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The Influence of Semi-Spherical Inhomogenity on the Linear Grounding System Characteristics

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Abstract: The influence of semi-spherical ground inhomogenity on quasistationary charachteristics of linear grounding systems (LGS) was analyzed. For that purpose, firstly electric scalar potential (ESP) distribution in the vicinity of wire electrodes grounding system was determined. Afterwards, corresponding system of integral equations (SIE) was formed having distribution of leakage current from conductor surfaces as unknown function. The SIE is solved using Method of Moments (MoM). In deriving procedure of quasistationar expressions for ESP, quasistationary image theory in flat and modified image theory in spherical semi-conducting mirror are used. The application of proposed model is illustrated with many different examples of grounding systems.

Keywords: Quasistationary EM field, linear grounding systems, semi-spherical inhomogenity, image theory.

1 Introduction

Modelling ground inhomogenity as semi-sphere can be useful in analysis of different grounding system problems. It can be applied to solving e.g. grounding system in the vicinity of vertical container (silage, reservoir) having semi-spherical bases with a lower one buried in the ground, or pillar ground electrode, where concrete foundation is approximated with semi-spherical ground inhomogenity. The model described and applied in this paper can be used also for analyzing the influence on grounding system of large holes in the ground (pond, small lake) filled with water. In this way, those holes are assumed as semi-spherical ground inhomogenities.

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In order to solve above-mentioned problems, general model for calculating ESP of the point current source (PCS) placed inside or outside semi-sphere inhomogenity is formed. This is carried out by combining quasistationary image theory for homogeneous ground and spherical inhomogenity. In limit case, when specific conductivity of semi-spherical inhomogenity tends to infinitely large value, the general model becomes known model for determining conductive semi-sphere influence on grounding system characteristics.

The analysis of non homogeneous ground on the grounding characteristics modelled with homogeneous layers or vertical sectors was research subject of many authors, e.g. [1–4], and also of the authors of this paper , e.g. [5–8].

Concerning spherical inhomogenity, authors could use models for determining influence of conductive ([9, 10]), and dielectric spherical bodies ([11–17]), on point source electric field distribution. The problem of this kind was solved in 1894 (Böcher) and an overview of the ESP solutions for point charge in the presence of dielectric sphere is given by Stratton in ([11], pp. 201-205). Stratton in [11] solved described problem by direct solving of Poisson, i.e. Laplace partial differential equation using method of separating variables. The obtained solution includes infinite series needed to be numerically summed. In recent years, many authors have given solution for the ESP based on the image theory of fictitious sources that model influence of the spherical inhomogenity. This StrattonXs solution is cited as exact and referent one.

Hanakkam also analyzed problems of this kind in [12], and obtained general solution for the ESP expressed in the form that includes class of integrals not having solution in a closed form.

Using different mathematical procedures Lindell et al. obtained same solution as in [12]. According to that, cases of point charges outside and inside inhomogenity are separately analyzed in [13] and [14].

In papers [18] and [19] author deduces the closed form solution for the ESP of the PCS current in the presence of spherical inhomogenity in two steps. In the first one, the author assumes a part of the solution that corresponds to images in the spherical mirror and approximately satisfies boundary conditions on the sphere surface. In the second step, assumed solutions are broadened by infinite sums that approximately correspond to the ones that occur as the exact general solution in [11], i.e. in other words, approximately satisfy the Laplace partial differential equation. Afterwards, unknown constants in the sum are obtained satisfying the boundary condition for ESP and for total current density normal component on the sphere surface. These solutions enable summing of infinite sums and presenting the ESP general solution in a closed form. They are confirmed theoretically and illustrated numerically in [20].

Finally, in [20] one improved approximate solution is proposed which also includes the contents presented in this paper. The obtained ESP solution fully satisfies Poisson, i.e. Laplace equation as well as boundary condition for ESP, and approximately satisfies boundary condition for total current density. In comparison with approximate solution from [18] and [19], rather better accordance is obtained comparing with the exact solution from [11].

The model used in this paper is based on the combination of quasistationary model for EM field of Hertz's dipole buried in homogeneous ground ([6], [8]) and expression for the ESP Green function of the PCS source in the presence of semiconducting sphere proposed in [18] and [19]. Using described models the general method is developed and expressions for the ESP of LGS in arbitrary ground point are derived. In order to calculate ESP value, it is necessary to determine unknown leakage current distribution (ULCD) from conductor surface in the surrounding ground. It is carried out with solving the SIE having leakage current distribution as unknown function. The SIE is formed using condition that electrode surface is approximately equipotential and it can be numerically solved applying Method of Moments (MoM, [21]).

The described model has been already applied for analyzing influence of the semi-spherical semi-conducting inhomogenity on characteristics of point ground electrode ([22]), contour circle wire ground electrode ([23]) as well as of single wire electrode characteristics ([24]).

