FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 19, no. 3, December 2006, 379-384

Low Frequency TM Plane-wave Scattering From a Two-layer Double-strip Grating With Equal Gaps

Dragan Filipović and Tatijana Dlabač

Abstract: This paper treats the low-frequency TM plane-wave scattering from a two-layer double-strip grating with equal gaps. The problem is solved by using a simplified network model which takes into account only the basic ($n = 0$) mode. Numerical results are given for the magnitude of the basic mode reflection coefficient versus the relative grating period.

Keywords: Scattering, grating, network model.

1 Introduction

The problem of plane-wave scattering from different gratings has always attracted attention of scientists and remains interesting in the present time. One of the most powerful methods for handling this problem is the Riemann-Hilbert method [1] which was developed mainly at the University of Harkov, Ukraine. The mathematical basis of this method is the complex function theory.

Guglielmi and Oliner [2] considered the problem of the plane-wave scattering from a single-strip grating (Fig. 1) and derived network models (Fig. 2) for this problem. The network model on Fig. 2(a) is valid for the so-called aperture formulation, when the electric field in the gap is taken as an unknown. In the alternative case, when the current density on the strip is an unknown quantity (the obstacle formulation) then the corresponding network model is the one shown on Fig. 2(b). The most important element of these two models is the coupling matrix which takes into

Manuscript received June 22, 2003. An earlier version of this paper was presented at seventh International Conference on Applied Electromagnetics ΠEC 2005, May 23-25, 2005, Niš, Serbia.

D. Filipović is with Faculty of Electrical Engineering, Cetinjski put bb, 81000 Podgorica, Montenegro (e-mail: draganf@cg.ac.yu). T. Dlabač is with Faculty of Maritime Studies, Dobrota 36, 85330 Kotor, Montenegro (e-mail: tanjav@cg.yu)

account interaction between the basic ($n = 0$) mode and the higher order harmonics ($n = \pm 1,\ \pm 2,\ \pm 3, \ldots$). This coupling matrix is in the form of an impedance or admittance matrix, and the elements of these matrices are derived in [3] in the form of some double integrals which have to be evaluated numerically. Very simple analytical formulas for the coupling matrix elements, related to the problem from Fig. 1, are derived in [4] and [5].

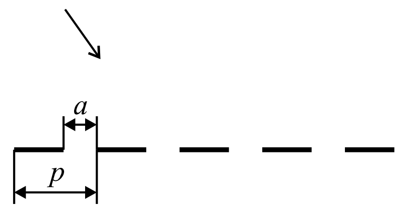

Fig. 1. Single-strip grating and plane wave incident at an arbitrary angle.

More comlex gratings are considered in [6] (double-strip grating with equal strips), in [7] (double-strip grating with equal gaps) and in [8] (asymmetrical double-strip grating). In all these cases, simple formulas for the coupling matrix elements are obtained. These formulas are applied in [9] to the analysis of plane-wave scattering from a two-layer single-strip grating in the low-frequency case (grating period $\ll \lambda$). The network model in this case is extremely simplified since all the higher order harmonisc are neglected and the coupling matrix is reduced to one single element.

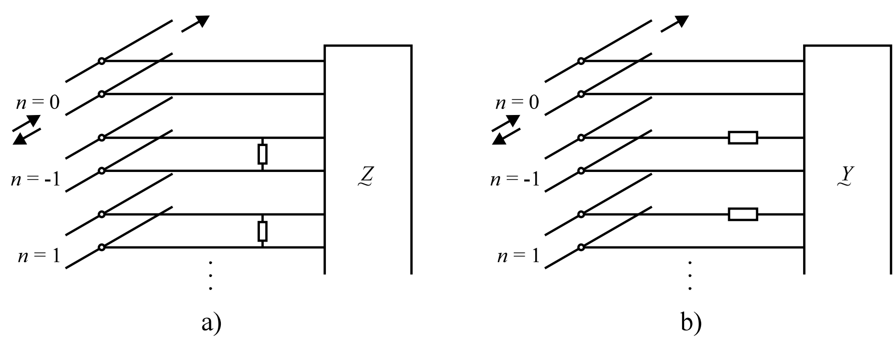

Fig. 2. Network models for the problem shown on Fig. 1.

In [10] the simplified network model is applied to the analysis of the low-frequency TE plane-wave scattering from a two-layer double-strip grating with equal strips. In the present paper we consider the dual problem, i.e. the low-frequency TM plane-wave scattering from a two-layer double-strip grating with equal gaps.

