

Reduction of Operational Amplifiers Finite Gain Effects in Switched-Capacitor Low-Pass Notch Biquads

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Abstract: An combined approach for reducing the errors in the notch frequency f_z , in the pole frequency f_p , in the quality factor Q_p and in the amplitude H_p at the pole frequency of switched-capacitor low-pass notch biquads is presented. At first, the conventional integrators in the biquads are replaced with gain- and offset-compensated integrators. Subsequently, the errors $\Delta f_z/f_z$, $\Delta f_p/f_p$, $\Delta Q_p/Q_p$ and $\Delta H_p/H_p$ are minimized by modifying the values of the integrating capacitances and of the appropriately chosen zero-forming and pole-forming capacitances. The effectiveness of this approach is demonstrated by designing two low-pass notch biquad topologies which realize the same transfer function.

Keywords: filters, gain- and offset- compensation, operational amplifiers, switched-capacitor integrators

1 Introduction

One of the important nonideal properties which influence the performance of the switched-capacitor (SC) circuits is the finite dc gain A of the operational amplifiers (op amps). In filters, the finite gain causes errors in both the amplitude and the phase responses [1, 2]. From the point of view of simplifying amplifier design and improving high-frequency capability several gain- and offset- compensated (GOC) SC building blocks (integrators, gain stages, sample-and-hold circuits) have been reported in the literature. The phase error $\theta(\omega)$ of the GOC integrators is proportional to $1/A^2$ (in a conventional integrator this is a simple inverse dependence $1/A$). In most of the GOC integrators proposed the reduction in phase error was obtained at the expense of increased gain error $m(\omega)$.

Manuscript received March 20, 2006.

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The SC low-pass notch (LPN) biquads are commonly realized using a feedback loop containing one inverting and one noninverting conventional integrators.

The gain errors of the integrators affect the notch frequency f_z and the pole frequency f_p of the biquads, while the phase errors affect the zero Q -factor Q_z , the pole Q -factor Q_p , the attenuation H_z at the notch frequency and the magnitude H_p of the biquad transfer function at the pole frequency.

A combined approach for minimization the effects of op amps finite gain in switched-capacitor bandpass biquads has been presented in [3].

In this paper the approach proposed in [3] is adapted for reducing the errors in the notch frequency f_z , in the pole frequency f_p , in the quality factor Q_p and in the amplitude H_p at the pole frequency of SC LPN biquads with low but precisely known and stable op amps dc gain [4]. The effectiveness of this approach is verified for two LPN biquad topologies which realize the same transfer function.

2 Proposed design approach

The z -domain transfer function of the LPN biquads has the general form

$$H(z) = \frac{N(z)}{D(z)} = k \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}, \quad (1)$$

which for any pairs of complex conjugate zeros and poles can be rewritten as

$$H(z) = k \frac{1 - 2R_z \cos \theta_z z^{-1} + R_z^2 z^{-2}}{1 - 2R_p \cos \theta_p z^{-1} + R_p^2 z^{-2}}. \quad (2)$$

Here, R_z is the radius and θ_z is the angle to the zero; R_p and Q_p correspond to the pole radius and angle, respectively. From (2) the following relationships for the notch frequency f_z , the zero Q -factor Q_z , the pole frequency f_p and the pole Q -factor Q_p can be derived:

$$f_z = \frac{f_s}{2\pi} \sqrt{\theta_z^2 + (\ln R_z)^2}, \quad Q_z = -\frac{\pi f_z / f_s}{\ln R_z} \quad (3)$$

and

$$f_p = \frac{f_s}{2\pi} \sqrt{\theta_p^2 + (\ln R_p)^2}, \quad Q_p = -\frac{\pi f_p / f_s}{\ln R_p}, \quad (4)$$

where f_s is the sampling frequency.

