it is necessary to use the product of Ricean and lognormal process. Using this statistical model shadowing effect affects both the direct LoS and scattered components. LCR and AFD are determined also for the case of Ricean fading with shadowing in the case when Doppler power spectral density psd is asimetric [6] and for the same case when Doppler power spectral density is simetrical [8, 9]. Those papers considered classical signal reception without applying combining techniques.

This paper has been written with the aim to determine second order statistical parameters for Ricean fading channels with lognormal shadowing for the case when selective combining technique is used. Independent and identical branches are observed and LCR and AFD are numerically determined using analytical expression presented in this paper.

2 Joint pdf of Signal Envelope and its Derivate

Received signal envelope $Z(t)$ can be modeled as a product of two independent processes, Ricean fading process $X(t)$ and lognormal shadow process $S(t)$, meaning $Z(t) = X(t)S(t)$ [6]. In order to obtain LCR and AFD we need to find not just probability density function of signal envelope but also probability density function of signal envelope derivative.

The pdf $p_x(x)$ of Ricean fading channel is given [10]:

$$p_X(x) = \frac{x}{\sigma^2} \exp\left\{-\frac{x^2+\rho^2}{2\sigma^2}\right\} I_0\left(\frac{x\rho}{\sigma^2}\right), \qquad x \geq 0, \tag{1}$$

where ρ is no centrality parameter, $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind and σ^2 is the variance of the Gaussian scattering components.

LoS component is characteried not only by amplitude ρ but also with Doppler frequencey f_ρ and phase θ_ρ. For the direct component to be independent of time it is necessary that it satisfies a sufficient condition [3, 11]:

$$\int_0^{2\pi} f_m \hat{p}(\theta) \cos\theta d\theta = f_\rho$$

where $\hat{p}(\theta)$ is the angle of arrival (AoA) distribution of scatter component and f_m is the maximum Doppler frequency. In the case of isotropic scatter and when $f_\rho = 0$ (corresponding to a LoS component with AoA of $\pi/2$ relative to the line of motion of the mobile) the variance of the time derivative of a Ricean process $\dot{X}(t)$ is determined only by the scattering components and maximum Doppler frequency [3, 11].

The derivative of signal anvelope $X(t)$ has Gausian distribution [11]:

$$p_{\dot{X}}(\dot{x}) = \frac{1}{\sqrt{2\pi\dot{\sigma}^2}} \exp\left\{-\frac{\dot{x}^2}{2\dot{\sigma}^2}\right\}, \qquad -\infty \leq \dot{x} \leq \infty, \tag{2}$$

where $\dot{\sigma}^2 = 2\pi^2 f_m^2 \sigma^2$. The processes $X(t)$ and $\dot{X}(t)$ are considered to be independent in the case of isotropic scattering and symmetrical power spectral density of signal envelope [8, 9], in which case $p_{X\dot{X}}(x,\dot{x}) = p_X(x)p_{\dot{X}}(\dot{x})$ is valid.

The lognormal process $S(t)$ is derived from Gaussian noise process $u(t)$ with zero mean and unit variance according to $S(t) = \exp\{m + nv(t)\}$. $v(t)$ is a real Gaussian noise process, which is the output of a low pass filter with transfer function $H(f) = \sqrt{\left(1/\sqrt{2\pi}\sigma_c\right)\exp\left\{-\left(f/\sigma_c\right)^2/4\right\}}$. The σ_c is related to the 3 dB cutoff frequency f_c according to $f_c = \sigma_c\sqrt{2\ln 2}$ [8].

Process $S(t)$ has lognormal distribution [8, 9]:

$$p_S(s) = \frac{1}{\sqrt{2\pi} sh\sigma_s} \exp\left\{ -\frac{1}{2} \left(\frac{\ln s - h\mu_s}{h\sigma_s} \right)^2 \right\}, \quad s \geq 0, \tag{3}$$

where $h = \ln(10)/20$, $m = h\mu_s$ and $n = h\sigma_s$. Equation (3) represents slow varying local mean signal level in a mobile radio environment where $h\mu_s$ and $(h\sigma_s)^2$ are respectively the mean and variance of $20\log_{10} S(t)$. Area mean signal level is S_0 and $\mu_s = 20\log_{10} S_0$. In distribution defined in such a way, μ_s, as well as σ_s (σ_s characterizes the decibel spread of the slowly varying shadowing component $S(t)$), given in decibels.

