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The Probability Stability Estimation of the System Based on the Quality of the Components

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Abstract: The simple and effective method for the probability stability estimation of the system with randomly selected parameters is presented in this paper. Parameter value can be changed under the influence of different factors. The effects of these changes depend on the component quality and can cause the failure of the component and system, as well. The influence of these changes to the system stability is analyzed in this paper, also.

Keywords: Probability stability, reliability, quality.

1 Introduction

Systems with stochastic parameters can be found in many branches of industry, such as chemical industry, plastic industry, and especially rubber industry. The values of the stochastic parameters do not coincide with the nominal values, so the properties of the system deviate from the desirable ones. For example, the value of the system parameter determines the stability of the system. In the case of determined parameters, system can be stable or unstable depending on parameter value. When the parameters of the system are random variables, the system can be stable, unstable or stable with some probability. The aim of this paper is to estimate that probability called probability stability of the system.

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The basic methods for the probability stability estimation of the system with randomly selected parameters are given in [1–4]. These methods relate to the continuous systems. However, the method can be applied to the discrete systems with the random parameters, too, [5–8]. The advantage of this method is in its practical use. This estimation is very important in relation to the system stability, the quality of system work and the system reliability.

The method enables the choice of such parameter value for which the system has the largest probability stability. The right choice of parameters value provides the correctness of system work, i.e., its reliability.

However, parameter value can be changed under the influence of different factors, exterior and interior, i.e., wear, corrosion, aging etc. The effects of these changes depend on the components quality and can cause the failure of the component and system, as well, i.e., can decrease the system reliability.

The reliability $R(t)$ is usually defined as the probability of the system work without failures in certain period of time and given environment conditions. Since, the moments of failures are random variables, the reliability theory is based on the probability theory, [9].

The influence of reliability to the system stability is analyzed in this paper, also.

2 The Probability Stability Estimation of the Discrete Systems

Let the discrete system is given by

$$\sum_{i=0}^{n} l_i x(k+n-i) = u(k), l_0 = 1 \tag{1}$$

The characteristic polynomial of the equation (1) is

$$z^n + l_1 z^{n-1} + \cdots + l_n = 0 \tag{2}$$

The coefficients of the characteristic equation are random variables with the probability density distributions $p_i(l_i)$. Since the parameters of the system are random variables, the system can be stable with probability stability P. The main goal is to determinate that probability stability.

The necessary and sufficient condition for the stability of the difference equation coefficients is that all zeros of its characteristic polynomial are located inside the unit circle in the z-plane.

First, the stability area of the system (1) in the parametric space must be obtained using some of the methods for the stability test, for example the Hurwitz criterion, Schur-Cohn Jury criterion, the bilinear transform method, etc.

If parameters of the systems are independent variables, then the total density distribution is given by

$$p(l_1,\ldots,l_n)=\prod_{i=1}^{n}p_i(l_i) \tag{3}$$

The probability stability of the system (1) is

$$P=\int_{S_n}\cdots\int p(l_1,\ldots,l_n)\mathrm{d}l_1\cdots\mathrm{d}l_n \tag{4}$$

where S_n is the stability area.

For the first order discrete system the stability area, S_1, is given by the next inequality

$$-1<l_1<1 \tag{5}$$

For the second order discrete system the stability area, S_2, in the parametric space l_1 ,l_2 is given by

$$\begin{array}{l} 1-l_1+l_2\geq 0 \\ 1+l_1+l_2\geq 0 \\ l_2\leq 1 \end{array} \tag{6}$$

The stability area S_2 is given on Fig.1 where N_2 presents the unstably area.

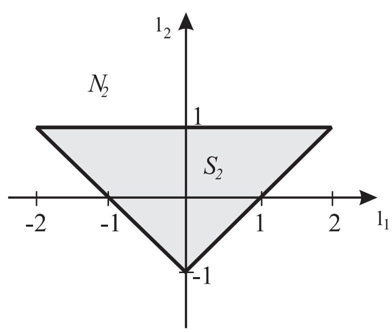

Fig. 1. The stability area S_2 of the second order discrete system

In the case of the second order discrete system, the stability area is the triangle.

For the third order discrete system the stability area is obtained in the same way as at the second order discrete system and is given by the following relations

$$\begin{array}{l} l_1+l_2+l_3>-1 \\ l_1-l_2+l_3<1 \\ l_1l_3+1>l_2+l_3^2 \end{array} \tag{7}$$

This stability area is given on Fig.2.

For the n-th order discrete system the calculation of stability area and probability stability is too complex. The limits of the stability area are usually complex mathematical relations and is difficult to determine the probability stability because it is necessary to integrate by the area of stability.

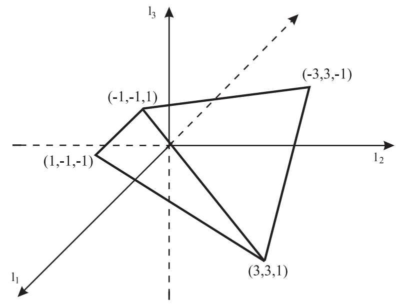

Fig. 2. The stability area S_3 of the third order discrete system

3 The Influence of Reliability to the System Stability

The scientific and technical progress of systems results the appearance and development of reliability theory. The complex systems should have high reliability considering the consequences of their failures influencing on further work and people security. The reliability $R(t)$ is usually defined as probability of system work without failures in certain time period and given environment conditions. The reliability theory is based on the application of probability theory since the moments of failures are random variables.

