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Iterative Successive MMSE Multi-User MIMO Transmit Filtering

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Abstract: In this paper we introduce a novel linear precoding technique. It was previously reported in the literature that when the user terminals are equipped with one antenna, minimum mean-squared-error (MMSE) in combination with successive interference cancellation is optimum on the uplink, while MMSE precoding in combination with Tomlinson-Harashima precoding (THP) is optimum on the downlink. The linear precoding technique introduced in this paper is based on the modified MSE criterion. It can serve the users that are equipped with arbitrary number of antennas with only limitation that the total number of users in the system has to be less than or equal to the rank of the combined multiple-input multiple-output (MIMO) channel matrix of all users. It was shown in the simulations that it extracts very high diversity gain and at low signal-to-noise ratios, when the total number of antennas at the user terminals is greater than the number of antennas at the base station, it approaches the maximum sum rate capacity of the broadcast channel. The technique introduced in this paper is favorable for practical implementation since it requires by an order of magnitude less operations than the techniques based on the singular value decomposition.

Keywords: MIMO systems, Multi-user MIMO, SDMA, transmit signal processing.

1 Introduction

Multiple-input, multiple-output (MIMO) systems are a key component of future wireless communication systems, because of their promising improvement in terms of performance and bandwidth efficiency [1], [2], [3], [4], [5]. Such systems have the potential to combine the high capacity achievable with MIMO processing with the benefits of space division multiple access (SDMA). It has been shown that time

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division multiple access (TDMA) systems cannot achieve a linear increase of sum-rate capacity of MU MIMO system in the number of transmit antennas [6], [7]. The solution to this problem is to serve users simultaneously using SDMA.

Miminum mean-squared-error (MMSE) filtering with successive interference cancellation (SIC) achieves the maximum sum rate capacity of a multiple-access channel and extracts the maximum antenna array and diversity gain [8]. MMSE transmit filtering in combination with Tomlinson-Harashima precoding (THP) provides high diversity on the downlink [9]. In a MU MIMO system employing MMSE precoding, if the user terminal is equipped with more than one antenna, the signal transmitted to each antenna needs to be precoded independently. This results in a significant performance loss.

Using a modified MSE cost function, a successive MMSE (SMMSE) precoding and decoding functions were introduced in [10] and [11], respectively. SMMSE does not have the dimensionality problem and provides higher array and diversity gain than other similar MU MIMO precoding techniques like [12],[9],[13], [14]. Although the techniques proposed in [15], like iterative regularized block diagonalization (IRBD), empirically achieve sum rate capacity of the broadcast channel, extract full diversity gain, and very high antenna array gain, they require very high computational effort.

In this paper we introduce an iterative SMMSE (iSMMSE) precoder that has lower computational complexity than techniques that require multiple calculations of the singular value decomposition (SVD), but still provides very good antenna and diversity gain. At low signal-to-noise ratios (SNRs) and when the number of antennas at the user terminals is greater than the number of antennas at the base station, iSMMSE approaches in simulations the sum rate capacity of the broadcast channels.

This paper is organized as follows. In Section 2 we introduce a MU MIMO system model. In Section 3, we describe the MU downlink system and the precoding techniques that will be compared. In Section 4, we present the results of simulations. A short summary follows in the Section 5.

2 System model

We consider a MU MIMO downlink channel, where M_T transmit antennas are located at the base station and M_{R_i} receive antennas are located at the i^{th} user terminal (UT), $i = 1, 2, \ldots, K$. There are K users (or UTs) in the system. The total number of receive antennas is

$$M_R = \sum_{i=1}^{K} M_{R_i}.$$

A block diagram of such a system is depicted in Fig. 1.