The derivative of signal anvelope $S(t)$ has Gausian distribution [6, 8]:

$$p_{\dot{S}}(\dot{s}|s) = \frac{1}{\sqrt{2\pi a} sh\sigma_s} \exp\left\{ -\frac{1}{2a} \left(\frac{\dot{s}}{sh\sigma_s} \right)^2 \right\}, \quad -\infty \leq \dot{s} \leq \infty, \tag{4}$$

where $a = (2\pi\sigma_c)^2$. Generally, frequency f_c is much smaller than f_m. Processes $S(t)$ and $\dot{S}(t)$ are independent, that is [6]:

$$p_{S\dot{S}}(s,\dot{s}) = p_S(s)\, p_{\dot{S}}(\dot{s}). \tag{5}$$

Received signal envelope $Z(t)$ can be modeled as a product of two random independed variables $Z(t) = S(t)X(t)$ with joint pdf $p_{XS}(x,s)$. This joint pdf can be obtained using equation [12]:

$$p_Z(z) = \int_{-\infty}^{\infty} \frac{1}{|y|} p_{XS}\left(\frac{z}{y}, y \right) dy. \tag{6}$$

By substituting equations (1) and (3) in equation (6), pdf for Ricean-lognormal process obtains following form:

$$p_Z(z) = \int_0^{\infty} \frac{1}{y^2} \frac{z}{\sigma^2} \exp\left\{ -\frac{\left(\frac{z}{y}\right)^2 + \rho^2}{2\sigma^2} \right\} I_0\left(\frac{z\rho}{y\sigma^2} \right) p_S(y) dy, \qquad z \geq 0. \tag{7}$$

In order to obtain LCR we need to find combined pdf of the received signal envelope $Z(t)$ and its derivative, which is determined using equation [6]:

$$p_{Z\dot{Z}}(z,\dot{z}) = \int_0^{\infty} \int_{-\infty}^{\infty} \frac{1}{y^2} p_{X\dot{X}}\left(\frac{z}{y}, \frac{\dot{z}}{y} - \frac{\dot{y}z}{y^2} \right) p_{S\dot{S}}(y,\dot{y})\, d\dot{y} dy. \tag{8}$$

Combining equations (1), (2), (3) and (4) into equation (8) we obtain:

$$
\begin{aligned}
p_{Z\dot{Z}}(z,\dot{z}) = {} & \frac{1}{2\pi h\sigma_s\sigma^2}\cdot\int\limits_0^{\infty}\frac{z}{y^3}\frac{1}{\sqrt{a\left(h\sigma_S\right)^2 z^2+\dot{\sigma}^2y^2}}\cdot \\
& \cdot\exp\left\{-\frac{\left(\frac{z}{y}\right)^2+\rho^2}{2\sigma^2}\right\} I_0\left(\frac{z\rho}{y\sigma^2}\right)\cdot \qquad z\geq 0,\ \ -\infty\leq\dot{z}\leq\infty \\
& \cdot\exp\left\{-\frac{\dot{z}^2}{2\left[a\left(h\sigma_S\right)^2 z^2+\dot{\sigma}^2y^2\right]}\right\}\cdot \\
& \cdot\exp\left\{-\frac{1}{2}\left(\frac{\ln y-h\mu_S}{h\sigma_S}\right)^2\right\}dy
\end{aligned}
\tag{9}
$$

3 LCR and AFD

The level crossing rate at envelope, R, is defined as the rate at which a fading signal envelope crosses level R in a positive, or negative going direction. Denoting the received signal envelope and its time derivative by r and $\dot{r}$, respectively, the average level crossing rate, N_R, is given by [1, 2]:

$$
N_R(r)=\int\limits_0^{\infty}\dot{r}\,p_{R\dot{R}}(r,\dot{r})\,d\dot{r}, \tag{10}
$$

where $p_{R\dot{R}}(r,\dot{r})$ is the joint probability density function of r and $\dot{r}$.