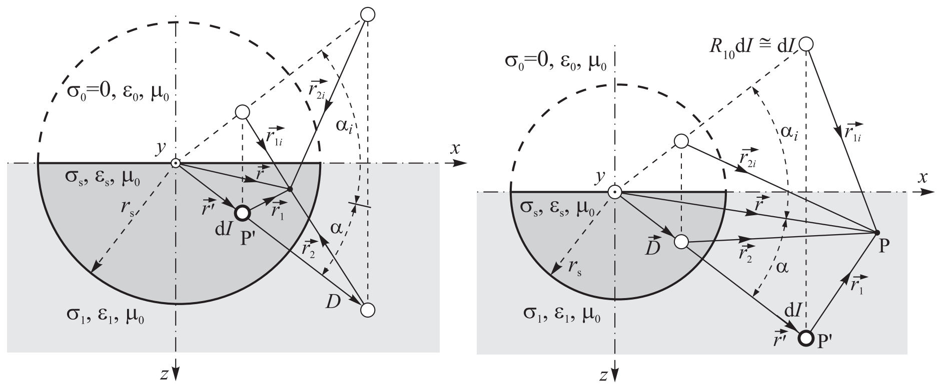
In addition, the model was used for determining grounding characteristics of thin plate electrode placed outside semi-spherical inhomogenity ([25]). In last mentioned case, the SIE was solved combining MoM and Equivalent Electrodes Method (EEM, [26]).

In this paper, developed model is applied on the analysis of the grounding systems formed by two ground wire electrodes as two represents of the LGS. The program package for numerical calculations based on the described procedure is realized and large number of numerical experiments was performed. Small part of the results is presented in fourth chapter of the paper.

2 Theoretical Basis of the Model

The non-homogeneous semi – conducting ground approximated with two linear, isotropic and homogeneous semi – conducting domains is considered in the paper. The first one is semi – sphere of the radius r_s and known electrical parameters σ_s, $\varepsilon_s = \varepsilon_0\varepsilon_{rs}$ and $\mu_s = \mu_0$ (σ_s – specific conductivity, $\varepsilon_s = \varepsilon_0\varepsilon_{rs}$ – permittivity, $\mu_s = \mu_0$ – permeability).

The second domain is homogeneous isotropic semi – conducting semi – space of known electrical parameters σ_1, $\varepsilon_1 = \varepsilon_0\varepsilon_{r1}$ and $\mu_1 = \mu_0$. In this paper are also used the following labels: $\underline{\sigma}_i = \sigma_i + \mathrm{j}\omega\varepsilon_i$ – complex conductivities, $\underline{\gamma}_i = (\mathrm{j}\omega\mu_0\underline{\sigma}_i)^{1/2}$ – complex propagation constants for $i = 0, 1, \mathrm{s}$; ω – angular frequency; and $\underline{n}_{1\mathrm{s}} = \underline{\gamma}_1/\underline{\gamma}_\mathrm{s}$, $\underline{n}_{i0} = \underline{\gamma}_i/\underline{\gamma}_0$, $i = 1, \mathrm{s}$ – refraction coefficients between ground/semi – sphere, ground/air and semi – sphere/air, respectively. Point ground electrode is placed in arbitrary point P' and fed by current of intensity dI and of very low frequency f. Descartes' coordinate system having origin at the semi – sphere centre is associated to the described geometry. Illustrations for two of four possible point ground electrode problem geometries and part of the images are presented in Fig.1.



(a) (b)

Fig. 1. Point current source outside (a) and inside (b) semi-conducting semi-spherical inhomogenity and part of discrete images for determining potential outside (a) and inside (b) inhomogenity.

Combining image theory models, final approximate expressions for the ESP Green functions at ground arbitrary point are derived for the PCS placed outside ($r' \geq r_\mathrm{s}$, Fig.1a) and inside semi-spherical inhomogenity ($r' \leq r_\mathrm{s}$, Fig.1b). It is carried out with successive application of the image theory in the flat semi – conducting mirror ([8]) and Green's functions for the PCS source placed outside/inside semi – conducting sphere. The last ones are used in the form that was proposed for the first time in [18] and [19]. This function satisfies boundary condition for potential at the boundary surface of spherical inhomogenity

$$\varphi_1(r = r_\mathrm{s} + 0) = \varphi_\mathrm{s}(r = r_\mathrm{s} - 0), \tag{1}$$

as well as the conduction for normal components of total current densities

$$\underline{\sigma}_1 \frac{\partial \varphi_1}{\partial r}|_{r=r_\mathrm{s}+0} = \underline{\sigma}_\mathrm{s} \frac{\partial \varphi_\mathrm{s}}{\partial r}|_{r=r_\mathrm{s}-0}, \tag{2}$$

and approximately satisfies Poisson, i.e. Laplace equation ([18–20]).

