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A Simple 2D Digital Calibration Routine for Transducers

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Abstract: A new practical algorithm for linearization and compensation of a two dimensional transducers transfer characteristic is described in this paper. The algorithm is suitable for smart sensor applications which incorporate a microprocessor. The calculating routine is simple and the algorithm requires initial calibration at nine points only.

Theoretical and practical results are given for the assumed and measured piezoresistive pressure sensor characteristics, respectively. After correction based on 3×3 calibration, the obtained error was less than one percent of the full scale output in both cases.

Keywords: Transducers, calibration routine, piezoresistive pressure sensor, microcontroller, three points method.

1 Introduction

Recent digital methods of transducer transfer characteristics linearization and correction are designed to be implemented by software in the smart sensor interfaces [1]. There are numerous digital methods and algorithms as linearization based on a look-up table [2], linearization based on piecewise-linear interpolation, linearization based on piecewise-polynomial or spline interpolation [3], [4], curve fitting linearization [5], linearization based on the error minimization [6], three-point calibration method [7] and so on. Transducer calibration techniques found in literature are practically all based on one of these techniques or on a combination of them. With respect to the number of calibration measurements the best linearization can be achieved with the spline interpolation and the curve-fitting techniques.

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The mentioned linearization techniques are mainly one-dimensional functions. Some of them as look-up table method can easily be expanded to achieve calibration of a two-dimensional transducer transfer curve. As the response of some semiconductor sensors and transducers, as piezorezistive pressure sensors, ceramic humidity sensors, silicon photo and magneto-sensors etc., depends on the influence of two or more nonelectrical quantities simultaneously, several two-dimensional or multi-dimensional complex calibration methods are developed [6]. Naturally, memory requirements and the number of calibration measurements will increase rapidly. This will determine the response time and the cost of calibration procedure, respectively.

A new simple two-dimensional algorithm, based on the one-dimensional general digital linearizing method for transducers [7], which is proposed in the presented paper, is a good compromise between the needed number of calibration measurements and the transducer response accuracy. 3×3 calibration values of the transducer output are transformed into a set of three cross-sensitivity dependent calibration coefficients. One of the three coefficients controls the zero offset, the second determines the scale factor and the third one serves for the transducer response linearization referred to the one input variable for the desired sensitivity. Simultaneously, all of three coefficients compensate the influence of the second cross-sensitivity variable. The proposed method is especially useful in data-acquisition systems based on the smart sensor interfaces.

2 Three Point Calibration Method

One practical three-point digital linearizing method for the general use has been described in the paper [7]. As shown, the method based on the second order approximation of a continued fraction given by

$$V(y) = C_0 + \frac{C_1 y}{1 + C_2 y} \tag{1}$$

where $y = f(x)$ presents the transducer transfer characteristic and $V(y)$ is the linearization function for the three-point calibration method. The calibration coefficients C_0, C_1 and C_2 should be calculated from three calibration measurements of the transducer response $y = f(x)$, namely $L = f(l)$, $M = f(m)$ and $H = f(h)$ with m in the middle of the interval $[l \ldots h]$ such that $h - m = m - l$. The coefficient C_0 serves to correct the offset error and can be determined from the condition $V(L) = n$, where n is the zero offset of the response V. The coefficient C_1 serves to obtain the correct scaling and can be determined from the equation $V(H) - V(L) = d$, where d is the scale factor of the response. $V(y)$ will be an approximately linear function of x if

$$\frac{V(H)-V(M)}{V(M)-V(L)} = \frac{h-m}{m-l} = 1 \tag{2}$$

The coefficient C_2 can be found directly from the last equation and it compensates nonlinear behavior of a transducer. The complete procedure of the calibration coefficients calculation has been shown in the paper [7]. Also, when in practice it will be impossible to realize the mid-range calibration exactly, the simple modified routine is shown in that paper. The results given for eight commonly used temperature transducers show that the method is superior to the second-order Lagrange interpolation.

