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Electrical Field Modelling at the Cable Joints

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Abstract: There is a large number of parameters that influence to the way of the producing cable joints, and therefore the greatest attention is paid on the electrical field density shaping. This is done in order to efficiently reduce electrical field density, especially the tangential electrical field component at the insulator surface. The described problem solving presents very important and complex task of high power technology for producing power cables and corresponding accessories. Finite elements method, finite difference method, charge simulation method, boundary relaxation method or boundary elements method can be applied for cable joints calculations, as well as equivalent electrodes method (EEM) [1, 2]. The simplest calculation can be carried out using equivalent electrodes method and it obtains very high accuracy of calculated values. Equivalent electrodes method is applied on non-modelled, as well as on some geometrically modelled cable joints, when they are assumed as deflectors of funnel form, which border lines can have elliptical shape, the shape of polynomial or exponential function, as well as the form generated as the combination of the shapes mentioned above. The results for the distribution of the electrical potential, radial and axial electrical field components are presented in the paper. The equipotential surfaces in the vicinity of cable joints are also presented.

Keywords: Equivalent electrodes method, Deflectors, Elliptical, Polynomial or exponential function, Electrical potential and field distributions.

1 Introduction

Security of the electric power systems strongly depends on cable networks reliability, involving the way of cable jointing and terminating [3–9].

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At the places of cable connecting, the exterior cover is removed and the radial character of electric field is disturbed. There is high intensity of axial electric field at the end of the cable isolation [10–17].

There are many approaches for solving the problem of minimizing electrical field density at the places of power cable splicing. The most frequently used method for minimizing electrical field density in the vicinity of the sharp end that is part of the insulator screen, is based on the application of the funnel-like, appropriately modelled screen extension. This way of electrical field shaping is known as geometrical modelling.

Far away from the cable break, charge distribution at its conductors is uniform. There is uniformly distributed positive charge at the interior, $\eta_a = q'/(2\pi a)$, and negative charge at the exterior conductor, $\eta_b = -q'/(2\pi b)$. If the distances between contact places of the interior conductors ($z = 0$) and area where the charge distribution at the coaxial cables conductors can be assumed as uniform are different, $L_1 \neq L_2$, approximate expression for potential is:

$$\frac{\varphi_{\text{apr}}(r,z)}{U} = \frac{1}{2\pi\ln\frac{b}{a}} \begin{cases} \int\limits_0^\pi \ln\dfrac{f(L_2-z,B)\,f(L_2+z,B)}{f(L_1-z,A)\,f(L_1+z,A)}\mathrm{d}\theta', & z \le L_1 \le L_2 \\ \int\limits_0^\pi \ln\dfrac{f(L_2-z,B)\,f(z-L_1,A)\,f(L_2+z,B)}{A^2 f(L_1+z,A)}\mathrm{d}\theta', & L_1 \le z \le L_2 \\ \int\limits_0^\pi \ln\dfrac{B^2 f(L_2+z,B)}{f(L_1-z,A)\,f(z-L_2,B)\,f(L_1+z,A)}\mathrm{d}\theta', & L_2 \le z \le L_1 \\ \int\limits_0^\pi \ln\dfrac{f(z-L_1,A)\,f(L_2+z,B)}{f(z-L_2,B)\,f(L_1+z,B)}\mathrm{d}\theta' + 2\pi I(r), & L_1 \le L_2 \le z \end{cases} \tag{1}$$

where U is the voltage the coaxial cable is supplied by, r, θ and z are cylindrical coordinates and:

$$A^2 = r^2 + a^2 - 2ar\cos\theta'; \tag{2}$$

$$B^2 = r^2 + b^2 - 2br\cos\theta'; \tag{3}$$

$$f(u,v) = u + \sqrt{v^2+u^2}; \tag{4}$$

$$I(r) = \begin{cases} \ln\dfrac{b}{a}, & 0 \le r \le a \\ \ln\dfrac{b}{r}, & a \le r \le b \\ 0, & r \ge b. \end{cases} \tag{5}$$