For small ratios f_z/f_s , f_p/f_s and high Q -factors the frequencies f_z , f_p and the Q -factors Q_z , Q_p are approximately given by

$$f_z \approx \frac{f_s}{2\pi} \sqrt{1 + a_1 + a_2}, \quad Q_z \approx \frac{\sqrt{1 + a_1 + a_2}}{1 - a_2} \quad (5)$$

and

$$f_p \approx \frac{f_s}{2\pi} \sqrt{1+b_1+b_2}, \quad Q_p \approx \frac{\sqrt{1+b_1+b_2}}{1-b_2}. \quad (6)$$

For standard design of LPN biquads the op amps gain values are assumed to be infinite. Then, the coefficients in (1) are functions of the capacitances only and $Q_z \rightarrow \infty$ ($a_2 = 1$). In this case, from (5), (6) and (1) the logarithmic sensitivities of the f_z , f_p , Q_p and of the amplitude H_p at the pole frequency to the capacitances can be obtained.

The proposed combined approach for minimization of the errors $\Delta f_z/f_z$, $\Delta f_p/f_p$, $\Delta Q_p/Q_p$, $\Delta H_p/H_p$ and for enhancement of Q_z consists in the following consecutive steps, in which the calculations are made for the nominal value A_o of the op amps dc gain A :

Step 1. First, to reduce the effect of op amp imperfections (dc gain A and offset voltage V_{OS}) the conventional integrators in the LPN biquad considered are replaced with Nagaraj-86 [5] and Ki-89 [6] GOC SC integrators. These simple bi-phase integrators form an excellent GOC integrator-pair without using extra clock phases or holding circuits to satisfy the sampling conditions. The reduced phase errors of the GOC integrators provide a reduction in the errors $\Delta Q_p/Q_p$, $\Delta H_p/H_p$ and enhance the quality factor Q_z .

Step 2. The gain error $m(\omega)$ of the integrators is equivalent to an element value variation ΔC_i of the integrating capacitance C_i . If the finite dc gain A_o is known, the value of C_i can be replaced with $C'_i = C_i(1+m)$ in the two integrators of the GOC biquad, thereby essentially reducing the gain errors $m(\omega)$ [7]. This prewarping technique automatically provides a reduction in the notch frequency error $\Delta f_z/f_z$ and in the pole frequency error $\Delta f_p/f_p$ of the biquad considered.

Step 3. For some LPN biquad configurations the compensation in Step 2 is not sufficiently effective. In this case the errors $(\Delta f_z/f_z)_2$ and $(\Delta f_p/f_p)_2$, obtained in the previous step, can be further reduced by modifying one zero-forming capacitance C_z and one pole-forming capacitance C_{p1} . The new capacitance values C'_z and C'_{p1} are the solutions of the equations

$$S_{C_z}^{f_z} \left(\frac{C'_z}{C_z} - 1 \right) = - \left(\frac{\Delta f_z}{f_z} \right)_2 \quad (7)$$

and

$$S_{C_{p1}}^{f_p} \left(\frac{C'_{p1}}{C_{p1}} - 1 \right) = - \left(\frac{\Delta f_p}{f_p} \right)_2. \quad (8)$$

The sensitivities $S_{C_z}^{f_z}$ and $S_{C_{p1}}^{f_p}$ are calculated using (5) and (6) for the initial values of the capacitances and infinite op amp dc gain.

Step 4. The errors $(\Delta Q_p/Q_p)_2$ and $(\Delta H_p/H_p)_2$ obtained in Step 2, or the errors $(\Delta Q_p/Q_p)_3$ and $(\Delta H_p/H_p)_3$ obtained in Step 3, can be further minimized by modifying one another pole-forming capacitance C_{p2} . This capacitance is chosen such that the following relations hold:

$$S_{C_{p2}}^{Q_p} \approx S_{C_{p2}}^{H_p}, \quad |S_{C_{p2}}^{f_p}| \ll 1, \quad |S_{C_{p2}}^{f_z}| \ll 1. \quad (9)$$

The new capacitance value C'_{p2} is calculated from the equations

$$S_{C_{p2}}^{Q_p} \left(\frac{C'_{p2}}{C_{p2}} - 1 \right) = - \left(\frac{\Delta Q_p}{Q_p} \right)_r \quad (10)$$

and

$$S_{C_{p2}}^{H_p} \left(\frac{C'_{p2}}{C_{p2}} - 1 \right) = - \left(\frac{\Delta H_p}{H_p} \right)_r. \quad (11)$$

The terms $(\Delta Q_p/Q_p)_r$ and $(\Delta H_p/H_p)_r$ on the right hand sides are the errors $(\Delta Q_p/Q_p)_2$ and $(\Delta H_p/H_p)_2$ obtained in Step 2, or the errors $(\Delta Q_p/Q_p)_3$ and $(\Delta H_p/H_p)_3$ obtained in Step 3.