3 A New 2D Algorithm

Taking into account the needed number of calibration measurements the method [7], shortly explained in the previous Section, can be extended to handle two-dimensional calibration. The basic principle is to select three lines in one dimension by fixing the values for one input variable and then along each selected line 1D calibration method [7] should be applied with three different values for the second variable. The values for the first variable must be selected according to the same procedure as for the one-dimensional method. By interpolating three by three obtained coefficients, on the same way as in the one-dimensional calibration method, we will get compensated and linearized response. To explain this principle mathematically we will use the uncalibrated sensor transfer function given by $y = f(x,z)$. To calibrate the cross-sensitivity for the signal z we assume that there is an additional sensor which senses z independent of other variables $z' = f(z)$. The uncalibrated sensor transfer function will be calibrated by the use of 3×3 calibration measurements of the sensor output at three fixed output values of an additional z sensor, namely for $z = z_l$

$$V_L(y) = C_0^{(L)} + C_1^{(L)} \frac{y}{1+C_2^{(L)}y} \tag{3}$$

for $z = z_m$

$$V_M(y) = C_0^{(M)} + C_1^{(M)} \frac{y}{1+C_2^{(M)}y} \tag{4}$$

for $z = z_h$

$$V_H(y) = C_0^{(H)} + C_1^{(H)} \frac{y}{1+C_2^{(H)}y} \tag{5}$$

The calibration coefficients $C_i^{(k)}$, $i=0,1,2,k=L,M,H$ should be calculated according to the mentioned 1D algorithm [7] by setting the equal offset n and the scale factor d for all of three cases. After this procedure the obtained coefficients $C_i^{(k)}$, $i=0,1,2,k=L,M,H$, $i=0,1,2,k=L,M,H$ are depended on the variable z. To calibrate the cross-sensitivity for the z signal each set of the three calculated coefficients $(C_0^{(L)}, C_0^{(M)}, C_0^{(H)})$, $(C_1^{(L)}, C_1^{(M)}, C_1^{(H)})$ and $(C_2^{(L)}, C_2^{(M)}, C_2^{(H)})$ should be interpolated across three nodes using expression (1)

$$C_0(z) = I_0 + J_0 \frac{z}{1+K_0 z} \tag{6}$$

$$C_1(z) = I_1 + J_1 \frac{z}{1+K_1 z} \tag{7}$$

$$C_2(z) = I_2 + J_2 \frac{z}{1+K_2 z} \tag{8}$$

The parameters I_s, J_s and K_s, $s=0,1,2$, can be determined from three linear systems of equations

$$C_0(z_l) = C_0^{(L)}; \qquad C_0(z_m) = C_0^{(M)}; \qquad C_0(z_h) = C_0^{(H)}; \tag{9a}$$

$$C_1(z_l) = C_1^{(L)}; \qquad C_1(z_m) = C_1^{(M)}; \qquad C_1(z_h) = C_1^{(H)}; \tag{9b}$$

$$C_2(z_l) = C_2^{(L)}; \qquad C_2(z_m) = C_2^{(M)}; \qquad C_2(z_h) = C_2^{(H)}. \tag{9c}$$

The interpolation nodes z_l, z_m and z_h should be chosen exactly or approximately equidistantly. Such access represents a guaranted way of the interpolation error minimisation in the whole range from z_l to z_h. The final calibration function is given by

$$V(y) = C_0(z) + C_1(z) \frac{y}{1+C_2(z)y} \tag{10}$$

4 Results of Simulation

The output of a pressure sensor, for example, is determined by the applied pressure and the operating temperature of the sensor simultaneously in such a way that the errors as offset, gain and non-linearity are dependent on the temperature [4], [6]. Such a sensor has to be calibrated for both pressure and temperature, hence the term two-dimensional calibration. The pressure sensor output signal is represented by $U = U(P,T)$ where U is the output voltage of the sensor, P is the applied pressure and T is the operating temperature. In order to estimate the measure of the error which proposed 2D algorithm produces, we will assume the arithmetic relation between the output voltage of the sensor and the pressure and the temperature in the following form

$$U(P,T) = -1.2\ln(1.7(1+0.01T)-P) \tag{11}$$

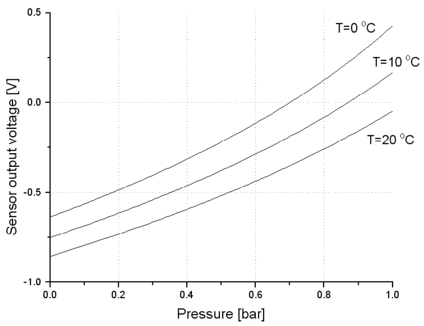

Fig. 1. Uncalibrated sensor transfer characteristics at the three temperatures.