2.1 The PCS outside semi – sphere

The electrical scalar potential at the point P in the surrounding of the PCS located outside semi-spherical inhomogenity, Fig.1(a) ([18–20, 22]), is expressed as

$$\begin{aligned} \mathrm{d}\varphi_{11}(\vec{r},\vec{r}') \cong \frac{\mathrm{d}I}{4\pi\underline{\sigma}_1}\Big\{ \Big[\frac{1}{r_1} + R_{1\mathrm{s}}\frac{r_\mathrm{s}}{r'}\Big(\frac{1}{r_2}-\frac{1}{r}\Big) + \frac{T_{1s}R_{1s}}{2}\frac{1}{r'}\ln\frac{r-D\cos\alpha+r_2}{2r}\Big] \\ + \Big[\frac{1}{r_{1i}} + R_{1\mathrm{s}}\frac{r_\mathrm{s}}{r'}\Big(\frac{1}{r_{2i}}-\frac{1}{r}\Big) + \frac{T_{1s}R_{1s}}{2}\frac{1}{r'}\ln\frac{r-D\cos\alpha_i+r_{2i}}{2r}\Big]\Big\}, r \geq r_\mathrm{s}, \end{aligned} \quad (3)$$

$$\begin{aligned} \mathrm{d}\varphi_{s1}(\vec{r},\vec{r}') \cong \frac{\mathrm{d}I}{4\pi\underline{\sigma}_1}\Big\{ \Big[\frac{T_{1\mathrm{s}}}{r_1} - \frac{R_{1\mathrm{s}}}{r'} + \frac{T_{1s}R_{1s}}{2}\frac{1}{r'}\ln\frac{r'-r\cos\alpha+r_1}{2r'}\Big] \\ + \Big[\frac{T_{1\mathrm{s}}}{r_{1i}} - R_{1\mathrm{s}}\frac{1}{r'} + \frac{T_{1s}R_{1s}}{2}\frac{1}{r'}\ln\frac{r'-r\cos\alpha_i+r_{1i}}{2r'}\Big]\Big\}, r \leq r_\mathrm{s}, \end{aligned} \quad (4)$$

where: $\mathrm{d}I$ – leakage current from the point ground electrode; $R_{1\mathrm{s}}$ and $T_{1\mathrm{s}}$ – quasi – stationary reflection and transmission coefficients, $R_{1\mathrm{s}} = T_{1\mathrm{s}} - 1 = (\underline{n}_{1\mathrm{s}}^2 - 1)/(\underline{n}_{1\mathrm{s}}^2 + 1)$; $\vec{r}'$ and $\vec{r}_i'$ are position vectors of the PCS and its image in the flat mirror, respectively; $r_1 = \sqrt{r^2 + r'^2 - 2rr'\cos\alpha}$, $r_2 = \sqrt{r^2 + D^2 - 2rD\cos\alpha}$, and $r_{1i} = \sqrt{r^2 + r'^2 - 2rr'\cos\alpha_i}$, $r_{2i} = \sqrt{r^2 + D^2 - 2rD\cos\alpha_i}$, $\cos\alpha = (\hat{r}\cdot\hat{r}')$, $\cos\alpha_i = (\hat{r}\cdot\hat{r}_i')$ – distances denoted in Fig.1; and $D = r_\mathrm{s}^2/r'$ Kelvin inversion factor. The rest of the parameters in the expressions (3) and (4) can be noticed in Fig.1a. Reflection and transmission coefficients in the application of semi – conducting flat mirror image theory, $R_{i0} = T_{i0} - 1 = (\underline{n}_{i0}^2 - 1)/(\underline{n}_{i0}^2 + 1) \cong 1$, $i = 1, \mathrm{s}$ are approximately equal to one, because $\underline{n}_{10}, \underline{n}_{\mathrm{s}0} >> 1$. This allows us to express ESP with finite number of images in spherical mirror.

Two indexes are used to label the potential. The first one denotes the medium in which potential is determined and the second one, the medium where the PCS is placed.

2.2 The PCS inside semi – sphere

In the way similar to those ones described in the previous chapter, the expressions for the ESP at the point P in the surroundings of the PCS located inside semi – spherical inhomogenity are derived, Fig.1b ([18–20]). Those expressions have the following form:

$$\begin{aligned} \mathrm{d}\varphi_{1s}(\vec{r},\vec{r}') \cong \frac{\mathrm{d}I}{4\pi\underline{\sigma}_1}\Big\{ \Big[\frac{T_{1\mathrm{s}}}{r_1} - \frac{R_{1\mathrm{s}}}{r} + \frac{T_{1s}R_{1s}}{2}\frac{1}{r_\mathrm{s}}\ln\frac{r-r'\cos\alpha+r_1}{2r}\Big] \\ + \Big[\frac{T_{1\mathrm{s}}}{r_{1i}} - \frac{R_{1\mathrm{s}}}{r} + \frac{T_{1s}R_{1s}}{2}\frac{1}{r_\mathrm{s}}\ln\frac{r-r'\cos\alpha_i+r_{1i}}{2r}\Big]\Big\}, r \geq r_\mathrm{s}, \end{aligned} \quad (5)$$

$$\mathrm{d}\varphi_{\mathrm{ss}}(\vec{r},\vec{r}') \cong \frac{\mathrm{d}I}{4\pi\underline{\sigma}_1}\Big\{\Big[\frac{\underline{n}_{1\mathrm{s}}^2}{r_1} - R_{1\mathrm{s}}\Big(\frac{r_\mathrm{s}}{r'}\frac{\underline{n}_{1\mathrm{s}}^2}{r_2} + \frac{1}{r_\mathrm{s}}\Big) + \frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\frac{1}{r_\mathrm{s}}\ln\frac{D - r\cos\alpha + r_2}{2D}\Big] + \Big[\frac{\underline{n}_{1\mathrm{s}}^2}{r_{1i}} - R_{1\mathrm{s}}\Big(\frac{r_\mathrm{s}}{r'}\frac{\underline{n}_{1\mathrm{s}}^2}{r_{2i}} + \frac{1}{r_\mathrm{s}}\Big) + \frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\frac{1}{r_\mathrm{s}}\ln\frac{D - r\cos\alpha_i + r_{2i}}{2D}\Big]\Big\}, r \le r_\mathrm{s}. \tag{6}$$