A SC combiner picks the input branch with the largest instantaneous signal-to-noise ration. Assuming the noise powers of the M input branches are the same, the joint pdf of SC output signal envelope and its time derivative is [13, 14]:

$$
p_{R\dot{R}}(r,\dot{r})=\sum_{i=1}^{M}p_{Z_i\dot{Z}_i}(r,\dot{r})\prod_{\substack{k=1\\k\neq i}}^{M}P_{Z_k}(r). \tag{11}
$$

Signal envelope distribution for every branch is $p_{Z_i}(r)$, $i=1,2,\ldots,M$ and it is defined with equation (6) for Ricean lognormal fading channels, while:

$$
P_{Z_k}(R)=\int\limits_0^{R}p_{Z_k}(r)dr. \tag{12}
$$

The average envelope fade duration is defined as the average time for which fading envelope remains below a specified level after crossing that level in a downward direction, and is given by [1, 2]

$$T_r = \frac{P_r\left(r \leq R\right)}{N_R}. \tag{13}$$

Cumulative distribution function of the SC combined output signal enevelope is:

$$P_r(r) = \prod_{i=1}^{M} P_{Z_i}(r). \tag{14}$$

Forward displayed analysis for LCR and AFD calculation is valid for diversity system with selection combining at the output and it can be applied for the system with nonidentical and independent branches. Numerical results are shown for the system with equal Ricean lognormal process parameters, which does not affects derived conclusions.

4 Numerical Results

Fig 1 illustrates the LCR, normalized to f_m, for SC for diferent number of diversity branches M, and for $\rho = 0$ and $\rho = 3$. The resultant signal envelope crosses lower levels at lower rates and as the number of diversity branches increases LCR is decreasing. The presence of a stronger LoS component generally causes a lower LCR. The result of evaluation of the AFD is shown in Fig 2 for various number of input branches. Fade duration is decreasing whereas the number of branches is increasing. Lower AFD is obtained for higher amplitude of LoS component.

Figs 3 and 4 illustrate numerical results for LCR and AFD, normalized to f_m, for dual diversity system, for various area mean signal levels μ. For lower levels when mean signal level increases, LCR decreases. Maximum LCR is smaller when LoS component is larger. AFD is decreasing for larger mean signal levels.

Observing Fig. 3 we can notice that maximum LCR value is nearly constant with the change of parameter μ for $\rho = const$. On the Fig. 5 dependence of the R_m level is shown for which we obtain maximum LCR value depending on parameter μ for dual diversity system and $\sigma_s = 4$dB. When $\rho = 0$, maximum LCR value is 0,935 and with the increase of parameter μ value of level which maximizes LCR is increased, for instance for $\mu = 1$dB, $R_m = 1.6$, while for $\mu = 10$dB, $R_m = 4.6$. For $\rho = 3$, maximum LCR value is 0.482 and it is obtained for $\mu = 1$dB on $R_m = 4.4$ while it is for $\mu = 10$dB on $R_m = 12.3$.

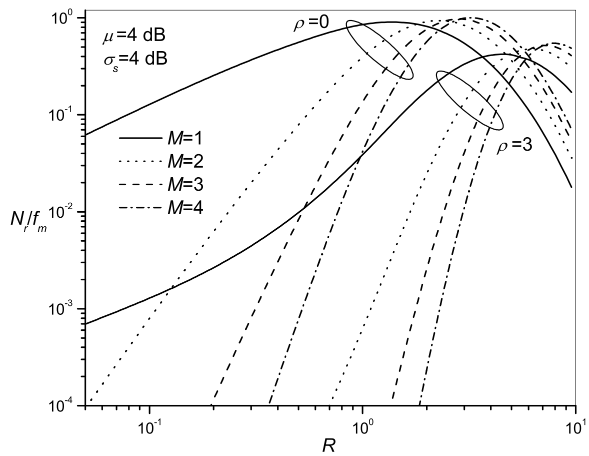

Fig. 1. Normalized LCR for Ricean fading channel with shadowing with SC combining for various number of diversity branches M, $f_m/f_c = 100$