2 Network Model of The Problem

Geometry of a two-layer double-strip grating with equal gaps is shown on Fig. 3.

The gratings are identical and their position is such that they would overlap when translated along the x-axis. The grating period is p, the distance between them – h. The gratings have two strips per period. Their widths are d_1 and $p-d$. All the gaps have the same width – $(d-d_1)/2$. A plane TM-polarized wave is incident perpendicularly. The medium is vacuum. For the low-frequency case the network model from Fig. 2(a) is extremely simplified, since only by the basic ($n=0$) mode is considered (Fig. 4). The coupling matrices reduce to two shunt impedances given by [1].

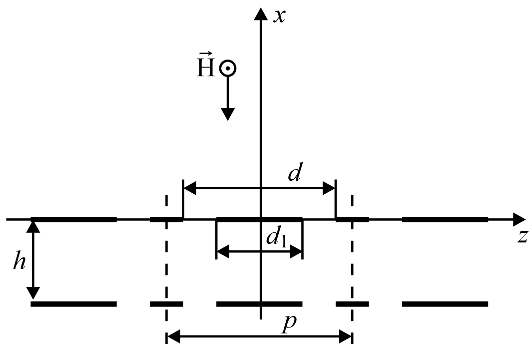

Fig. 3. Geometry of the problem

$$Z_{00} = \frac{jZ_0}{2\frac{p}{\lambda}\ln\frac{v-u}{2}} \tag{1}$$

where

$$u = \cos\frac{\pi d}{p} \tag{2}$$

$$v = \cos\frac{\pi d_1}{p} \tag{3}$$

and $Z_0 = 120\pi$ (Ω) is the free-space wave impedance.

The reflection coefficient (for the $n=0$ mode) can be found from Fig. 4. It is

$$S_{11} = \frac{-\left(2\hat{Z}_{00} + j\mathrm{tg}\beta h\right)}{2\hat{Z}_{00}^2 + 2j\hat{Z}_{00}^2\mathrm{tg}\beta h + 2\hat{Z}_{00} + 2j\hat{Z}_{00}\mathrm{tg}\beta h + j\mathrm{tg}\beta h} \tag{4}$$

where

$$\hat{Z}_{00} = \frac{Z_{00}}{Z_0} = \frac{Z_{00}}{120\pi} \tag{5}$$

and

$$\beta = \frac{2\pi}{\lambda} \tag{6}$$

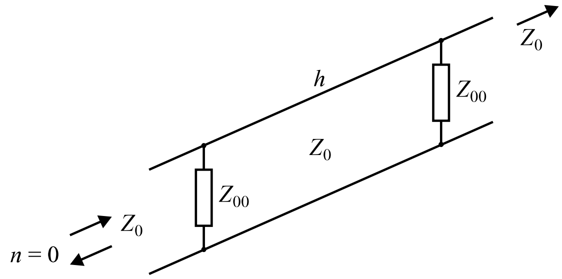

Fig. 4. Simplified network model of the problem

3 Numerical Results

Figs 5-7 show the magnitude of the basic mode reflection coefficient $|S_{11}|$ versus the relative period p/λ, for different values of d_1/p, d/p and h/p.

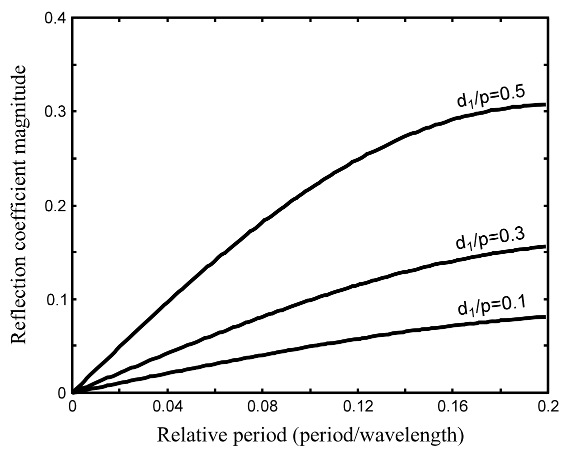

Fig. 5. Reflection coefficient magnitude of the basic ($n = 0$) mode versus relative period for $d/p = 0.7$ and $h/p = 0.5$

We note that if

$$d_1 = p - d \tag{7}$$

the double-strip grating from Fig. 3 becomes a single-strip grating, whose period is one half of the original period. The corresponding results in this case should