We use the notation $\{M_{R_1},\ldots,M_{R_K}\}\times M_T$ to describe the antenna configuration of the system. First, we assume frequency flat slow fading channels. In case of frequency selective channels, we assume transmission using OFDM where the same MIMO processing is performed on each subcarrier. Let the MIMO channel of user i be denoted as $\boldsymbol{H}_i \in \mathbb{C}^{M_{R_i}\times M_T}$. Then, the combined channel matrix is given by

$$\boldsymbol{H} = \left[\begin{array}{cccc} \boldsymbol{H}_1^T & \boldsymbol{H}_2^T & \cdots & \boldsymbol{H}_K^T \end{array}\right]^T \in \mathbb{C}^{M_R\times M_T}. \tag{1}$$

The data vectors $\boldsymbol{x}_k \in \mathbb{C}^{r_k\times 1}$, $k=1,\ldots,K$, for the K UTs are stacked in the vector $\boldsymbol{x} = \left[\boldsymbol{x}_1^T,\ldots,\boldsymbol{x}_K^T\right]^T \in \mathbb{C}^{r\times 1}$. The received vector is given by

$$\boldsymbol{y} = \boldsymbol{G}\left(\boldsymbol{H}\boldsymbol{F}\boldsymbol{x}+\boldsymbol{n}\right) \tag{2}$$

where

$$\boldsymbol{y} = \left[\begin{array}{ccc} \boldsymbol{y}_1^T & \cdots & \boldsymbol{y}_K^T \end{array}\right]^T \in \mathbb{C}^{r\times 1}$$

is the received data vector,

$$\boldsymbol{n} = \left[\begin{array}{ccc} \boldsymbol{n}_1^T & \cdots & \boldsymbol{n}_K^T \end{array}\right]^T \in \mathbb{C}^{M_R\times 1}$$

is the stacked vector of the zero mean additive white Gaussian noise at the input of the receive antennas. The joint precoding and decoding matrices are denoted by $\boldsymbol{F}$ and $\boldsymbol{G}$, respectively.

Let us define the joint precoder matrix as

$$\boldsymbol{F} = \left[\begin{array}{cccc} \boldsymbol{F}_1 & \boldsymbol{F}_2 & \cdots & \boldsymbol{F}_K \end{array}\right] \in \mathbb{C}^{M_T\times r} \tag{3}$$

where $\boldsymbol{F}_i \in \mathbb{C}^{M_T\times r_i}$ is the i^{th} user's precoder matrix. Moreover,

$$r = \sum_{i=1}^{K} r_i \leq \operatorname{rank}\left(\boldsymbol{H}\right) \leq \min\left(M_R, M_T\right)$$

is the total number of the transmitted data streams, whereas r_i is the number of data stream sequences transmitted to the i^{th} user.

3 Iterative SMMSE transmit filter

The precoding matrix $\boldsymbol{F}$ is designed in two steps. We separate the multi-user interference (MUI) suppression and the system performance optimization. In the first step we balance the MUI suppression which is achieved by reducing the overlap of the row spaces spanned by the effective channel matrices of different users and

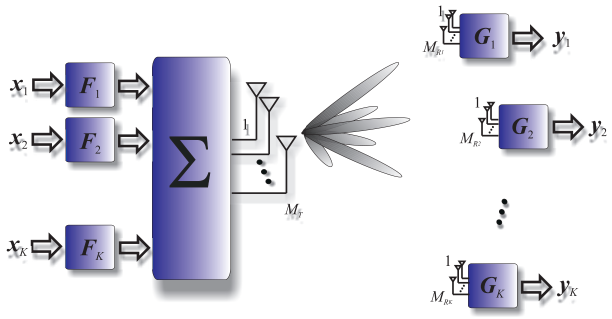

Fig. 1. Block diagram of multi-user MIMO downlink system.