The labels used in expressions (5)–(6) correspond to those used in expressions (3)–(4) and can be noticed from Fig. 1b. In the text that follows, corresponding Green's functions , $G_{ij}(\vec{r},\vec{r}'), i, j = 1, \mathrm{s}$, defined on the basis of expressions (3)–(6) will be used,

$$\mathrm{d}\varphi_{ij}(\vec{r},\vec{r}') = \mathrm{d}I G_{ij}(\vec{r},\vec{r}'), \quad i, j = 1, \mathrm{s}. \tag{7}$$

2.3 Accuracy of the proposed ESP expression

As it has been already explained in previous text, ESP Green's function for PCS outside/inside semi-spherical inhomogenity (expressions (3)–(6)) are derived by approximate expressions for ESP Green's function of the PCS localized outside /inside spherical inhomogenity (proposed in [18, 19]) and using semi – conducting flat mirror image theory ([5–8]).

The exact ESP Green's function for the same problem can be also derived using exact solution for Green's functions of the PCS placed outside/inside spherical inhomogenity. These solutions satisfy boundary conditions (1) and (2) as well as Poisson, i.e. Laplace differential equation ([11], [20]) and for system shown in Fig.1a is given with two following expressions:

$$\begin{aligned}\mathrm{d}\varphi_{11}(\vec{r},\vec{r}') = \frac{\mathrm{d}I}{4\pi\underline{\sigma}_1}\Big\{&\Big[\frac{1}{r_1} + R_{1\mathrm{s}}\frac{r_\mathrm{s}}{r'}\Big(\frac{1}{r_2} - \frac{1}{r}\Big) - \frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\sum_{n=0}^{\infty}\frac{1}{n + \frac{T_{1\mathrm{s}}}{2}}\frac{r_\mathrm{s}^{2n+1}}{(r'r)^{n+1}}P_n(\cos\alpha)\Big]\\ &+ \Big[\frac{1}{r_{1i}} + R_{1\mathrm{s}}\frac{r_\mathrm{s}}{r'}\Big(\frac{1}{r_{2i}} - \frac{1}{r}\Big) - \frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\sum_{n=0}^{\infty}\frac{1}{n + \frac{T_{1\mathrm{s}}}{2}}\frac{r_\mathrm{s}^{2n+1}}{(r'r)^{n+1}}P_n(\cos\alpha_i)\Big]\Big\}, r \ge r_\mathrm{s},\end{aligned} \tag{8}$$

$$\begin{aligned}\mathrm{d}\varphi_{\mathrm{s}1}(\vec{r},\vec{r}') = \frac{\mathrm{d}I}{4\pi\underline{\sigma}_1}\Big\{&\Big[\frac{T_{1\mathrm{s}}}{r_1} - \frac{R_{1\mathrm{s}}}{r'} - \frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\frac{1}{r'}\sum_{n=0}^{\infty}\frac{1}{n + \frac{T_{1\mathrm{s}}}{2}}\Big(\frac{r}{r'}\Big)^n P_n(\cos\alpha)\Big]\\ &+ \Big[\frac{T_{1\mathrm{s}}}{r_{1i}} - \frac{R_{1\mathrm{s}}}{r'} - \frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\frac{1}{r'}\sum_{n=0}^{\infty}\frac{1}{n + \frac{T_{1\mathrm{s}}}{2}}\Big(\frac{r}{r'}\Big)^n P_n(\cos\alpha_i)\Big]\Big\}, r \le r_\mathrm{s}.\end{aligned} \tag{9}$$

The exact ESP Green's function are for the point source inside inhomogenity, Fig.1b,

$$\begin{aligned}\mathrm{d}\varphi_{1\mathrm{s}}(\vec{r},\vec{r}') =& \frac{\mathrm{d}I}{4\pi\underline{\sigma}_1}\Big\{\Big[\frac{T_{1\mathrm{s}}}{r_1}-\frac{R_{1\mathrm{s}}}{r}-\frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\frac{1}{r'}\sum_{n=0}^{\infty}\frac{1}{n+\frac{T_{1\mathrm{s}}}{2}}\Big(\frac{r}{r'}\Big)^{n+1}P_n(\cos\alpha)\Big]\\ &+\Big[\frac{T_{1\mathrm{s}}}{r_{1i}}-\frac{R_{1\mathrm{s}}}{r}-\frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\frac{1}{r'}\sum_{n=0}^{\infty}\frac{1}{n+\frac{T_{1\mathrm{s}}}{2}}\Big(\frac{r}{r'}\Big)^{n+1}P_n(\cos\alpha_i)\Big]\Big\}, r\geq r_{\mathrm{s}},\end{aligned} \tag{10}$$