2 Outline of the Method

For electric field, $E_0 = U/a$,

$$K(\frac{\pi}{2}, m) = \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\alpha}{\sqrt{1 - m\sin^2\alpha}} \tag{6}$$

the complete elliptic integral of the first kind, m its squared modulus, and

$$m(r, r', z, z') = k^2(r, r', z, z') = \frac{4rr'}{(r + r')^2 + (z - z')^2}, \tag{7}$$

axial component of electrical field is

$$\begin{aligned} \frac{E_{z\,\mathrm{sp_{apr}}}}{E_0} = \frac{a}{\pi \ln \dfrac{b}{a}} \Bigg(& \frac{K\left(\frac{\pi}{2}, m(r,b,z,L)\right)}{\sqrt{(r+b)^2 + (L-z)^2}} - \frac{K\left(\frac{\pi}{2}, m(r,a,z,L)\right)}{\sqrt{(r+a)^2 + (L-z)^2}} \\ & + \frac{K\left(\frac{\pi}{2}, m(r,b,z,-L)\right)}{\sqrt{(r+b)^2 + (L+z)^2}} - \frac{K\left(\frac{\pi}{2}, m(r,a,z,-L)\right)}{\sqrt{(r+a)^2 + (L+z)^2}} \Bigg). \end{aligned} \tag{8}$$

The charge distributions mentioned above do not coincide with the real ones, on the places where cables are merged, because the boundary conditions are not satisfied, so consequently the conductors are not of constant potential. Due to this, additional expressions are superposed to the previous ones, where equivalent electrodes are used as additional elements.

Toroidal electrodes are employed as equivalent electrodes, having cross section radii $a_{e1} = L_1/4N_1$ and $a_{e2} = (L_2 - z_d)/4N_2$, and which medium lines are located at the places

$$z_{an} = (2n-1)\frac{l_1}{2}; \qquad r = a; \tag{9}$$

$$z_{bn} = z_d + (2n-1)\frac{l_2}{2}; \qquad r = b. \tag{10}$$

3 Numerical Results

There are many means of modeling cable joints exterior conductor ends. The best results are obtained when shape of exterior conductors ”follows” one of the equipotential surfaces, although they cannot be presented analytically, using known mathematical functions. In order to provide simplicity in producing cable joints, geometrical modeling is realizing by using some of known mathematical functions.

Using the equivalent electrodes method it is possible to determine potential and electrical field distribution in arbitrary chosen point of cable end region. The calculation is done for cable joints having funnel shape, which axial sections are either polynomial or exponential functions and when the exterior conductor end is modeled by ellipse.

The equivalent electrodes, which replace various segments of interior conductor ends, have toroidal shapes. Their cross sections radii are determined as:

- The first step is to divide the curve modeling is done at, on N_4 segments and to connect these points;
- Equivalent radius (a_{e4}) is determined as the forth of smallest distance among the points;
- Centers of the equivalent electrodes are placed on the medium line ($r = r_{pn}$, $z = z_{pn}$) among the points, but on the vary surface on the exterior conductor.

In order to determine unknown EE charges, the system of linear equations is formed, using boundary condition that electrodes' surfaces are on the same potential. Afterwards, it is possible to determine all necessary electrical field parameters using standard procedures.

3.1 Geometrically modelled cable joint by power or exponential functions

The appearance of cable joint realized in this way is presented in Fig. 1.