The following two cases are considered:

- a) The capacitance C'_{p2} is the solution of equation (10), i.e.,

$$C'_{p2} = C_{p2} \left[1 - \frac{(\Delta Q_p/Q_p)_r}{S_{C_{p2}}^{Q_p}} \right]. \quad (12)$$

- b) The capacitance C'_{p2} is the average of the solutions of (10) and (11), i.e.,

$$C'_{p2} = C_{p2} \left\{ 1 - 0.5 \left[\frac{(\Delta Q_p/Q_p)_r}{S_{C_{p2}}^{Q_p}} + \frac{(\Delta H_p/H_p)_r}{S_{C_{p2}}^{H_p}} \right] \right\}. \quad (13)$$

3 Application of the proposed approach

The approach proposed is illustrated by means of two LPN biquad topologies. For comparative purposes the two biquads are designed to fulfil the same specifications:

$$f_p \cong 1 \text{ kHz}, \quad f_z \cong 2.5 \text{ kHz}, \quad Q_p \cong 30, \quad H_p \cong 28 \text{ dB}, \quad f_s \cong 12.5 \text{ kHz}.$$

Fig.1 shows the circuit schema of the Martin and Sedra's type-I LPN biquad with conventional integrator [8]. The relative capacitance values are: $C_1=2.027$, $CF_1=1$, $CB_1=1$, $CB_2=1$, $C_2=59.207$, $CF_2=29.945$ and $CF_3=10.687$.

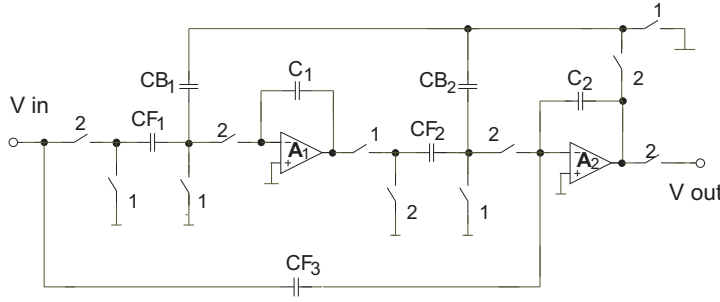


Fig. 1. Martin and Sedra's type-I LPN biquad with conventional integrators.

The ideal z -domain transfer function is

$$H_{id}^{22}(z) = -\frac{CF_3}{C_2} \frac{z^{-2} - \left[2 - \frac{CF_1CF_2}{C_1CF_3}\right] z^{-1} + 1}{z^{-2} - \left[2 + \frac{CB_2}{C_2} - \frac{CB_1CF_2}{C_1C_2}\right] z^{-1} + \frac{CB_2}{C_2} + 1}. \quad (14)$$

The frequencies f_z and f_p , and the quality factor Q_p are approximately given by

$$f_z \approx \frac{f_s}{2\pi} \sqrt{\frac{CF_1CF_2}{C_1CF_3}}, \quad f_p \approx \frac{f_s}{2\pi} \sqrt{\frac{CB_1CF_2}{C_1(C_2 + CB_2)}} \quad (15)$$

$$Q_p \approx \sqrt{\frac{CB_1CF_2(C_2 + CB_2)}{C_1CB_2^2}}.$$

It was found that for nominal op amps gains $A_{01} = A_{02} = 100$ the deviations of f_z , f_p , Q_p and H_p from the ideal case are:

$$(\Delta f_z/f_z)_c = -0.852\%, \quad (\Delta f_p/f_p)_c = -1.427\%,$$

$$(\Delta Q_p/Q_p)_c = -47.33\%, \quad (\Delta H_p/H_p)_c = -47.25\%.$$

The zero Q -factor and the attenuation at f_z are $Q_{zc} = 128.18$ and $H_{zc} = -56.78$ dB.

According to the approach proposed the first integrator in the conventional biquad (Fig.1) is replaced with the Nagaraj-86 integrator and the second integrator with the Ki-89 integrator. The resulting filter is shown in Fig.2, where $C_{h1} = 1$ and $C_{h2} = C_2$.