$$\begin{aligned}\mathrm{d}\varphi_{\mathrm{ss}}(\vec{r},\vec{r}') =& \frac{\mathrm{d}I}{4\pi\underline{\sigma}_1}\Big\{\Big[\frac{\underline{n}_{1\mathrm{s}}^2}{r_1}-R_{1\mathrm{s}}\Big(\frac{r_\mathrm{s}}{r'}\frac{\underline{n}_{1\mathrm{s}}^2}{r_2}+\frac{1}{r_\mathrm{s}}\Big)\\ &-\frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\sum_{n=0}^{\infty}\frac{1}{n+\frac{T_{1\mathrm{s}}}{2}}\frac{(r'r)^n}{r_\mathrm{s}^{2n+1}}P_n(\cos\alpha)\Big]\\ &+\Big[\frac{\underline{n}_{1\mathrm{s}}^2}{r_{1i}}-R_{1\mathrm{s}}\Big(\frac{r_\mathrm{s}}{r'}\frac{\underline{n}_{1\mathrm{s}}^2}{r_{2i}}+\frac{1}{r_\mathrm{s}}\Big)\\ &-\frac{T_{1\mathrm{s}}R_{1\mathrm{s}}}{2}\sum_{n=0}^{\infty}\frac{1}{n+\frac{T_{1\mathrm{s}}}{2}}\frac{(r'r)^n}{r_\mathrm{s}^{2n+1}}P_n(\cos\alpha_i)\Big]\Big\}, r\leq r_\mathrm{s}.\end{aligned} \tag{11}$$

As it can be noticed from the exact expressions for ESP Green's function, calculating of those functions is ballasted with infinite sums, which can present significant and complex numerical problem in the case of linear ground electrodes.

The parameters in expressions (8)–(11) correspond to those ones used in expressions (3)–(6). Real part of the normalized values of the Green's functions $G_{ij}(\vec{r},\vec{r}'), i,j=1,\mathrm{s}$, at the ground surface for the system from Fig.1 are presented in Fig.2a, for the PCS is outside, and in Fig.2b for the PCS inside semi – spherical inhomogenity. Parameter values are $r_\mathrm{s}=1\mathrm{m}$, $\theta=45^0$, $\sigma_1=0.01\mathrm{S/m}$, $\varepsilon_{\mathrm{r}1}=\varepsilon_{\mathrm{rs}}=10$, $p_\mathrm{s}=\sigma_1/\sigma_\mathrm{s}=0.1$ and $r'=1.1\mathrm{m}$ (Fig.2a), i.e. $r'=0.9\mathrm{m}$ (Fig.2b). Good agreement can be noticed between the results obtained using approximate expressions (3)–(6) and exact expressions (8)–(11). Very detailed analysis of described Green's function can be found in [20].

3 Model Application on Grounding Systems

The described model can be applied for analysis of two grounding systems made of linear conductors, one placed inside and the second outside sphere. In the limit cases of homogeneous or sectoral ground very good result and satisfactory agreement is obtained. It is explained in detail e.g. in [24].

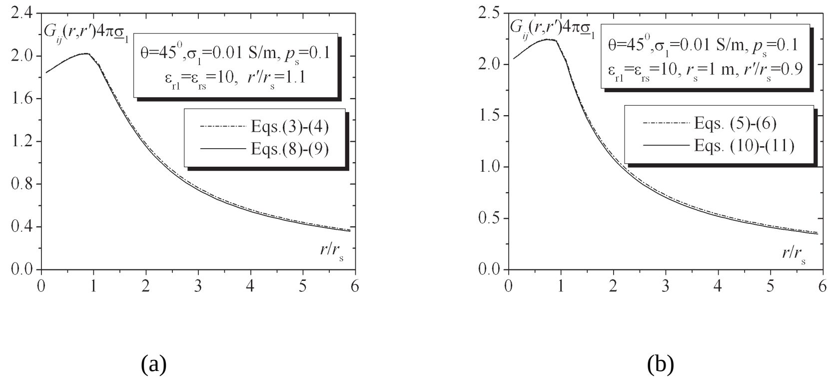


Fig. 2. ESP distribution at the ground surface for the point source outside (a) and inside (b) semi-spherical inhomogenity.

3.1 The LGS formed by two wire straight electrodes

Grounding system formed by two wire electrodes, or two grounding systems, one placed inside and the other outside inhomogenity is observed (Fig.3). Assuming electrodes as clusters of point sources, and superposing their influence, the ESP in the surroundings of the system can be expressed as

$$\varphi_1(\vec{r}) = \int_{l_1} \mathrm{d}I(\vec{r}_1')G_{11}(\vec{r},\vec{r}_1') + \int_{l_2} \mathrm{d}I(\vec{r}_2')G_{1\mathrm{s}}(\vec{r},\vec{r}_2'), \tag{12a}$$

$$\varphi_\mathrm{s}(\vec{r}) = \int_{l_1} \mathrm{d}I(\vec{r}_1')G_{\mathrm{s}1}(\vec{r},\vec{r}_1') + \int_{l_2} \mathrm{d}I(\vec{r}_2')G_{\mathrm{ss}}(\vec{r},\vec{r}_2'), \tag{12b}$$