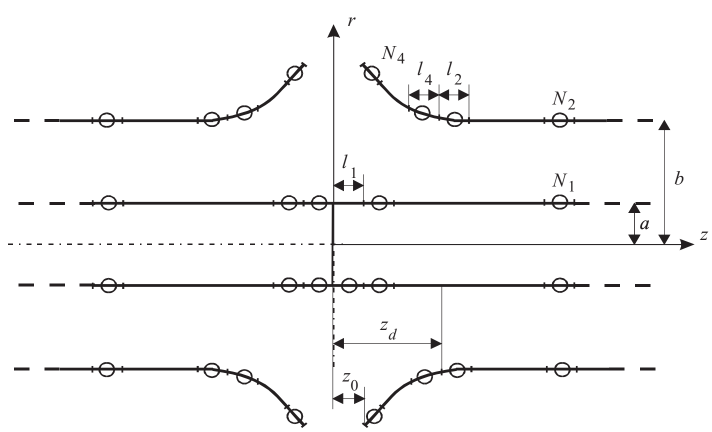

Fig. 1. Geometrically modelled cable joint using power (polynomial) or exponential functions.

The parametric expressions of the power function used for ends modelling of the coaxial cable exterior conductors are:

$$z(t) = z_d - t(z_d - z_0), \quad r(t) = b + t^k(r_0 - b), \tag{11}$$

and parametric expressions for exponential function have the form:

$$z(t) = z_0 + t(z_d - z_0), \quad r(t) = b\left(\frac{r_0}{b}\right)^{1-t}, \tag{12}$$

where k is the degree, and t is parameter having value between 0 and 1.

Centers of toroidal equvalent electrodes, which replace ends of outer conductors, are:

$$r_n = r(t_n); \quad z_n = z(t_n), \tag{13}$$

where

$$t_n = \frac{2n-1}{2N_4}. \tag{14}$$

To illustrate EEM application for calculating the cable joints modelled in this way, the joint having parameters $b = 3a$, $z_d = 6a$, $r_0 = 4a$, $z_0 = 3a$ and $L_1 = L_2 = 8a$ is observed. Number of EE used for substituting interior and exterior conductors are chosen to be $N_1 = 40$, $N_2 = 20$, $N_3 = 50$, and polynomial degree $k = 3$.

The numerical solution of the cable joints has been obtained by employing the algorithm described in the previous section. Table 1 shows obtained convergence of the results for electric field, normalized to E_0, when the number of equivalent electrodes and the length of a cable joint which is being modeled by equivalent electrodes, are used as parameters. An accurate determination of the fields in the immediate neighborhood of the shield end has to take into account the details of the local geometry and is in general done using EEM. The electrical field is calculated at the segments center, which is modeled by equivalent electrodes. In the case when n_1 equivalent electrodes are placed on the cable ends it is obtained an accurate result at one decimal figure. When n_1n_2 equivalent electrodes are placed on the cable ends it is obtained an accurate result at two decimal figures, and similarly when $n_1n_2\ldots n_l$ are used the accuracy of the results are at l decimal numbers.

The equipotential lines of observed cable joint are presented in Fig. 2 and Fig. 3. It can be notice that the highest density of equipotential curves is very close to outer conductor, inside the cable joint. This is the region with the strongest electrical field strength. Outside the outer conductor, far away from the cable break, there is a region with zero potential. In this place equipotential surfaces are changed. Dielectric breakdown is energetic phenomenon, so that it is very important to know a surfaces shape with equal energy (equienergetic surfaces).

Equipotential lines of geometrically modelled cable joint are shown in the next two figures (Fig. 4 and Fig. 5), for exponential function and parameters values: $b = 3a$, $z_d = 6a$, $r_0 = 4a$, $z_0 = 3a$ and $L_1 = L_2 = 8a$.

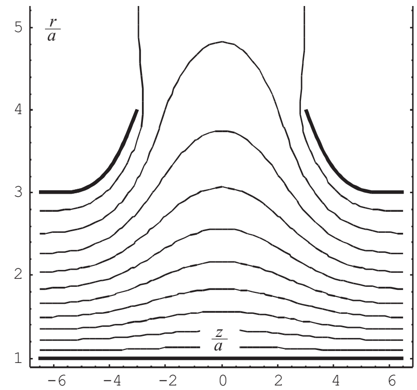

Fig. 2. Equipotential lines of geometrically modelled cable joint, when power function is used, for parameters values: $k = 3$, $b = 3a$, $z_d = 6a$, $r_0 = 4a$, $z_0 = 3a$ and $L_1 = L_2 = 8a$.