any MIMO processing gain which requires that the users use as much as possible the available subspaces. In the second step we optimize the system performance assuming parallel SU MIMO channels. Thus, the precoding matrix in equation (3) is rewritten as

$$\boldsymbol{F} = \beta \boldsymbol{F}_a \cdot \boldsymbol{F}_b, \tag{4}$$

where

$$\boldsymbol{F}_a = \begin{bmatrix} \boldsymbol{F}_{a_1} & \boldsymbol{F}_{a_2} & \cdots & \boldsymbol{F}_{a_K} \end{bmatrix} \in \mathbb{C}^{M_T \times M_x},$$

and

$$\boldsymbol{F}_b = \begin{bmatrix} \boldsymbol{F}_{b_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{F}_{b_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{F}_{b_K} \end{bmatrix} \in \mathbb{C}^{M_x \times r},$$

with $\boldsymbol{F}_{a_i} \in \mathbb{C}^{M_T \times M_{x_i}}$ and $\boldsymbol{F}_{b_i} \in \mathbb{C}^{M_{x_i} \times r_i}$, $M_{x_i} \leq r$, and $M_x = \sum_{i=1}^{K} M_{x_i}$ depending on the specific choice of the precoding algorithm. The matrix $\boldsymbol{F}_a$ is used to suppress the MUI interference first, and then the matrix $\boldsymbol{F}_b$ is used to optimize the system performance according to a specific criterion assuming that the MU MIMO channel has been transformed into a set of parallel SU MIMO channels. The parameter β is chosen to set the total transmit power to P_T.

The successive MMSE (SMMSE) precoding filter $\boldsymbol{F}_a$ is derived from the linear transmit MMSE precoding optimization by neglecting the mutual contribution of the elements of one user's channel matrix to this users' MSE. Since each user can coordinate the processing over all of its antennas, we can combine the signals of different spatial streams transmitted to one user in order to extract higher diversity and array gain. The interference of other co-channel users to the signal arriving at the i^{th} user's j^{th} antenna is suppressed independently from the other antennas at the same terminal. This is done for each antenna at the same user terminal successively.

The iterative SMMSE filter exploits the fact that all users do not transmit data over the entire available subspaces. The other co-channel users can then transmit in this unused subspace in order to improve the performance without causing any additional interference.

Therefore, the j^{th} column of the i^{th} user's precoding matrix $\boldsymbol{F}^{(l)}_{a_i}$ in the l^{th} iteration, corresponding to the i^{th} user's j^{th} receive antenna, is equal to the first column of the matrix $\boldsymbol{F}^{(l)}_{a_{i,j}}$ which is obtained from the following optimization

$$\boldsymbol{F}^{(l)}_{a_{i,j}} = \arg\min_{\boldsymbol{F}^{(l)}_{a_{i,j}}} \mathrm{E}\left\{ \left\| \overline{\boldsymbol{H}}^{(l)}_{i,j} \boldsymbol{F}^{(l)}_{a_{i,j}} \overline{\boldsymbol{z}}^{(l)}_{i,j} - \overline{\boldsymbol{z}}^{(l)}_{i,j} \right\|^2_{\mathscr{F}} + \frac{\|\boldsymbol{n}\|^2_{\mathscr{F}}}{\beta^2} \right\} \tag{5}$$

such that $\beta^2 \|\boldsymbol{F}_a \boldsymbol{F}_b \boldsymbol{x}\|^2_{\mathscr{F}} \leq P_T$, $\forall i, j$. The matrix $\overline{\boldsymbol{H}}^{(l)}_{i,j}$ and the vector $\overline{\boldsymbol{z}}^{(l)}_{i,j}$ corresponding to the i^{th} user's, $i = 1, \ldots, K$, j^{th} receive antenna, $j = 1, \ldots, M_{R_i}$, are defined as

$$\overline{\boldsymbol{H}}^{(l)}_{i,j} = \begin{bmatrix} \boldsymbol{h}^{(l)\ T}_{i,j} \\ \boldsymbol{H}^{(l)}_1 \\ \vdots \\ \boldsymbol{H}^{(l)}_{i-1} \\ \boldsymbol{H}^{(l)}_{i+1} \\ \vdots \\ \boldsymbol{H}^{(l)}_K \end{bmatrix}, \text{ and } \overline{\boldsymbol{z}}^{(l)}_{i,j} = \begin{bmatrix} z^{(l)}_{i,j} \\ \boldsymbol{z}^{(l)}_1 \\ \vdots \\ \boldsymbol{z}^{(l)}_{i-1} \\ \boldsymbol{z}^{(l)}_{i+1} \\ \vdots \\ \boldsymbol{z}^{(l)}_K \end{bmatrix}$$