where: $\mathrm{d}I(\vec{r}_k') = I_f(\vec{r}_k')\mathrm{d}l_k$, $I_f(\vec{r}_k')$, $k = 1,2$, – leakage current density per unit length of k-th wire electrode, $\mathrm{d}l_k$ – corresponding differential length element and $G_{ij}(\vec{r},\vec{r}'), i, j = 1,\mathrm{s}$ – the ESP Green functions given with (3)–(6). The expressions (12a)–(12b) are valid for the observed points placed outside, $\varphi_1(\vec{r})$, i.e. inside inhomogenity $\varphi_\mathrm{s}(\vec{r})$.

After applying expressions (12a)–(12b) for determining potential at the conductor surfaces, the SIE is formed, since $\varphi_1 \cong U_1$, i.e. $\varphi_\mathrm{s} \cong U_2$ at the electrodes 1 and 2 surfaces, respectively. With U_1 and U_2 are denoted feeding voltages.

For numerical solving of obtained SIE the MoM is used. The conductor 1 is divided in N segments of length $\triangle_1$ and conductor 2 in M segments having length $\triangle_2$. Potential values $\varphi_1 = U_1$, and $\varphi_\mathrm{s} = U_2$ given by (12a)–(12b) are matched in the points at the surface of conductor 1 and conductor 2, respectively. In this way

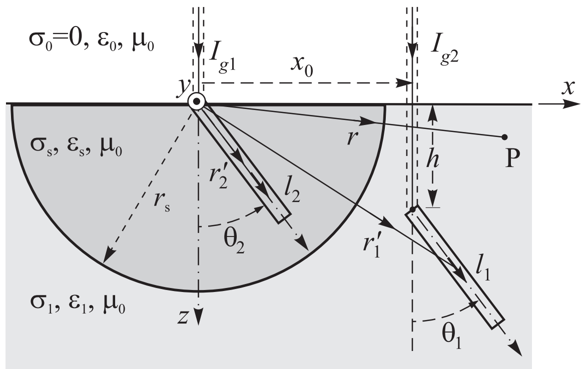


Fig. 3. Illustration of grounding system formed by two wire electrodes placed inside and outside inhomogenity.

N equations are formed having form

$$U_1 = \sum_{n=1}^{N} \frac{I_{1n}}{\Delta_1} \int_{\Delta_1} G_{11}(\vec{r}_{pm1}, \vec{r}_1{}')\mathrm{d}l_1 + \sum_{m=1}^{M} \frac{I_{2m}}{\Delta_2} \int_{\Delta_2} G_{1\mathrm{s}}(\vec{r}_{pm1}, \vec{r}_2{}')\mathrm{d}l_2, \qquad (13a)$$

and M equations of the following form

$$U_2 = \sum_{n=1}^{N} \frac{I_{1n}}{\Delta_1} \int_{\Delta_1} G_{\mathrm{s}1}(\vec{r}_{pm2}, \vec{r}_1{}')\mathrm{d}l_1 + \sum_{m=1}^{M} \frac{I_{2m}}{\Delta_2} \int_{\Delta_2} G_{\mathrm{ss}}(\vec{r}_{pm2}, \vec{r}_2{}')\mathrm{d}l_2, \qquad (13b)$$

where $\vec{r}_{pm1}$ and $\vec{r}_{pm2}$ are position vectors of points matched on the first, i.e. of the second conductor, respectively, and I_{1n}, $n = 1, 2, \ldots, N$ and I_{2m}, $m = 1, 2, \ldots, M$ are unknown leakage currents of the n-th i.e. m-th segment. The last ones are obtained as solutions of the system (13a)–(13b). The SIE is solved for two regimes, so-called symmetric ($U_1 = U_2 = U_\mathrm{s} = 1\mathrm{V}$) and asymmetric ($U_1 = -U_2 = U_\mathrm{a} = 1\mathrm{V}$) regime of feeding ([27]). Using the same procedure as in [27], after determining "Y" parameters, corresponding "Z" parameters of LGS can be obtained. "Y" and "Z" parameters represent integral LGS characteristics. Its analysis provides reliable estimation of the validity of the model and used methods, as well as estimation of accuracy of numerically determined leakage current in the system. The total leakage currents from conductors 1 and 2 are $I_{g1}^{s/a} = \sum_{n=1}^{N} I_{1n}$ and $I_{g2}^{s/a} = \sum_{m=1}^{M} I_{2m}$, respectively. Labels "$s/a$" denote solution for the currents corresponding to the supplying potential U_s – symmetric, i.e. U_a – asymmetric. In the general case the known following relations between electrodes voltage and their feeding currents are,

$$U_1 = Z_{11}I_{g1} + Z_{12}I_{g2}, \tag{14a}$$
$$U_2 = Z_{21}I_{g1} + Z_{22}I_{g2}, \tag{14b}$$