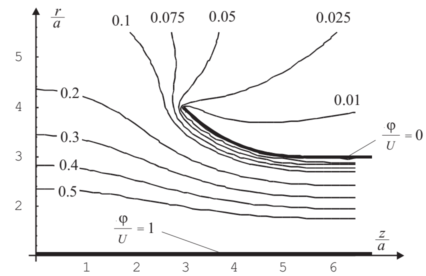

Fig. 3. Equipotential lines of exactly defined potential, for geometrically shaped cable joint, when power function is used, for parameters values: $k = 3$, $b = 3a$, $z_d = 6a$, $r_0 = 4a$, $z_0 = 3a$ and $L_1 = L_2 = 8a$.

Table 1. Electric field, E/E_0, at outer conductor surface in points: $r(t_n)$, $z(t_n)$.

$t_n = \frac{2(n+i)-1}{2N_4}$	$n = 1$	$n = 3$	$n = 5$
$i = 45$	0.637962	0.296344	0.272487
$i = 40$	0.269218	0.268255	0.270089
$i = 35$	0.271206	0.272982	0.273511
$i = 30$	0.273167	0.271144	0.267335
$i = 25$	0.264797	0.258593	0.251132
$i = 20$	0.247035	0.238327	0.229222
$i = 15$	0.224623	0.215512	0.206727
$i = 10$	0.202523	0.194593	0.187404
$i = 5$	0.184115	0.178176	0.17309
$i = 0$	0.170843	0.166721	0.161189

3.2 Geometrically modelled cable joint using ellipse

The ellipse used for deflectors modelling is defined by expressions:

$$z(\theta) = z_c - a_{\text{po}} \sin\theta; \quad r(\theta) = r_c - b_{\text{po}} \cos\theta. \tag{15}$$

The angle α is determined as

$$\alpha = \arccos\left(\frac{r_{\text{kr}} - r_0}{r_{\text{kr}} - b}\right). \tag{16}$$

For example, the cable joint having parameters, $b = 3a$, $z_d = 0.5a$ and $L_1 = L_2 = 5a$ is analysed.

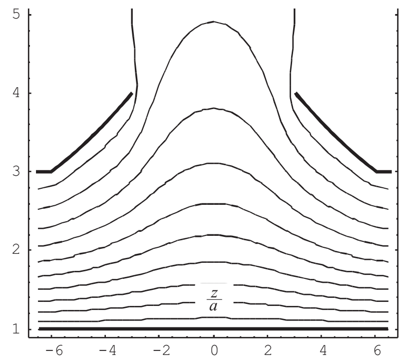

Fig. 4. Equipotential lines of geometrically modelled cable joint, when exponential function is used, for parameters values: $b = 3a$, $z_d = 6a$, $r_0 = 4a$, $z_0 = 3a$ and $L_1 = L_2 = 8a$.

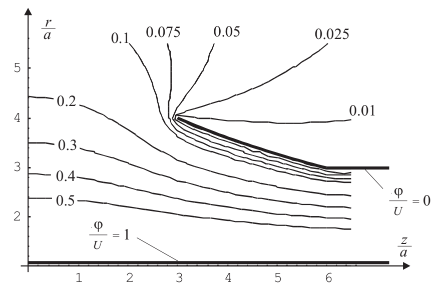

Fig. 5. Equipotential lines of exactly defined potential, for geometrically shaped cable joint, when exponential function is used, for parameters values: $b = 3a$, $z_d = 6a$, $r_0 = 4a$, $z_0 = 3a$ and $L_1 = L_2 = 8a$.