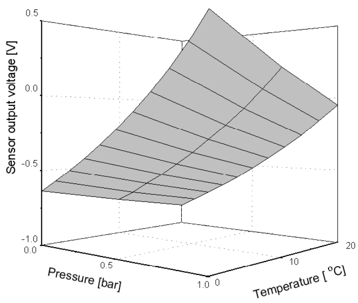

Fig. 2. Surface plot of the sensor response in the temperature range from $0°C$ to $20°C$.

Several fictive sensor responses were generated by using the mathematical description of the two-dimensional transfer function (11) with different kinds of nonlinearity and cross-sensitivity. The uncalibrated sensor responses used for testing and the appropriate 3D surface plot are shown in Fig. 1 and Fig. 2 respectively. Three equidistant pressure points $P_l = 0$bar, $P_m = 0.5$bar, $P_h = 1$bar and temperature points $T_l = 0°$C, $T_m = 10°$C, $T_h = 20°$C present the selected calibration nodes.

Table 1. The output voltage in [V] of the pressure sensor.

P(bar) / $T(°C)$	0	0.5	1.0
0	-0.63675	-0.21879	0.42801
10	-0.75113	-0.37777	0.16711
20	-0.85554	-0.51814	-0.04706

Table 1. shows the discrete pressure sensor responses at the different pressure and temperature values calculated by Eq. (11). In order to calculate calibration coefficients, according to Eq. (3), (4) and (5), we apply the one dimensional algorithm [7] by fixing three temperature values successive. Additionally, we will adopt the equal offset $n = 0$ and the scale factor $d = 1000$ for the Eq. (3), (4) and (5). The selected offset and scale factor represent the calibrated sensor response-digital word. In Table 2. are given the calculated values of the calibration coefficients at three different temperatures for the selected offset and scale factor.

Table 2. Coefficients calculated by the algorithm [7] at three temperatures with the adopted offset $n = 0$ and the scale factor $d = 1000$.

	$T_l = 0°C$	$T_m = 10°C$	$T_h = 20°C$
C_0	$C_0^{(L)} = 697.1711$	$C_0^{(M)} = 867.7201$	$C_0^{(H)} = 1041.018$
C_1	$C_1^{(L)} = 824.8303$	$C_1^{(M)} = 839.6594$	$C_1^{(H)} = 857.3526$
C_2	$C_2^{(L)} = 0.387355$	$C_2^{(M)} = 0.3636727$	$C_2^{(H)} = 0.3452811$

Now, across the three temperature nodes by using the pairs of the equations (6) and (9a), (7) and (9b), (8) and (9c), the nonlinear interpolations of the coefficients C_0, C_1 and C_2 are performed, respectively. In that way the temperature dependent calibration coefficients are obtained

$$\begin{aligned} C_0(T) &= 697.1711 + 16.9196\frac{T}{1-7.931\times10^{-4}T} \\ C_1(T) &= 824.8303 + 1.362887\frac{T}{1-8.094\times10^{-3}T} \\ C_2(T) &= 0.3873 - 2.709\times10^{-3}\frac{T}{1+1.438\times10^{-2}T} \end{aligned} \tag{12}$$

Ultimately, the linearised and compensated sensor response is given by

$$V(U) = C_0(T) + C_1(T)\frac{U}{1+C_2(T)U} \tag{13}$$

The calibrated transfer characteristics of the pressure sensor at the different temperatures 5, 7, 15 and 18°C and the appropriate 3D surfaces are shown in Figure 3 and Figure 4, respectively.