where Z_{11} and Z_{22} are self-impedances of LGS–1 (conductor 1) and LGS–2 (conductor 2), while $Z_{12} = Z_{21}$ are mutual-impedances between LGSs, i.e. electrodes 1 and 2. If conductors in Fig.3 form the unique LGS, substituting $U_1 = U_2 = U_s = 1\text{V}$ in (14), grounding impedance can be determined as

$$Z_g = R_g + jX_g = \frac{1}{(I^s_{g1} + I^s_{g2})}. \tag{15}$$

3.2 The LGS formed by contour electrode and wire conductor

The procedure described in previous chapter can be applied for solving of the LGS formed by wire electrode placed inside semi – conducting semi – spherical inhomogenity (e.g. armature in concrete foundation) and contour circle conductor placed outside inhomogenity, as shown in Fig.4. The ESP of this system can be determined using expression (13a)–(13b). Integration in the first addenda of the expressions given with (13a)–(13b) is done along the contour electrode length. Wire conductor and contour electrode are divided in the N and M segments and when procedure completely analogue to the one described in 3.1 is applied, the "Z" – parameters of observed grounding system are also determined.

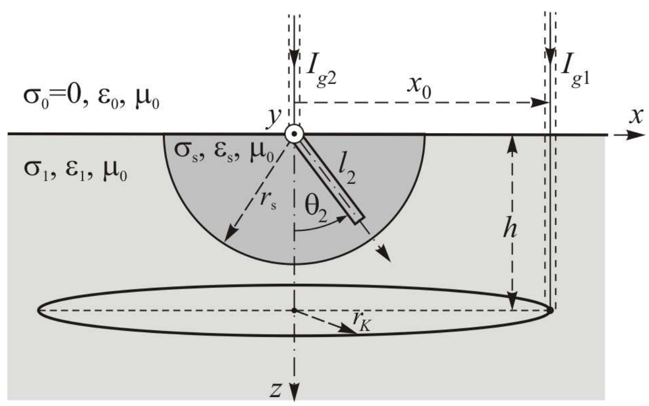
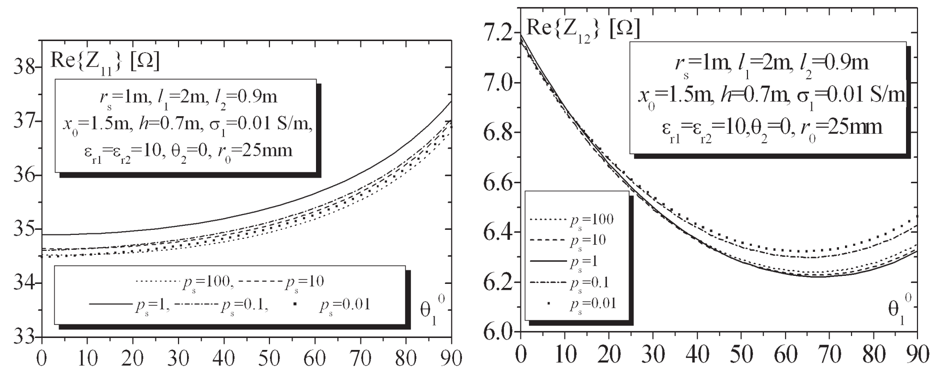


Fig. 4. Illustration of the tower grounding system modelled by wire conductor placed in semi-spherical foundation and contour electrode feeding with isolated earthing conductor.

4 Numerical Results

4.1 The LGS formed by two wire ground electrodes

The "*Z*" – parameters of the grounding system formed by two wire electrodes, Fig.3, are determined applying procedure from 3.1.



(a) (b)

Fig. 5. Real parts of grounding impedance Z_{11} and mutual impedance $Z_{12} = Z_{21}$ of the system shown in Fig.3, versus angle θ_1 for ratio $p_s = \sigma_1/\sigma_s$ as parameter.

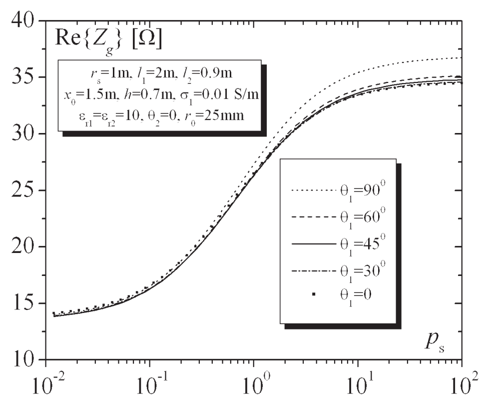


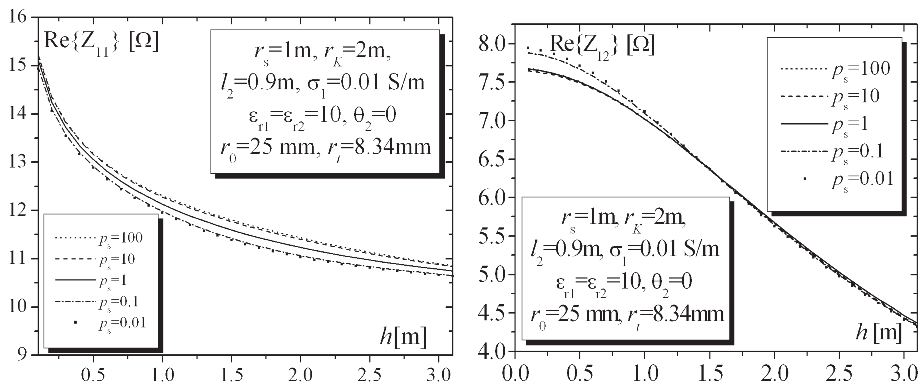
Fig. 6. Real part of the impedance Z_g of the system from Fig.3, versus ratio $p_s = \sigma_1/\sigma_s$ for angle θ_1 as parameter.