The performance parameters of the GOC biquad for $A_{01} = A_{02} = 100$ are

$$(\Delta f_z/f_z)_1 = -1.4033\%, \quad (\Delta f_p/f_p)_1 = -2.0818\%,$$

$$(\Delta Q_p/Q_p)_1 = -1.4044\%, \quad (\Delta H_p/H_p)_1 = -1.1374\% \quad (16)$$

$$Q_{z1} = 13000, \quad H_{z1} = -96.91 \text{ dB}, \quad H(2500 \text{ Hz}) = -45.87 \text{ dB}.$$

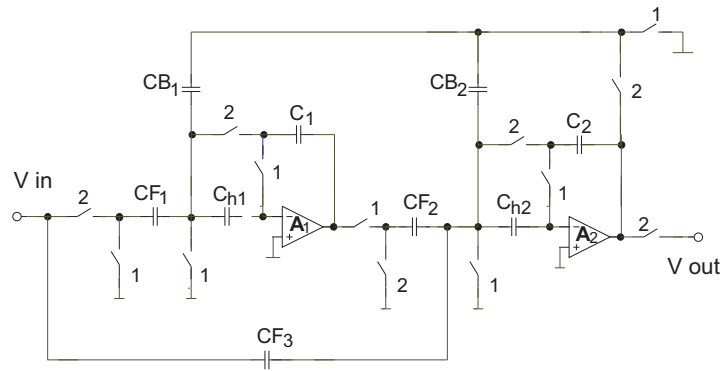


Fig. 2. Martin and Sedra's type-I LPN biquad with GOC integrators.

First, for reducing the errors $(\Delta f_z/f_z)_1$ and $(\Delta f_p/f_p)_1$ of the GOC biquad, the integrating capacitances C_1 and C_2 are modified according to the expressions [9]

$$C'_1 = \left(C_1 - \frac{CF_1 + CB_1 + C_{h1}}{A_{01}} \right) \left(1 + \frac{1}{A_{01}} \right)^{-1} \quad (17)$$

and

$$C'_2 = \left(C_2 - \frac{CB_2 + CF_2 + CF_3}{A_{02}} \right) \left(1 + \frac{1}{A_{02}} \right)^{-1}. \quad (18)$$

For $A_{01} = A_{02} = 100$ one obtains $C'_1 = 1.977228$ and $C'_2 = 58.208594$. The corresponding performance parameters of the biquad are

$$\begin{aligned} (\Delta f_z/f_z)_2 &= -3.25 \cdot 10^{-5}\%, & (\Delta f_p/f_p)_2 &= -2.075 \cdot 10^{-3}\%, \\ (\Delta Q_p/Q_p)_2 &= -0.98282\%, & (\Delta H_p/H_p)_2 &= -0.98410\%, \\ Q_{z_2} &= 12867, & H_{z_2} &= -96.68 \text{ dB}, & H(2500\text{Hz}) &= -84.35 \text{ dB}. \end{aligned} \quad (19)$$

From (15) and (14) one finds

$$S_{CB_2}^{Q_p} = -\frac{2C_2 + CB_2}{2(C_2 + CB_2)} = -0.99170, \quad S_{CB_2}^{f_p} = -\frac{CB_2}{2(C_2 + CB_2)} = -8.305 \cdot 10^{-3},$$

$$S_{CB_2}^{H_p} = -1.00162.$$

Nearly the same errors $(\Delta Q_p/Q_p)_2$ and $(\Delta H_p/H_p)_2$, on one hand and nearly the same sensitivities $S_{CB_2}^{Q_p}$ and $S_{CB_2}^{H_p}$ on the other hand suggest that the errors $(\Delta Q_p/Q_p)_2$ and $(\Delta H_p/H_p)_2$ can be further reduced by modifying the capacitance CB_2 . The choice of this capacitance is based also on the low sensitivity $S_{CB_2}^{f_p}$.