where $\boldsymbol{h}^{(l)\ T}_{i,j}$ is the j^{th} row of the i^{th} user's channel matrix $\boldsymbol{H}^{(l)}_i \in \mathbb{C}^{r_i \times M_T}$ and $z^{(l)}_{i,j}$ is the j^{th} element of the i^{th} user's vector $\boldsymbol{z}^{(l)}_i \in \mathbb{C}^{r_i \times 1}$. The elements of the vector $\boldsymbol{z}^{(l)}_i$ are zero mean, unit variance i.i.d. complex uniform random variables. The elements of the vector $\boldsymbol{n}$ are zero mean complex Gaussian random variables with variance σ_n^2. Note that the vectors $\boldsymbol{z}^{(l)}_i = \boldsymbol{F}^{(l)}_{b_i} \boldsymbol{x}_i$, $i = 1, \ldots, K$, are the i^{th} user's encoded data. The statistical properties of the elements of the vector $\boldsymbol{z}^{(l)}_i$, in general depend on the matrix $\boldsymbol{F}^{(l)}_{b_i}$. However, when we generate matrices $\boldsymbol{F}^{(l)}_{a_i}$ we assume that the matrices $\boldsymbol{F}^{(l)}_{b_i}$ are unitary. This assumption is true if each user is receiving independent data streams over all of the receive antennas. In that case the statistics of the elements of the vectors $\boldsymbol{z}^{(l)}_i$ are the same as the statistics of the elements of the vectors $\boldsymbol{x}_i$.

The i^{th} user's equivalent channel matrix in the l^{th} iteration is equal to:

$$\boldsymbol{H}^{(l)}_i = \boldsymbol{U}^{(r_i)\ (l-1)\ H}_i \boldsymbol{H}_i \tag{6}$$

where $\boldsymbol{U}_i^{(r_i)\ (l-1)}$ contains the first r_i vectors of $\boldsymbol{U}_i^{(l-1)}$ which is obtained from the following SVD

$$\boldsymbol{H}_i\boldsymbol{F}_a^{(l-1)} = \boldsymbol{U}_i^{(l-1)}\boldsymbol{\Sigma}_i^{(l-1)}\boldsymbol{V}_i^{(l-1)\ H}. \tag{7}$$

The first r_i vectors of $\boldsymbol{U}_i^{(l-1)}$ correspond to the r_i strongest singular values of $\boldsymbol{H}_i\boldsymbol{F}_a^{(l-1)}$. In the first iteration $\boldsymbol{H}_i^{(0)} = \boldsymbol{H}_i$ and $\boldsymbol{z}_i^{(0)} \in \mathbb{C}^{M_{R_i}\times 1}$.

The columns in the precoding matrix $\boldsymbol{F}_{a_i}^{(l)}$, each corresponding to one receive antenna or data stream, are calculated successively. The corresponding column of the precoding matrix $\boldsymbol{F}_{a_i}^{(l)}$ is equal to the first column of the following matrix:

$$\boldsymbol{F}_{a_{i,j}}^{(l)} = \left(\overline{\boldsymbol{H}}_{i,j}^{(l)\ H}\overline{\boldsymbol{H}}_{i,j}^{(l)} + \alpha \boldsymbol{I}_{M_T}\right)^{-1}\overline{\boldsymbol{H}}_{i,j}^{(l)\ H} \tag{8}$$

The parameter α is equal to $\alpha = \sigma_n^2 K/P_T$.