Real parts of self – impedance Z_{11} and mutual impedance Z_{12} value of the grounding system from Fig.3, versus angle θ_1, for ratio $p_s = \sigma_1/\sigma_s$, are shown in Fig.5. The parameter values are $r_s = 1\text{m}$, $h = 0.7\text{m}$, $l_1 = 2\text{m}$, $l_2 = 0.9\text{m}$, $x_0 = 1.5\text{m}$, $\sigma_1 = 0.01\text{S/m}$ and $\varepsilon_{r1} = \varepsilon_{r2} = 10$. The circle cross-section radius of both conductors is $r_0 = 25$mm, while procedure explained in 3.1 is applied for $N = 10$ and $M = 10$ segments.

If grounding system from Fig.3 is treated as unique electrode, its impedance can be obtained from the expression (15). Real part of the total impedance Z_g of the system, versus ratio $p_s = \sigma_1/\sigma_s$, for angle θ_1 as parameter is shown in Fig.6. System parameters are $r_s = 1$m, $h = 0.7$m, $l_1 = 2$m, $l_2 = 0.9$m, $x_0 = 1.5$m, $\sigma_1 = 0.01$S/m and $\varepsilon_{r1} = \varepsilon_{r2} = 10$. The circle cross – section radius of both conductors is , while procedure explained in 3.1 is applied for $N = 10$ and $M = 10$ segments.

4.2 The LGS formed by contour electrode and wire conductor

The Z – parameters of the grounding system formed by wire electrode placed inside and contour electrode placed outside inhomogenity, as it is shown in Fig.4, are determined applying procedure from 3.2.



(a) (b)

Fig. 7. Real parts of self-impedance Z_{11} and mutual impedance $Z_{12} = Z_{21}$ of the system from Fig.4 versus depth h, for ratio $p_s = \sigma_1/\sigma_s$ as parameter.

Real part of self – impedance Z_{11} and mutual impedance Z_{12} for the system in Fig.3 versus depth h of buried contour conductor is shown in Fig.7. In this example are $r_s = 1$m, $r_k = 2$m, $l_2 = 0.9$m, $\sigma_1 = 0.01$S/m, $\varepsilon_{r1} = \varepsilon_{r2} = 10$ and $\theta_2 = 0$, while ratio $p_s = \sigma_1/\sigma_s$ is parameter. The radius of circle cross-section of wire conductor is $r_0 = 25$mm and equivalent radius of conductor strip used for contour electrode is $r_i = 8.34$mm. The procedure described in 3.2 is applied for $N = 10$ and $M = 10$ segments on wire conductor and contour electrode, respectively.

5 Conclusion

The general method for determining influence of the semi – conducting semi – spherical inhomogenity on grounding systems characteristics is proposed for analyzing and solving different types of the LGS. The analytical part of the presented method is based on quasistationary image theories for plate and spherical semi – conducting mirror. The solution of SIE formed in that way is leakage current distribution from the electrodes surface. After that, SIE for the ULCD is created and numerically solved using Method of Moments. After that, all other parameters (grounding impedance and Z – parameters) are determined using usual procedures. The method has been already applied for the limit cases, as for the LGS in the presence of homogeneous and sectoral ground inhomogenity. The presented results indicate the good validity of the method.

The grounding system formed by two wire straight conductors or contour electrode are observed as a two coupled LGSs. The influence of the semi-sphere electrical parameters as well as system geometry on grounding characteristics is analyzed.

Using developed model of the LGS in the presence of semi – spherical semi – conducting ground inhomogenity, characteristics of few practical problems can be theoretically and numerically modeled. It is realized by simple choice of the semi – sphere and ground electrical parameters and illustrated geometry. Practical problems of this kind are, for example:

- Arbitrary configuration for the grounding system where electrode is placed at the coast or sunked in the semi – sphere which approximates pound or lake;
- Ferro concrete foundation grounding system of the pillar approximated with semi – sphere with, or without especially placed electrodes in the foundation surrounding.
- Ground electrode in the presence of ideal conducting semi-sphere ($\sigma_s \to \infty$), or in the presence of the semi – spherical cavity ($\sigma_s = 0$ and $\varepsilon_s = \varepsilon_0$);
- In the limit case of geometry model for $x_0 - r_s << r_s$, when $r_s \to \infty$, the model can be used for solving problems of grounding systems in the inhomogenity of sectoral type ([5,7]); etc.

Described methodology can be also applied on the problems of semi – spherical geometries on the flat ground surface, and supplying conductors can be in the ground or in semi – sphere or in the air.

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