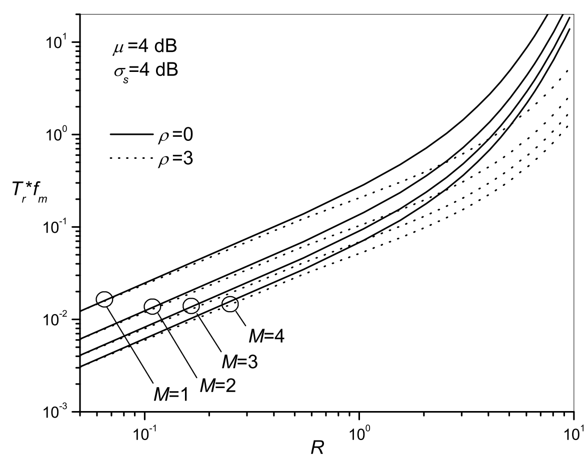

Fig. 2. Normalized AFD for Ricean fading channel with shadowing with SC combining for various number of diversity branches M, $f_m/f_c = 100$

The LCR and AFD have been computed for various σ_s and are presented in Figs 6 and 7. In Fig 6 we observe a smaller σ_s leading to a smaller LCR. This indicates that when the instantaneous signal suffers less shadowing, it crosses the

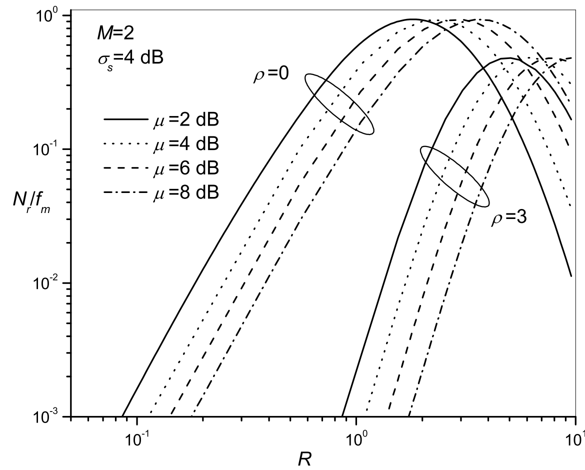

Fig. 3. Normalized LCR for Ricean fading channel with shadowing with SC combining for various of area mean signal level μ, $f_m/f_c = 100$

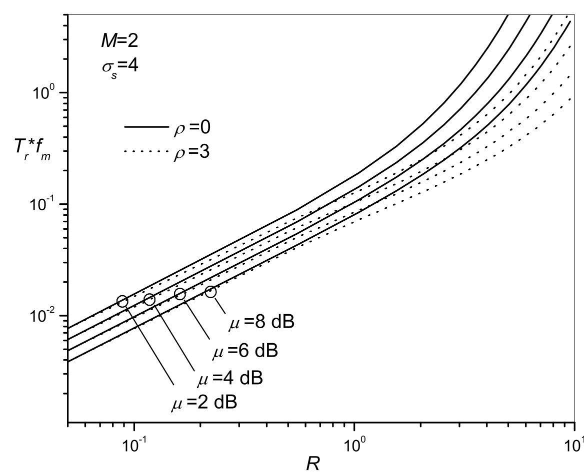

Fig. 4. Normalized AFD for Ricean fading channel with shadowing with SC combining for various of area mean signal level μ, $f_m/f_c = 100$

threshold R less frequently. Maximum LCR value during change of the parameter σ_s is obtained at the approximately same level R_m.

On the Fig. 8 dependence of the maximum LCR value from the parameter σ_s

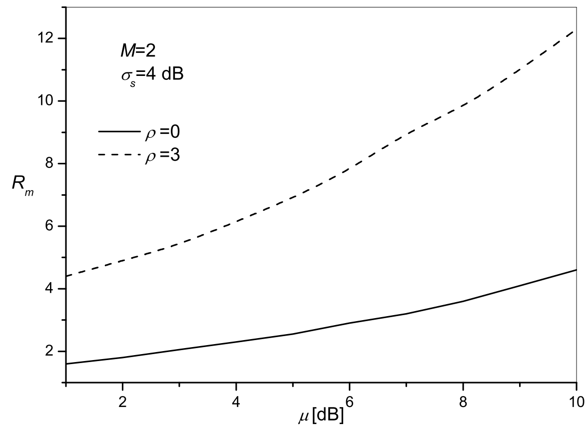

Fig. 5. Level R_m providing maximum LCR for Ricean fading channel with shadowing with SC combining depending on area mean signal level μ, $f_m/f_c = 100$