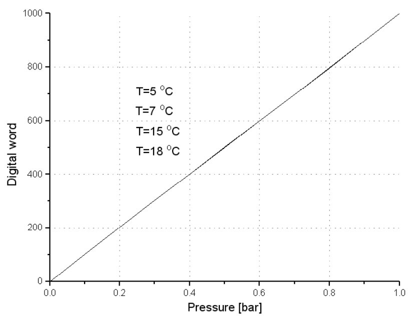

Fig. 3. Linearised and temperature compensated characteristics.

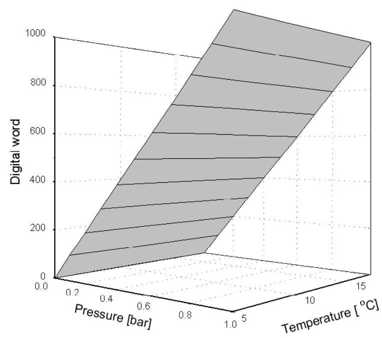

Fig. 4. Surface plot of the sensor response after the calibration.

Taking into account the one-dimensional algorithm [7] the ideal linear sensor response can be shown in the following form

$$V(U(P,T))\Big|_{T\varepsilon(T_l-T_h)} = \frac{d}{P_h - P_l}(P - P_l) + n = 1000P \tag{14}$$

Maximum deviations regard to the ideal sensor response should be expected in the middle of the two calibration nodes $(P_m + P_l)/2$ and $(P_h + P_m)/2$. The error curves, calculated as the difference between the Eq. (14) and (13) at the different operating temperatures, are shown in Figure 5.

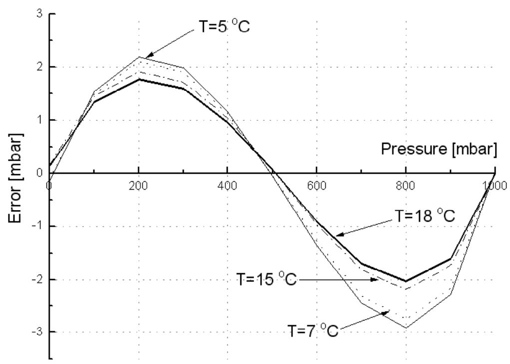

Fig. 5. Error curves at the different operating temperatures.

As can be seen, the maximum absolute deviations are less than 0.3 percent of the full scale output in the adopted temperature range. Furthermore, the reduction of the error can be expected by the temperature or the pressure or the temperature and pressure range segments.

5 Experimental Results

The very good results obtained by the algorithm simulation are verified across the experimental procedure for the piezoresistive pressure sensor NPT-0.3, made in the Institute of Chemistry, Technology and Metallurgy-Centre for Microelectronic's Technologies and Mono-crystals (IHTM-CMTM) in Belgrade. The sensor is realized for pressures up to 0.3 bar. In this range the sensor response is approximately linear at the constant temperature of the sensor. However, the sensor is sized for a greater pressure than specified but its transfer characteristics are nonlinear outside the specified range. Our measurements are performed in the extended pressures and temperatures range for the given sensor, from 0*bar* to 1 bar and from $-19.5°$C to $59.9°$C, respectively. The measurements are performed under the laboratory conditions (IHTM-CMTM) by using the digital pressure controller MENSOR-USA which resolves pressure to 0.1 mbar and the insulated temperature chamber with a precise digital thermometer. The sensor response is monitored by a digital voltmeter. In Table 3 are shown the measurement responses of the sensor in [mV] in the extended temperatures and pressures range. Figure 6 and Figure 7 show 2D and 3D plot of the uncalibrated sensor response at the different temperatures, respectively.

Table 3. Measured data in [mV] of the pressure sensor NPT-0.3.