After calculating the precoding vectors for all receive antennas in this fashion, the equivalent combined channel matrix of all users is equal to $\boldsymbol{H}\boldsymbol{F}_a^{(l)} \in \mathbb{C}^{M_R\times r}$ after the precoding. For high SNR ratios and when $M_R \leq M_T$, this matrix is also block diagonal. We can now apply any other previously defined SU MIMO technique on the i^{th} user's equivalent channel matrix $\boldsymbol{H}_i\boldsymbol{F}_{a_i}^{(l)}$.

Thus matrices $F_{b_i}^{(l)}$, $i = 1,\ldots,K$, are equal to [16]

$$\boldsymbol{F}_{b_i}^{(l)} = \boldsymbol{V}_i^{(l)}\boldsymbol{\Phi}_i^{(l)} \tag{9}$$

where the matrix $\boldsymbol{V}_i^{(l)}$ is obtained from the SVD given in equation (7) and the matrix $\boldsymbol{\Phi}_i^{(l)}$ is a diagonal power loading matrix which depends on the specific choice of optimization [16], [8], e.g., maximum information rate, minimum MSE, minimum bit error rate (BER), etc.

4 Simulation results

In this section we compare the performance of a system employing the precoding technique introduced in this paper to SMMSE, SMMSE THP [17] and BD [12]. To this end we simulate a purely stochastic spatially white channel $\boldsymbol{H}_w$ and the second is a frequency selective MIMO channel with a power delay profile as defined by IEEE802.11n - D with non-line of sight conditions [18]. The elements of the channel matrices on each subcarrier are zero mean, unit variance complex Gaussian variables. We assume data transmission using an OFDM system with DFT size $N = 64$, a subcarrier spacing of 150 kHz and a cyclic prefix that is $N_{\text{pre}} = 4$ samples long. The data is encoded using a convolutional code rate $1/2$ $(561,753)_{\text{oct}}$. After coding the data is mapped using BPSK and QAM modulation. Coded and modulated symbols are transmitted using $N_c = 48$ subcarriers and $N_{\text{symb}} = 2$ OFDM symbols.

In the second channel model we also consider antenna correlation at the BS and UTs. Antenna correlation is modeled in the delay domain using the Kronecker model such that the channel of each user's l^{th} path component is modeled as

$$\boldsymbol{H}_i^{(l)} = \boldsymbol{R}_{R_i}^{(l)\,1/2} \boldsymbol{H}_{w_i}^{(l)} \boldsymbol{R}_{T_i}^{(l)\,1/2} \tag{10}$$

where $\boldsymbol{H}_{w_i}^{(l)}$ is a spatially white unit variance flat fading MIMO channel of dimension $M_{R_i} \times M_T$, whereas $\boldsymbol{R}_{R_i}^{(l)}$ and $\boldsymbol{R}_{T_i}^{(l)}$ are receive and transmit covariance matrices with $\operatorname{tr}\left(\boldsymbol{R}_{R_i}^{(l)}\right) = M_{R_i}$ and $\operatorname{tr}\left(\boldsymbol{R}_{T_i}^{(l)}\right) = M_T$.

For the simulations we assume a scenario where the MS is surrounded by a rich scattering environment and the BS/AP antennas are separated by less than the coherence distance. These propagation conditions correspond to a cellular communication systems typically characterized by a low angular spread at the BS/AP. On the other hand, the angular spread at the mobile is often very large and thus low spatial correlation can be achieved with relatively small antenna separation. Hence, we can write

$$\boldsymbol{R}_{R_i}^{(l)} = \boldsymbol{I}_{M_{R_i}}, \;\; \boldsymbol{R}_{T_i}^{(l)} = \frac{M_T}{\operatorname{tr}\left(\boldsymbol{A}^{(l)*} \boldsymbol{A}^{(l)T}\right)} \boldsymbol{A}^{(l)*} \boldsymbol{A}^{(l)T} \tag{11}$$