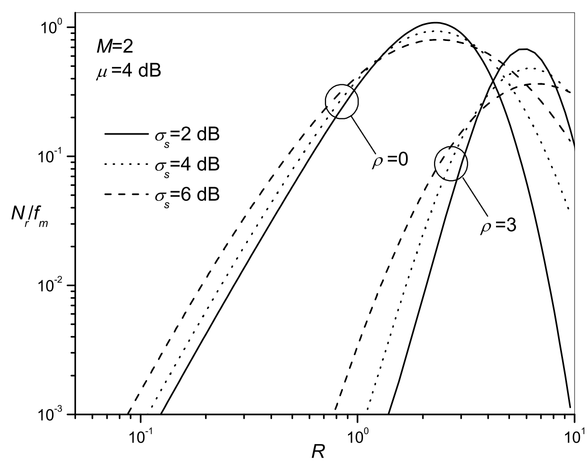

Fig. 6. Normalized LCR for Ricean fading channel with shadowing with SC combining for various decibel spread σ_s, $f_m/f_c = 100$

for $\mu = 4$dB, $M = 2$ and for $\rho = 0$ and $\rho = 3$. For $\rho = 0$, $\sigma_s = 1$dB at $R_m = 2.3$ we obtain maximum LCR $\max(N_r/f_m) = 1.145$ while for the $\sigma_s = 5$dB, $\max(N_r/f_m) = 0.865$. When $\rho = 3$ maximum LCR value for $\sigma_s = 1$dB is obtained for $R_m = 5.8$ and is equal $\max(N_r/f_m) = 0.813$, while for the $\sigma_s = 6$dB, $\max(N_r/f_m) = 0.365$ is at $R_m = 6.7$.

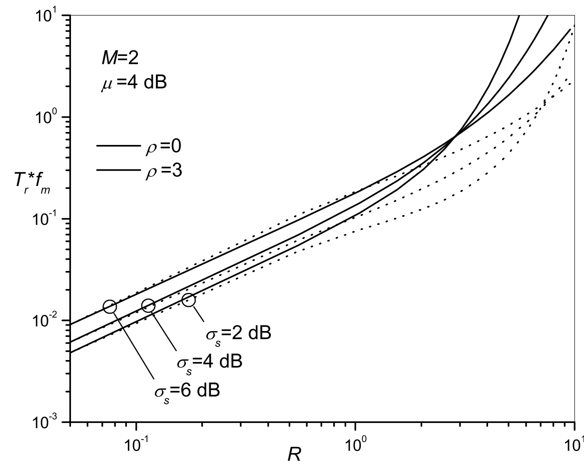

Fig. 7. Normalized AFD for Ricean fading channel with shadowing with SC combining for various decibel spread σ_s, $f_m/f_c = 100$

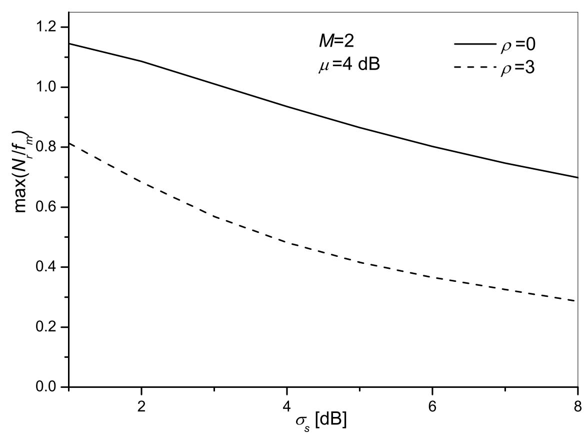

Fig. 8. Maximum normalized LCR for Ricean fading channel with shadowing with SC combining depending of decibel spread σ_s, $f_m/f_c = 100$

5 Conclusion

Numerical results for level crossing rate and average fade duration for M branch selection combining in Ricean-lognormal environment have been obtained. Influence of lognormal shadowing parameters on LCR and AFD for dual branch diversity system was also considered in this paper. The use of selection diversity combining techniques improves the reception with deep fades being smoothed out.

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