$T\,(^\circ C)$ / P [bar]	-19.5	-10	2.1	11.5	20.4	29.5	40.5	49.6	59.9
0	-3.4	0.95	6.1	10.4	15.85	20	25.2	29.3	34.1
0.125	46.7	50.2	54.6	58.25	63	66.8	71.2	74.7	78.85
0.25	96.05	98.8	102.5	105.45	109.4	112.7	116.4	119.2	122.7
0.375	144.15	146.25	149.1	151.4	154.65	157.3	160.2	162.4	165.15
0.5	190.75	192.05	194.5	195.7	198.15	200.2	202.3	203.85	205.9
0.625	235	235.75	236.95	237.95	239.7	241.2	242.5	243.4	244.75
0.75	277.35	277.3	277.8	278.1	279.2	280.05	280.6	280.9	281.65
0.875	317.25	316.6	316.4	316	316.5	316.8	316.6	316.4	316.5
1.0	355	353.7	352.8	351.9	351.7	351.5	350.7	349.9	349.4

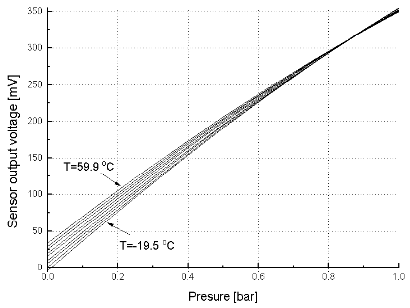

Fig. 6. Uncalibrated transfer characteristics of the pressure sensor NPT-0.3 at nine temperatures.

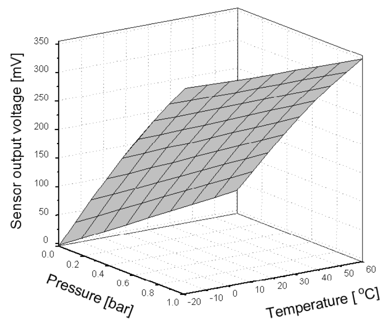

Fig. 7. NPT-0.3 sensor surface plot in the temperature range from $-19.5^\circ C$ to $59.9^\circ C$.

Three equidistant pressure points $P_l = 0$bar, $P_m = 0.5$bar, $P_h = 1$bar and three approximately equidistant temperature points $T_l = -19.5^\circ$C, $T_m = 20.4^\circ$C, $T_h = 59.9^\circ$C are selected as the calibration nodes. By fixing the selected three temperature values successive, the new calibration coefficients are calculated by the Eq. (3), (4) and (5) with the adopted offset $n = 0$ and scale factor $d = 10000$. The calculated values of the new calibration coefficients at the three different temperatures for the selected offset and scale factor are given in Table 4.

To calibrate the cross-sensitivity for a temperature variable, each set of the three calculated coefficients $(C_0^{(L)}, C_0^{(M)}, C_0^{(H)})$, $(C_1^{(L)}, C_1^{(M)}, C_1^{(H)})$ and $(C_2^{(L)}, C_2^{(M)}, C_2^{(H)})$

Table 4. New coefficients calculated by the algorithm [7] at the three temperatures with the adopted offset $n = 0$ and scale factor $d = 10000$.

	$T_l = -19.5°C$	$T_m = 20.4°C$	$T_h = 59.9°C$
C_0	$C_0^{(L)} = 80.3737$	$C_0^{(M)} = -394.5724$	$C_0^{(H)} = -887.5445$
C_1	$C_1^{(L)} = 23.6739$	$C_1^{(M)} = 24.7102$	$C_1^{(H)} = 25.5721$
C_2	$C_2^{(L)} = -4.3033 \times 10^{-4}$	$C_2^{(M)} = -4.6611 \times 10^{-4}$	$C_2^{(H)} = -5.1323 \times 10^{-4}$

is interpolated across the three temperature nodes by using the pairs of the Eq. (6) and (9a), (7) and (9b), (8) and (9c). So, the new temperature dependent calibration coefficients are obtained

$$
\begin{aligned}
C_0(T) &= -148.9545 - 11.8955\frac{T}{1 - 5.8886 \times 10^{-4}T} \\
C_1(T) &= 24.204 + 2.5973 \times 10^{-2}\frac{T}{1 + 2.2905 \times 10^{-3}T} \\
C_2(T) &= -4.4662 \times 10^{-4} - 8.8999 \times 10^{-7}\frac{T}{1 - 3.3467 \times 10^{-3}T}
\end{aligned} \tag{15}
$$

Finally, the linearized and temperature compensated sensor response is given by the equation

$$
V(U) = C_0(T) + C_1(T)\frac{U}{1 + C_2(T)U} \tag{16}
$$

where U is the pressure sensor output voltage.