and the l-th path of i-th user channel is modeled as

$$\boldsymbol{H}_i^{(l)} = \sqrt{\frac{M_T}{\operatorname{tr}\left(\boldsymbol{A}^{(l)*} \boldsymbol{A}^{(l)T}\right)}} \boldsymbol{H}_{w_i}^{(l)} \boldsymbol{A}^{(l)T} \tag{12}$$

where $\boldsymbol{A}^{(l)} \in \mathbb{C}^{M_T \times N}$ is an array steering matrix containing N array response vectors of the transmitting antenna array corresponding to N directions of departure [19], and $\boldsymbol{H}_{w_i}^{(l)} \in \mathbb{C}^{M_R \times N}$ is a spatially white unit variance flat fading MIMO channel.

First, we show the 10 % outage capacity as a function of the ratio of the total transmit power P_T and the power of additive white Gaussian noise at the input of every antenna, σ_n^2. The capacity is calculated using the results on the capacity of MIMO broadcast channels in [3]. We also present capacity results for a TDMA system and the ”dirty-paper” code (DPC) bound [20] as a comparison. From Figure 2 we can see that when the total number of antennas at the UTs is less than or equal to the number of antennas at the base station, iSMMSE and SMMSE provide the same capacity. SMMSE and iSMMSE have higher capacity than BD at low SNRs.

However, when the total number of antennas at the user terminals is greater than the number of antennas at the base station, SMMSE experiences a capacity floor

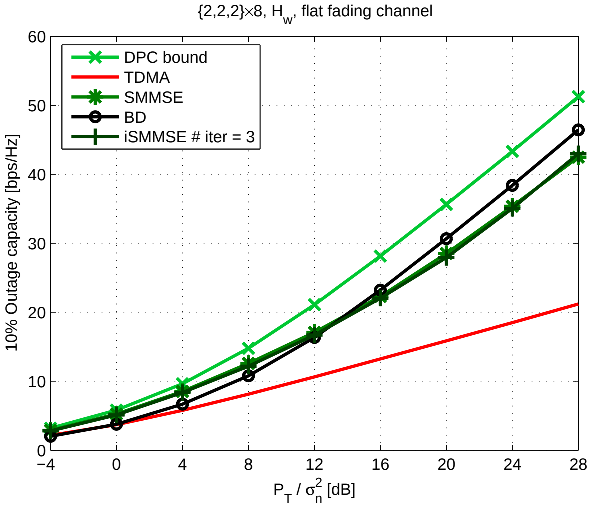

Fig. 2. 10 % outage capacity as a function of SNR.

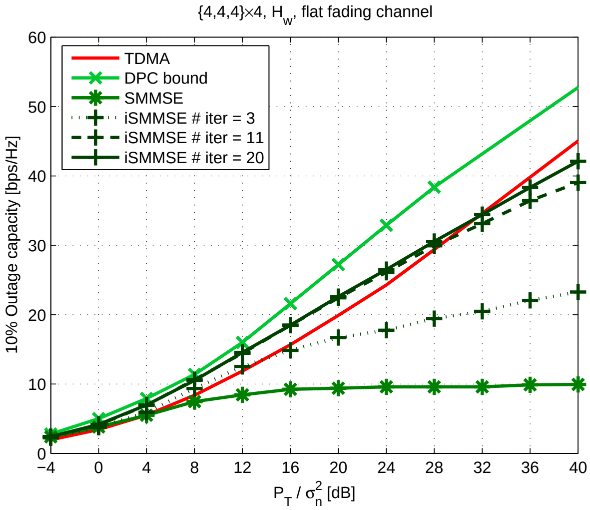

Fig. 3. 10 % outage capacity as a function of SNR.

lower than the capacity of a TDMA system. iSMMSE in this case provides higher capacity than SMMSE and by increasing the number of iterations we approach the DPC bound at low SNRs as it can be seen from Figure 3.