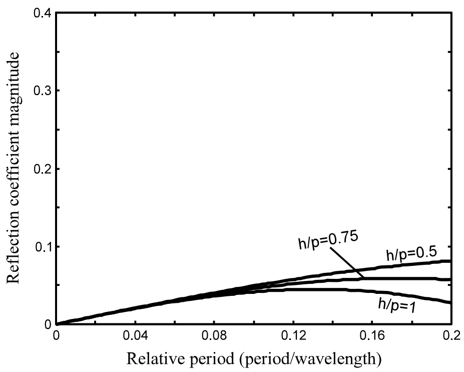

Fig. 6. Reflection coefficient magnitude of the basic ($n = 0$) mode versus relative period for $d_1/p = 0.1$ and $d/p = 0.7$

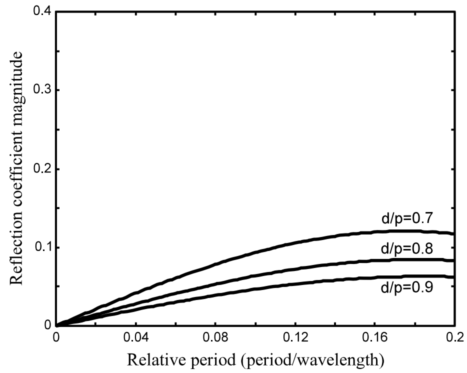

Fig. 7. Reflection coefficient magnitude of the basic ($n = 0$) mode versus relative period for $d_1/p = 0.3$ and $h/p = 0.7$

coincide with those presented in [9]. This can be verified analytically by comparing eqn. (1), where d_1 is given by (7), with eqn. (2) in [9], where p is replaced by $p/2$ and the gap width a is given by

$$a = \frac{d - d_1}{2} = \frac{d - (p - d)}{2} = d - \frac{p}{2} \tag{8}$$

4 Conclusion

This paper treats the low-frequency TM plane wave scattering from a two-layer double-strip grating with equal gaps. The problem is solved by using the simplified network model which takes into account only the basic ($n = 0$) mode. The numerical results include magnitude of the reflection coefficient shown versus the relative period p/λ, for different grating geometries. In the special case, when the double-strip grating becomes a single strip-grating, the obtained results are identical to those obtained earlier [9].

References

[1] В. П. Шестопалов, "Метод задачи Римана-Гилберта в теории дифракции и распространения злектромагнитных волн". Харьков: Харьковский университет, 1971.

[2] M. Guglielmi and A. A. Oliner, "Multimode network description of a planar periodic metal-strip grating at a dielectric interface - Part I," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 534–541, Mar. 1989.

[3] M. Guglielmi and H. Hochstadt, “Multimode network description of a planar periodic metal-strip grating at a dielectric interface - Part III: Rigorous solution,” *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 902–909, May 1989.

[4] D. Filipović, “New explicit expressions for the coupling matrix elements related to scattering from a planar periodic single-strip grating,” *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 1540–1544, 1995.

[5] ——, “New closed-form expression for coupling matrix elements related to scattering from planar periodic single-strip grating,” in *Proc. IEE-Microw. Antennas Propag.*, vol. 142, 1995, pp. 492–494.

[6] ——, “A new solution for te plane-wave scattering from a symmetric double-strip grating composed of equal strips,” *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 1990–1996, 1996.

[7] D. Filipović and A. Vujačić, “Closed-form expressions for the compling matrix elements related to tm-scattering from a symmetric double-strip grating,” in *Proc. of the 26th European Microwave Conference*, 1996, pp. 893–894.

[8] D. Filipović, “Prilog mrežnom modelu difrakcije ravanskog talasa na planarnoj periodičnoj metalnoj rešetki,” Ph.D. dissertation, Elektrotehnički fakultet, Podgorica, Montenegro, 1996.

[9] T. Dlabač and D. Filipović, “Low-frequency plane wave diffraction on a two-layer grating,” *Serbian Journal of Electrical Engineering*, vol. 1, no. 3, pp. 1–6, Nov. 2004.

[10] D. Filipović and T. Dlabač, “Niskofrekventna difrakcija TE polarizovanog ravanskog talasa na dvoslojnoj rešetki sa dvije jednake trake po periodi,” in *Zbornik radova XLIX Konferencije ETRAN*, vol. II, Budva, Srbija i Crna Gora, June 5–10, 2005, pp. 287–288.