During the exploitation, the system is undergone to the different factors influencing to its work. These factors can change the values of system parameters and its reliability and in the limit case can bring system to the instability or failure.

The choice of appropriate parameter value for which the system has the largest probability stability provides the correctness of system work, i.e., its high reliability.

The method presented in Section 2 can be used for the analysis of reliability using probability stability of the system with random parameters. If parameters of the system have stationary values in certain time period, then the system is reliable in that period of time. It can be noticed that the reliability is in correlation with probability stability.

The reliability is defined by the next relation

$$R(t) = \int_{t_1}^{\infty} p(t)\, dt \tag{8}$$

where $p(t)$ is probability density distribution. $R(t)$ presents the probability that the system in the observed moment will work correctly.

The reliability of the system is changing depending on the quality of the system components. The quality represents the set of all properties determining individual parameters of available worth including the conditions in which the system should work and the necessary working time. The basic elements of the quality are

- Functionality,
- Reliability,
- Economy.

The research of reliability indirectly includes the problems of functionality and economy so this is the research of the system quality, as well.

The quality of component can determinate the lifetime of its. For example, the aging of components is one of the reasons of changing their values and system reliability.

4 The Calculation Results

The main goal of this paper is to calculate the probability of system stability for randomly selected parameters in given time period. First, using the method presented in Section 2, the value of the parameter for which the system has the largest probability stability, P_1, is randomly selected. During the time this value changes due to different factors, for example the aging of component and the reliability of the system decreases and the probability stability, as well. This change can be presented graphical, for example by Poisson distribution. Finally, the total probability that the system will be stable for randomly selected parameters in given time period is defined by the next relation

$$P = 1 - (1 - P_1)(1 - R) \tag{9}$$

where P_1 is the probability stability of the system with randomly selected parameters and R is the system reliability, i.e., the probability that the system will be stable during a certain time for randomly selected parameters. If the values of parameters are stationary then the system is both reliable and stable.

Let the parameters $l_1, \ldots, l_n$ have normal distribution

$$p_i = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(l_i - \bar{l}_i)^2}{2\sigma_i^2}} \tag{10}$$

where $\overline{l_i}$ are mathematical expectations and σ_i standard deviations.

If the parameters are independent random variables, then the multidimensional distribution density is given by

$$p(l_1,\ldots,l_n) = \frac{1}{\left(\sqrt{2\pi}\right)^n \prod\limits_{i=1}^{n} \sigma_i} e^{-\sum\limits_{i=1}^{n} \frac{(l_i-\bar{l}_i)^2}{2\sigma_i^2}} \tag{11}$$

The probability stability is calculated using the Eq.(4). In Table 1 and Table 2 the result for the probability stability of the second and the third order discrete systems for the normal distribution of parameters are shown. This method can be applied to the other probability distribution, such as exponential, uniform, Poisson distribution of parameters, etc.

Table 1. Computational Results for the Second Order Disrete System

$\bar{l}_1$	$\bar{l}_2$	σ_1	σ_2	P_1
0.4	0.5	0.8	0.9	0.646
0.1	0.2	0.8	0.9	0.712
0.1	0.2	0.5	0.6	0.886
0.1	0.2	0.3	0.4	0.976
0.1	0.2	0.2	0.3	0.996
0.05	0.1	0.1	0.2	0.999

Table 2. Computational Results for the Third Order Disrete System

$\bar{l}_1$	$\bar{l}_2$	$\bar{l}_3$	σ_1	σ_2	σ_3	P_1
0.1	0.2	0.3	0.4	0.5	0.6	0.863
0.1	0.2	0.3	0.6	0.7	0.8	0.757
0.3	0.4	0.5	0.7	0.8	0.9	0.663
0.3	0.4	0.5	0.1	0.2	0.3	0.952
0.2	0.3	0.4	0.1	0.2	0.3	0.997
0.05	0.1	0.11	0.1	0.2	0.3	0.998

From these tables we can chose values of parameters for which the system has the largest probability stability.For these values the system reliability in given time period is calculated. If these values are stationary in certain time, the system is stable and reliable. However, these values change during the time because of the aging of components or other factors. The reliability decreases in time under, for example, the Poisson distribution, $p(l_i,a) = \frac{l_i^a e^{-l_i}}{a!}$. Let us observe the system in the period of, for example, 10 months. Then the reliability of the second and the third order discrete system can be calculated and the total probability that the system will be stable for randomly selected parameters in given time period of 10 months.

Example: Let the probability stability of the second order discrete system selected from the Table I is 0.976. For given parameters value reliability of this system is calculated, R=0.923. Using the relation (9), the total probability stability of the system is P=0.998.

5 Conclusion

The method for the probability stability estimation of the discrete systems based on the quality of the components is presented in this paper. This method can be applied both to the discrete and continuous systems. For its simplicity, this method can be used in practice, for example, in the industry of chemical products, industry of plastic materials and the rubber industry where some technological parameters have stochastic character. Random selection of the right parameter value enables the largest probability of the system stability and if this value is stationary, the system is reliable, too. In this paper the influence of parameter change to the system stability and reliability is analyzed. The results of calculation are shown in tables.

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