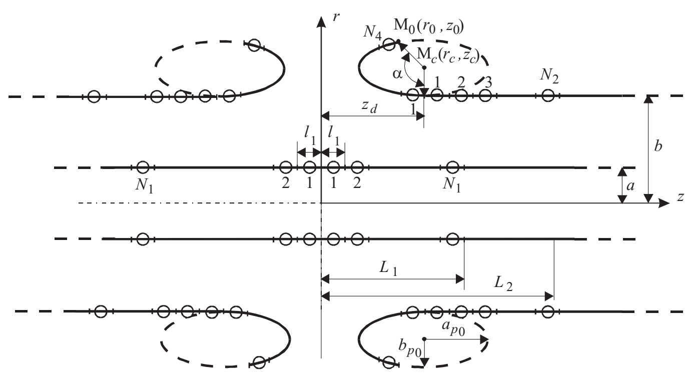

Fig. 6. Geometrically modelled cable joint using ellipse.

Distributions of the axial component of electrical field (Fig. 8) and electrical field intensity (Fig. 9) in axial direction are presented.

Axial cross-section of equipotential surfaces, for geometrically modelled cable joint by using ellipse, with parameter values $b = 3a$, $z_d = 7a$, $r_0 = 3.75a$, $r_{\mathrm{kr}} = 3.5a$, $z_{\mathrm{kr}} = 4a$ and $L_1 = L_2 = 8a$, is shown in Fig. 7. Density of equipotential curves is less then density at cable joints modeled with power or exponential functions. Result of this is better reduction of strong electrical field near the cable break. For practical applications when cable joints are geometrically modeled, maximal value of the axial component of the electrical field is mostly considered. Due to these reasons distribution of electrical fields axial component is like plotted.

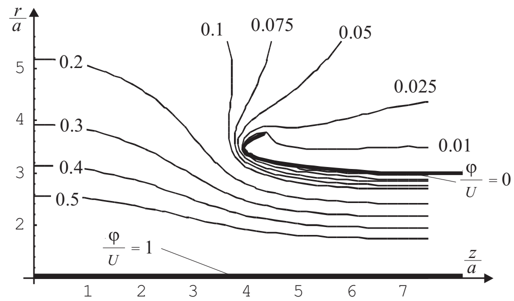

Fig. 7. Equipotential lines of geometrically modelled cable joint using ellipse, for parameters values $b = 3a$, $z_d = 7a$, $r_0 = 3.75a$, $r_{\mathrm{kr}} = 3.5a$, $z_{\mathrm{kr}} = 4a$ and $L_1 = L_2 = 8a$.

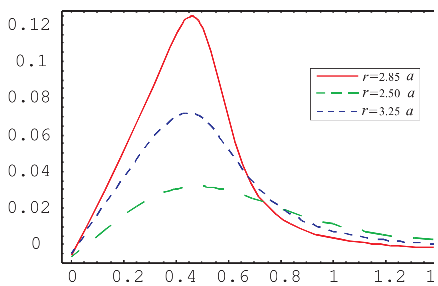

Fig. 8. Axial electrical field component (E_z/E_0) in axial direction (z/a), for different radial coordinate values.

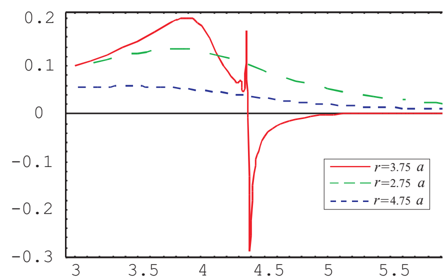

Fig. 9. Electrical field intensity (E/E_0) in axial direction (z/a), for different radial coordinate values.

3.3 Geometrically modelled cable joint using ellipse and power function

The best results and the maximum reduced electrical fields are obtained when cable joints are modelled with both ellipse and power functions (Fig. 10).