In Figure 4 we compare the BER performance of iSMMSE to the BER performance of SMMSE, SMMSE THP, BD and IRBD. iSMMSE provides higher diversity and array gain than nonlinear precoding technique SMMSE THP and at

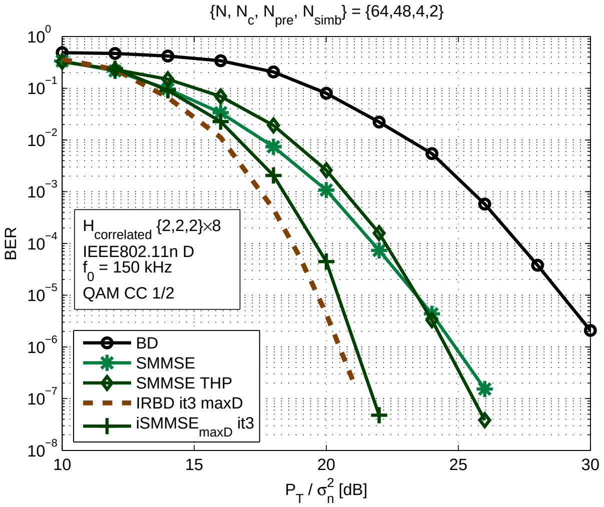

Fig. 4. BER performance comparison of iSMMSE and SMMSE, SMMSE THP, BD, and IRBD in a system with the antenna configuration $\{2,2,2\} \times 8$.

high SNRs it approaches the performance of IRBD which is much more complex.

In Figure 5 we compare the BER performance of iSMMSE with the BER performance of IRBD and SMMSE when the users subspaces significantly overlap, e.g., when $M_R > M_T$. As a reference we show also the BER curve for a similar ”genie aided” system where the users are assumed perfectly orthogonal in order to show the diversity inherent in this type of system. IRBD outperforms SMMSE. However, iSMMSE has the same performance as IRBD at low SNRs. By increasing the number of iterations we improve the diversity gain of the system and further approach the performance of IRBD. At high SNRs and with more iterations, iSMMSE extracts very high diversity gain. Even with more iterations, iSMMSE still requires less computational effort and energy than IRBD. Therefore, a good performance with relatively low complexity makes iSMMSE very attractive for practical implementation.

5 Conclusion

In this paper we have introduced a novel linear precoding technique iSMMSE. SMMSE provides higher diversity and array gain than MMSE by suppressing the co-channel interference to each antenna at one user terminal independently. By iterating the closed form solution, we improve the array and diversity gain, especially in case of high MUI when the total number of antennas at the user termi-

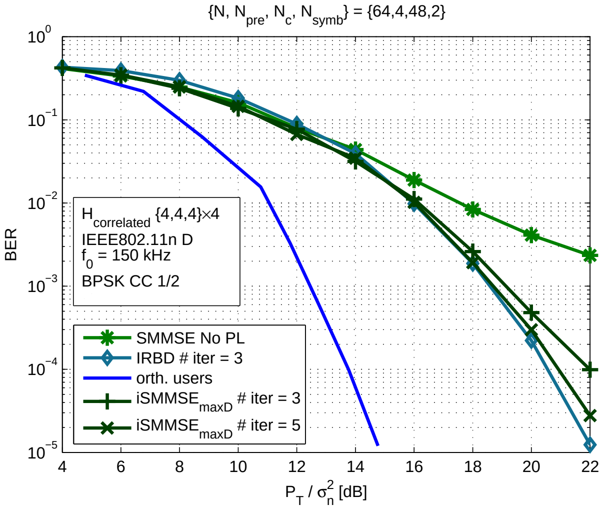

Fig. 5. BER performance comparison of iSMMSE and SMMSE and IRBD in a system with the antenna configuration $\{4,4,4\} \times 4$.

nals is greater than the number of antennas at the base station. The performance of iSMMSE improves as we increase the number of iterations and it is similar to the performance of other more complex precoding techniques that require multiple calculation of singular value decomposition. iSMMSE provides very good performance regardless of the antenna configuration with relatively low computational load and is therefore a very good candidate for practical implementation in the future multi-user MIMO systems.

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