The new capacitance value CB'_2 is given by the expression

$$CB'_2 = CB_2 \left[1 - \frac{(\Delta Q_p/Q_p)_2}{S_{CB_2}^{Q_p}} \right]. \quad (20)$$

One obtains $CB'_2 = 0.9900895$. The corresponding performance parameters of the biquad are

$$\begin{aligned} (\Delta f_z/f_z)_3 &= -3.171 \cdot 10^{-5}\%, & (\Delta f_p/f_p)_3 &= 6.355 \cdot 10^{-3}\%, \\ (\Delta Q_p/Q_p)_3 &= -5.928 \cdot 10^{-3}\%, & (\Delta H_p/H_p)_3 &= 6.873 \cdot 10^{-7}\%, \\ Q_{z3} &= 12867, & H_{z3} &= -96.68 \text{ dB}, & H(2500 \text{ Hz}) &= -84.35 \text{ dB}. \end{aligned}$$

The capacitance CB'_2 can be made equal to the unit capacitance. Then, the new values of the capacitances C_2 , CF_2 and CF_3 are

$$C_2 = 58.791247, \quad CF_2 = 30.244742 \quad \text{and} \quad CF_3 = 10.793974.$$

By rounding-off the values of the capacitances to the third digit after the decimal point we finally obtain

$$\begin{aligned} C_1 &= 1.977, & CF_1 &= 1, & CB_1 &= 1, & C_{h_1} &= 1, & C_2 &= 58.791, \\ CB_2 &= 1, & CF_2 &= 30.245, & CF_3 &= 10.794, & C_{h_2} &= 58.791. \end{aligned}$$

Table 1 summarizes the performance parameters of the GOC Martin and Sedra's type-I LPN biquad with rounded-off capacitances and gain variation $A_{01} = A_{02} = 100 \pm 8$.

Table 1. Performance parameters of the GOC Martin and Sedra's type-I LPN biquad with rounded-off capacitances

A	92	100	108
$(\Delta f_z/f_z)$ [%]	-0.1164	$6.884 \cdot 10^{-3}$	0.1122
$(\Delta f_p/f_p)$ [%]	-0.1699	0.0128	0.1690
$(\Delta Q_p/Q_p)$ [%]	-0.2156	$1.727 \cdot 10^{-4}$	0.1734
$(\Delta H_p/H_p)$ [%]	-0.1860	$6.113 \cdot 10^{-3}$	0.1596
Q_z	10906	12865	14997
$H(f_z)$ [dB]	-95.26	-96.68	-97.99
$H(2.5 \text{ kHz})$ [dB]	-68.42	-81.29	-66.31

The GOC-version of the conventional Huang and Sansen's LPN biquad (Fig.5 from [10]) is shown in Fig.3. The relative capacitance values are: $C_1 = 2.02$, $C_{1S} = 27.79$, $CF_1 = 1$, $CB_1 = 1$, $CB_2 = 1$, $C_2 = 5.633$, $CF_2 = 2.79$, $CF_3 = 1$, $C_{h_1} = 1$ and $C_{h_2} = C_2$.

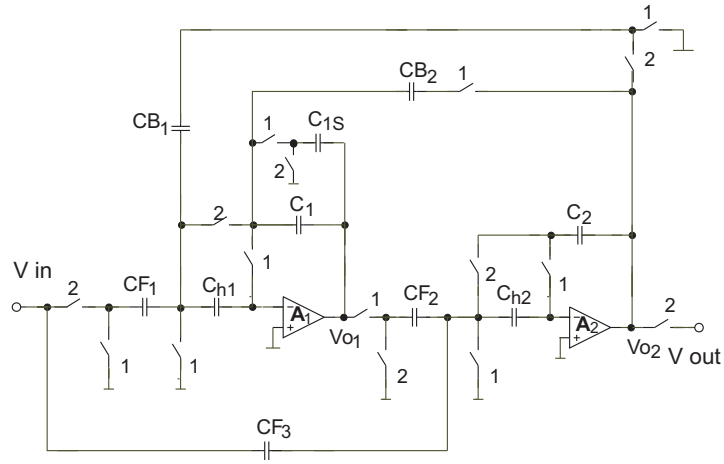


Fig. 3. Huang and Sansen's LPN biquad with GOC integrators.