Radial and axial coordinates of ellipse's center are determined by using condition of ”flat contact” between ellipse and power function shaped part of deflector. In these points the first derivates of functions are the same. Respecting this condition, coordinates of center of ellipse are

$$z_c = z_0 + \frac{a_{\mathrm{po}}}{\sqrt{1 + \frac{1}{r'(r_0, z_0)^2} \frac{b_{\mathrm{po}}^2}{a_{\mathrm{po}}^2}}}; \tag{17}$$

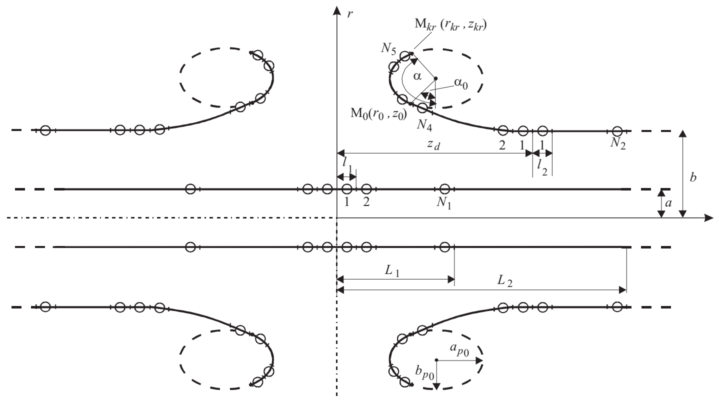

Fig. 10. Geometrically modelled cable joint using ellipse together with power function.

$$r_c = r_0 - \frac{1}{r'(r_0,z_0)^2}\frac{b_{\text{po}}^2}{a_{\text{po}}}\frac{1}{\sqrt{1+\frac{1}{r'(r_0,z_0)}\frac{b_{\text{po}}^2}{a_{\text{po}}^2}}}, \tag{18}$$

where

$$r'(r_0,z_0) = -k\frac{r_0-b}{z_d-z_0} \tag{19}$$

is the first derivate in ”contact” points.

This part of deflector is modeled by toroidal equivalent electrodes, placed in points with coordinates $r_{\text{pe}n} = r(\theta_{\text{en}})$ and $z_{\text{pe}n} = z(\theta_{en})$, where

$$\theta_{\text{en}} = \alpha_0 + (2n-1)\frac{\alpha_{\text{pod}}}{2}; \qquad \alpha_{\text{pod}} = \frac{\alpha-\alpha_0}{N_5}. \tag{20}$$

The cable joints, modelled in this way, with: $b = 3a$, $z_d = 8a$, $r_0 = 4a$, $z_0 = 5a$, $a_{\text{po}} = 2a$, $b_{\text{po}} = a$, $\alpha = 3\pi/4$ and $L_1 = L_2 = 9a$, are observed. Polynomial degree is $k = 2$. Calculated center has parameters: $r_c = 4.6$, $z_c = 6.6$.

Electric field distribution on outer conductor is shown in Table 2. Maximum value of electric field intensity, in this case is $E = 0.225780E_0$. This value is about tree times less then maximum value of electric field intensity at cable joint when only power function is used ($E = 0.6379620E_0$-in Table 1). Numerical evaluation of the electric field in a cable joint is carried out by adopting a model based on the electro-quasi-static approximation of Maxwell equations.

Distributions of the axial component of electrical field in radial direction (Fig. 11), and in axial direction (Fig. 12) are presented.

The equipotential lines of observed cable joint are shown in Fig. 13.Axial cross-section of equipotential surfaces is presented in the case when cable joint is geo-

Table 2. Electric field, E/E_0,on outer conductor surface in points: $r(\theta_{\text{en}})$, $z(\theta_{\text{en}})$.