The calibrated 2D transfer characteristics and the appropriate 3D surface plot of the pressure sensor NPT-0.3 response at the nine temperatures -19.5, -10, 2.1, 11.5, 20.4, 29.5, 40.5, 49.6 and 59.9°C are shown in Figure 8 and Figure 9 respectively.

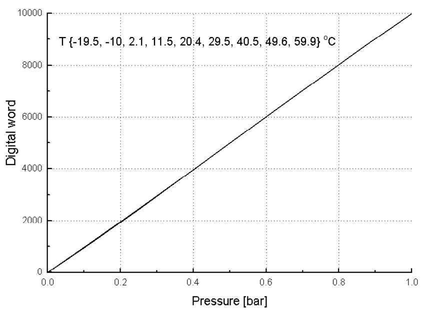

Fig. 8. Linearized and temperature compensated transfer characteristics of the NPT-0.3 pressure sensor.

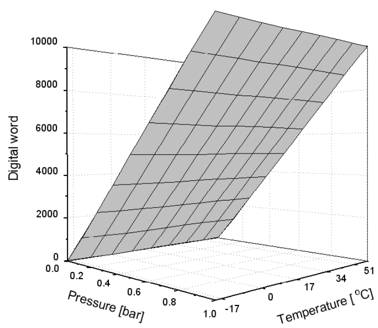

Fig. 9. Surface plot of the sensor NPT-0.3 response after the calibration.

The ideal linear and temperature compensated sensor response can be shown in the following form

$$V(U(P,T))\Big|_{T\varepsilon(-19.5-59.9)^{\circ}C} = \frac{d}{P_h - P_l}(P - P_l) + n = 10000P \qquad (17)$$

so the error curves, calculated as the difference between the Eq. (17) and (16) at the different operating temperatures, are as in Figure 10.

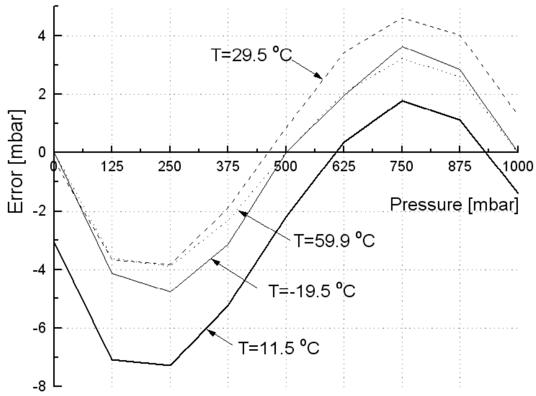

Fig. 10. Error curves at the nine different operating temperatures of the NPT-0.3 pressure sensor.

As can be seen, in the extended pressures and temperatures range of the sensor NPT-0.3 the maximum absolute deviations are not greater than 0.73 percent. In that way the practical validity of the proposed routine is confirmed.

6 Conclusion

The presented two-dimensional algorithm provides the practical method for the non-linearity correction and cross-sensitivity compensation of a transducer output. The calculating routine is simple, limited by a set of only nine calibration data. It is not necessary to know the transfer curve of a transducer explicitly. The mentioned limited set of calibration points will be sufficient. The algorithm is suitable for smart sensor applications which incorporate a microprocessor. The algorithm has been tested by the mathematical computer program MicroCal Origin. The analytical description of a piezoresistive pressure sensor response is assumed. This gives the obvious advantage when we study the behavior of the calibrated sensor response. Moreover the experimental laboratory measurements are performed by the piezoresistive pressure sensor NPT-0.3 made in IHTM-CMTM. By applying the proposed algorithm the satisfactory linear and temperature compensated response is obtained in the wider temperature and pressure ranges than specified for the sensor NPT-0.3. The achieved results have shown the method conveniences as accuracy, small number of measurements and short response time.

Acknowledgments

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