The performances parameters of the GOC biquad for $A_{01} = A_{02} = 100$ are

$$\begin{aligned} (\Delta f_z/f_z)_1 &= -1.6892\%, & (\Delta f_p/f_p)_1 &= -2.3416\%, \\ (\Delta Q_p/Q_p)_1 &= -0.2035\%, & (\Delta H_p/H_p)_1 &= 0.058\% \\ Q_{z1} &= 6622, & H_{z1} &= -91.04 \text{ dB}, & H(2500 \text{ Hz}) &= -44.02 \text{ dB}. \end{aligned} \quad (21)$$

For the modified values of the integrating capacitances $C'_1 = 1.970297$ and $C'_2 = 5.539703$ the performance parameters of the biquad are

$$\begin{aligned} (\Delta f_z/f_z)_2 &= -0.28253\%, & (\Delta f_p/f_p)_2 &= -0.25599\%, \\ (\Delta Q_p/Q_p)_2 &= 0.07455\%, & (\Delta H_p/H_p)_2 &= 0.0660\%, \\ Q_{z2} &= 6625, & H_{z2} &= -90.91 \text{ dB}, & H(2500 \text{ Hz}) &= -58.52 \text{ dB}. \end{aligned} \quad (22)$$

The errors $(\Delta f_z/f_z)_2$ and $(\Delta f_p/f_p)_2$ can be further reduced by modifying the zero-forming capacitance CF_1 and the pole-forming capacitance CB_1 . The new capacitance values CF'_1 and CB'_1 are the solutions of the equations

$$S_{CF_1}^{f_z} \left(\frac{CF'_1}{CF_1} - 1 \right) = - \left(\frac{\Delta f_z}{f_z} \right)_2, \quad S_{CB_1}^{f_p} \left(\frac{CB'_1}{CB_1} - 1 \right) = - \left(\frac{\Delta f_p}{f_p} \right)_2. \quad (23)$$

One obtains $CF'_1 = 1.0056506$ and $CB'_1 = 1.0051199$, for which the performance parameters of the biquad are

$$\begin{aligned} (\Delta f_z/f_z)_3 &= 3.961 \cdot 10^{-2}\%, & (\Delta f_p/f_p)_3 &= 1.792 \cdot 10^{-3}\%, \\ (\Delta Q_p/Q_p)_3 &= 0.3300\%, & (\Delta H_p/H_p)_3 &= 0.3972\%, \\ Q_{z3} &= 6629, & H_{z3} &= -90.92 \text{ dB}, & H(2500 \text{ Hz}) &= -88.19 \text{ dB}. \end{aligned} \quad (24)$$

The errors $(\Delta Q_p/Q_p)_3$ and $(\Delta H_p/H_p)_3$ can be reduced by modifying the capacitance CB_2 . The new capacitance value CB'_2 is the average of the capacitance $(CB'_2)_{Q_p}$ and $(CB'_2)_{H_p}$ calculated from the equations

$$S_{CB_2}^{Q_p} \left(\frac{(CB'_2)_{Q_p} - CB_2}{CB_2} \right) = - \left(\frac{\Delta Q_p}{Q_p} \right)_3 \quad (25)$$

and

$$S_{CB_2}^{H_p} \left(\frac{(CB'_2)_{H_p} - CB_2}{CB_2} \right) = - \left(\frac{\Delta H_p}{H_p} \right)_3. \quad (26)$$

This results in $CB'_2 = 1.0036366$.

Table 2 summarizes the performance parameters of the GOC Huang and Sansen's LPN biquad with rounded-off capacitances and gain variation $A_{0_1} = A_{0_2} = 100 \pm 8$.

Table 2. Performance parameters of the GOC Huang and Sansen's LPN biquad with rounded-off capacitances

A	92	100	108
$(\Delta f_z/f_z)$ [%]	-0.0800	0.0682	0.1949
$(\Delta f_p/f_p)$ [%]	-0.2032	$2.133 \cdot 10^{-3}$	0.1777
$(\Delta Q_p/Q_p)$ [%]	-0.22606	-0.062	0.0603
$(\Delta H_p/H_p)$ [%]	-0.0791	0.0622	0.1653
Q_z	5619	6631	7728
$H(f_z)$ [dB]	-89.50	-90.93	-92.25
$H(2.5\text{kHz})$ [dB]	-67.44	-77.25	-64.24

4 Conclusion

A combined approach for reducing the effects of op amps finite gain in switched-capacitor low-pass notch biquads has been presented. The effectiveness of the approach proposed has been demonstrated by two low-pass notch biquad topologies which realize the same transfer function. The filters with modified capacitances have approximately an order smaller of magnitude relative errors

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