$\theta_{\text{en}} = \alpha_0 + (2(i+n)-1)\dfrac{\alpha\text{pod}}{2}$	$n=1$	$n=2$	$n=3$	$n=4$
$i=35$	0.224139	0.115076	0.109933	0.106496
$i=30$	0.107376	0.109444	0.112112	0.115198
$i=25$	0.122152	0.125851	0.129610	0.133379
$i=20$	0.140825	0.144489	0.148141	0.151827
$i=15$	0.159559	0.163737	0.116317	0.131409
$i=10$	0.160278	0.173266	0.184897	0.195003
$i=5$	0.210443	0.215875	0.219938	0.222789
$i=0$	0.225540	0.225780	0.225520	0.224273

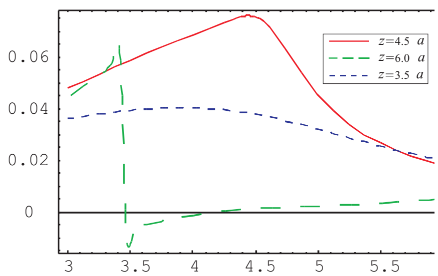

Fig. 11. Axial electrical field component distribution (E_z/E_0) in radial direction (r/a), for different axial coordinate values.

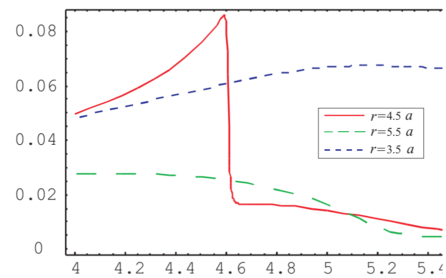

Fig. 12. Axial electrical field component distribution (E_z/E_0) in axial direction (r/a), for different radial coordinate values.

metrically modelled by using ellipse together with power function, for parameter values:$b = 3a$, $z_d = 8a$, $r_0 = 4a$, $z_0 = 5a$, $a_{\text{po}} = 2a$, $b_{\text{po}} = a$, $\alpha = 3\pi/4$, $L_1 = L_2 = 9a$ and $k = 2$.

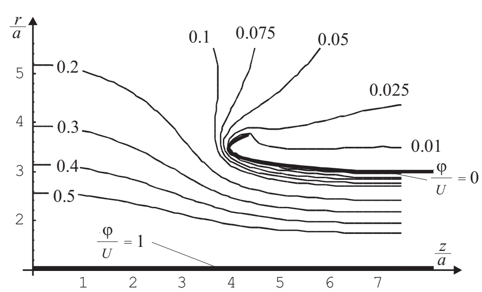

Fig. 13. Equipotential lines of geometrically modelled cable joint using ellipse together with power function, for parameters values: $b = 3a$, $z_d = 8a$, $r_0 = 4a$, $z_0 = 5a$, $a_{\text{po}} = 2a$, $b_{\text{po}} = a$, $\alpha = 3\pi/4$, $L_1 = L_2 = 9a$ and $k = 2$.

4 Conclusion

The above considerations are applicable to a wide class of HV electrical devices such as insulators, bushings, spacers, voltage divider, cable accessories (joints and terminations), etc. Cable joints and terminations represent the weakest components of a HV cable power line. The problem of electric field grading in such components, having both theoretical and technological implications, can be framed in the more general topic of field control in HV equipment. The using geometrically modeled cable accessories for terminating and jointing various shielded power cable types has been well documented in the literature, but the problem still remains incompletely solved. Cable joints and terminations are supposed to have small dimensions and very good service characteristics. There is a number of joints developed in last few years. Cable failures still happen, causing a great economic loss, mainly because of a cable joint breakdown. For that reason any improvement in the cable joint construction is of interest.

Electric field of highest strength exists at cable end. Engineers and cable producers main aim is to reduce strong axial fields at the ends of outer cables. It is possible to fulfil it by applying geometrical modelling of cable ends. For joint modelling, the exponential function, power function, ellipse, as well as the combination of the ellipse and power or exponential functions are used.

The calculation is carried out using equivalent electrodes method. The EEM application advantage, compared to other methods, is high accuracy, even when the number of used EE is relatively small.

Equivalent electrodes are appointed at the end of coaxial cable, where the edge effect exists. At great distances from the joints, inside the cable, it may be considered that the field is approximately homogeneous and the charge distribution is continuous. It is possible to determine the potential as superposition of two components: the first one originates from continuous distribution of electrical charge, and the second one from equivalent electrodes.

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